using the matrix inversion lemma [14], (20) can be rewritten as

\[ S_{k+1} = [\alpha^2 - (\alpha \beta)^2]A^T \sum_{j=1}^{q} S_j p_j A_{j} + Q_{k} + \]

\[ + ([\sum_{j=1}^{q} S_j p_j]^{-1} + \beta B_{i} R^{-1} B_{i}^{T})^{-1} A_{j} + Q_{k}. \]  

(21)

Compare this to the following equation:

\[ S_{k+1} = [\alpha^2 - (\alpha \beta)^2]A^T \sum_{j=1}^{q} S_j p_j A_{j} + Q_{k}. \]  

(22)

It can be proved by induction that \( S_{k} \geq S_{k} \). Since (22) can be written as

\[ i_{k+1} = [\alpha^2 - (\alpha \beta)^2]i_{k} + q \]

(23)

where \( i_{k} = [s_{k}^{T}(S_{k})], s_{k}^{T}(S_{k}), \ldots, s_{k}^{T}(S_{k})]^{T} \), and \( q = [s_{k}^{T}(Q_{k}), s_{k}^{T}(Q_{k}), \ldots, s_{k}^{T}(Q_{k})]^{T} \), by using stacking operator and Kronecker product properties as before, if \( [\alpha^2 - (\alpha \beta)^2]i_{k} \geq 1 \), then \( S_{k} \) will diverge and so will \( S_{k} \). The above result states that

\[ [\alpha^2 - (\alpha \beta)^2]i_{k} \geq 1 \]

(24)

is necessary for the convergence of (20). It is shown below that (24), which will be called the "generalized uncertainty threshold principle," is also sufficient for the existence of the steady-state solution for some systems and choice of weighting matrices in (2).

**Theorem 3:** Let \( B_{i} \) be square and full rank (as assumed in [9]); then the condition (24) is both necessary and sufficient for the convergence of (20) to \( S^{\infty} > 0 \) which satisfies

\[ S^{\infty} = \Phi(S^{\infty}, j = 1, \ldots, q). \]

(25)

**Proof:** The necessity part simply follows from the previous theorem. Sufficiency is shown as follows. Due to the hypothesis on \( B_{i} \), one can find a real number \( b_{i} > 0 \) such that \( \beta B_{i} R^{-1} B_{i}^{T} \geq b_{i} I_{i} \). It follows from (20) by induction that \( S_{k} \leq S_{k} \), where

\[ S_{k+1} = [\alpha^2 - (\alpha \beta)^2]A^T \sum_{j=1}^{q} S_j p_j A_{j} + \beta (\alpha \beta)^2 A^T A_{j} + Q_{k}, \]

(26)

Operating by the stacking operator on both sides and using properties of the Kronecker product as before will reveal that (24) is sufficient for the boundedness of \( S^{\infty} \) and, therefore, of \( S_{k} \).

From (21), we have \( S_{k} \leq \Phi(S^{\infty}, j = 1, \ldots, q) \). Assuming \( S_{k-1} \leq S_{k} \), it can easily be shown that \( S_{k} \leq S_{k+1} \). So by induction, we can arrive at the monotonic increasing property \( S_{k} \). Combining this with the boundedness of this positive-definite matrix sequence will imply the existence of the positive-definite limit which satisfies the algebraic version of (20) given by (25).

Theorems 2 and 3 state that the uncertainty threshold which must not be exceeded for the existence of the steady-state solutions to the Riccati-like matrix equations (20) or equivalently (9) in the quadratic optimal control of systems with both jump-Markov and white noise parameters. When this threshold is exceeded, then it only makes sense to make short-term decisions [9] where \( N \) is finite and small. If the positive-definite steady-state solutions of (9) exist, then it is possible to show that system (1) can be MS stabilized by these solutions used in feedback gains (8). This way, condition (24) can also be interpreted as an MS stabilizability condition.

One can see that the generalized uncertainty threshold condition replaces the system matrix \( A \) for a single-mode system in (12) by the matrix \( \psi \) which contains all the possible system operation modes and transition probabilities. The general uncertainty threshold condition, therefore, displays the effects of both the stochastic stability of plant modes reflected by \( A \) (system matrix corresponding to the \( n \) operation mode) and \( \rho \) (likelihood of stay in the mode or transition to another) and the noise powers of multiplicative noises \( \alpha^2 \), \( \beta \), and their correlation \( \alpha \beta \).

**References**


**Upper and Lower Covariance Bounds for Perturbed Linear Systems**

J.-H. XU, R. E. SKELTON, AND G. ZHU

Abstract—Both upper and lower bounds are established for state covariance matrices under parameter perturbations of the plant. The motivation for this study lies in the fact that many robustness properties of linear systems are given explicitly in terms of the state covariance matrix. Moreover, there exists a theory for control by covariance assignment. Hence, the results herein provide robustness properties of these covariance controllers.

I. INTRODUCTION

The main idea of the covariance control theory developed in [1]-[4] is to choose a state covariance \( X_{k} \) according to the different requirements on the system performance, and then to design a controller so that the specified state covariance is assigned to the closed-loop system. For the continuous-time case,

\[ x(t) = A_{k} x(t) + D_{k} w(t) \]

(1.1)

where \( A_{k} \) is stable, \( w(t) \) is a white noise signal of unit intensity, and the

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The state covariance $X_0$ is defined as follows:

$$X_0 = \lim_{t \to \infty} E[x(t)x^T(t)].$$  \hfill (1.2)

The state covariance $X_0$ is $\geq 0$ defined in (1.2) is the unique solution of the Lyapunov equation

$$A_0X_0 + X_0A_0^T + D_0D_0^T = 0.$$  \hfill (1.3)

Perturbations in the system matrix $A_0$ are inevitable in practice. As a result, the real system matrix is actually $A_0 + \Delta A$ instead of $A_0$ where the perturbation $\Delta A$ describes the uncertainty of the system and it belongs to a specified set $\Omega$. In this case, the state covariance of the perturbed system $X = X^T \geq 0$ is the unique solution of the following equation:

$$(A_0 + \Delta A)X + X(A_0 + \Delta A)^T + D_0D_0^T = 0$$  \hfill (1.4)

as long as $A_0 + \Delta A$ remains stable.

For the system designed with the covariance control theory, of course, it is desired to know the perturbed state covariance. The main purpose of this paper is to find the upper and lower bounds of the perturbed state covariance $X$. Hence, we should seek the solution which corresponds to the smallest perturbed system covariance. For the continuous-time system, the upper bound problem is not discussed the multiple solutions of the corresponding equation, and as long as $A_0X_0$ is asymptotically stable

$$X = X^T \geq 0$$  \hfill (1.5)

which satisfies (2.3) and $A_0 = \Delta A$ is asymptotically stable $\forall \Delta A \in \Omega$ and $X \leq \bar{X}$, $\forall \Delta A \in \Omega$ where $X$ satisfies (1.4).

Consider that the symmetric and nonnegative-definite solution of (2.1) is not unique, which we shall show later. Hence, we should seek the smallest one which corresponds to the best upper bound. This is our main purpose in the rest of this section.

**II. Upper Bound of Perturbed State Covariance**

**A. Formulation of the Upper Bound Problem**

The problem of finding the upper bound can be converted to that of solving the following algebraic Riccati equation:

$$A_0X + XA_0^T + \frac{X}{\beta} - \beta A + W_0 = 0$$  \hfill (2.1)

where $W_0 = D_0D_0^T$ and $A$ characterizes the system uncertainty set $\Omega$ in the following way:

$$\Omega \triangleq \{ \Delta A : \Delta A\Delta A^T \leq A \}$$  \hfill (2.2)

and $\beta$ is a positive real number. From [5], [6], we obtain the following result.

**Theorem 2.1:** Suppose $A_0$ is stable and $(A_0 + \Delta A, W_0)$ is stabilizable for $\Delta A \in \Omega$, and there exists a real $\beta$ and $X \geq 0$ satisfying (2.1); then $A_0 + \Delta A$ is asymptotically stable $\forall \Delta A \in \Omega$ and $X \leq \bar{X}$, $\forall \Delta A \in \Omega$ where $X$ satisfies (1.4).

Consider that the symmetric and nonnegative-definite solution of (2.1) is not unique, which we shall show later. Hence, we should seek the smallest one which corresponds to the best upper bound. This is our main purpose in the rest of this section.

**B. Preliminaries for Solving the Upper Bound Problem**

Consider the algebraic Riccati equation

$$XA + A^TX + XBB^TX + C^TC = 0.$$  \hfill (2.5)

The following theorem shows that the symmetric and nonnegative-definite solution of (2.3) is not unique.

**Theorem 2.2:** Suppose $A$ is stable; then the following statements are equivalent:

i) There exists a unique $X_t = X_t^T \geq 0$ which satisfies (2.3) and $(A + BB^TX_t)$ is stable.

ii) There exists a unique $X_t = X_t^T \geq 0$ which satisfies (2.3) and $(A + BB^TX_t)$ is completely unstable.

To prove this theorem, we need the following lemmas.

**Lemma 2.1** [7], [8]: Let $G(s) = C(sI - A)^{-1}B$ be a stable transfer matrix, and define the related Hamiltonian matrix

$$H \triangleq \begin{bmatrix} A & BB^T \\ -C^TC & -A^T \end{bmatrix}.$$  \hfill (2.4)

Then $\|G\|_\infty < 1$ iff $H$ has no eigenvalues on the $j\omega$ axis.

**Lemma 2.2a** [9], [10]: Let $A$, $Q$, and $R$ be real $n \times n$ matrices with symmetric $Q$ and $R$, and define the $2n \times 2n$ Hamiltonian matrix

$$H \triangleq \begin{bmatrix} A & R \\ Q & -A^T \end{bmatrix}.$$  \hfill (2.5)

Suppose either $R \geq 0$ or $R \leq 0$, $(A, R)$ stabilizable, and $H$ has no eigenvalues on the $j\omega$ axis. Then, there exists a unique $X$ which satisfies

i) $X$ is symmetric

ii) $X$ is a solution of the algebraic Riccati equation

$$A^TX + AX + XBB^TX - Q = 0.$$  \hfill (2.6)

iii) $A + RX$ is stable.

Further, there exists a nonsingular matrix $T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$ with which (2.5) can be transformed to an upper block triangular matrix via a similarity transformation:

$$T^{-1} A + RX = X A + T^{-1} R X.$$  \hfill (2.5)

with $H_{11}$ being stable. Then the $X$ satisfying the above three conditions can be obtained as

$$X = T_{21}T^{-1}_{22},$$

In a manner similar to the proof of Lemma 2.2a, we obtain the following.

**Lemma 2.2b:** Under the same assumption on $A$, $Q$, $R$, and $H$ as in Lemma 2.2a, there exists a unique $X$ which satisfies

i) $X$ is symmetric

ii) $X$ is a solution of (2.6)

iii) $A + RX$ is completely unstable.

It is straightforward to show the following.

**Lemma 2.3:** The following statements are equivalent:

i) $(A, B)$ is controllable (stabilizable)

ii) $(A, BB^T)$ is controllable (stabilizable)

iii) $(A, \sqrt{BB^T})$ is controllable (stabilizable).

Now, we can prove Theorem 2.2 as follows.

**Proof (Theorem 2.2):** The equivalency of i) and ii) is evident because of Lemma 2.1.

Now we prove that statement ii) implies statement iii). Because of i) and the stabilizability of $(A, BB^T)$ (see Lemma 2.3), there exists a unique $X_t = X_t^T$ which solves (2.3), and $(A + BB^TX_t)$ is stable according to Lemma 2.2a. Further, $X_t \geq 0$ because $A$ is stable, and in (2.3), $(XBB^TX_t + C^TC) \geq 0$.

Now we show that iii) implies i). Define

$$T \triangleq \begin{bmatrix} I & 0 \\ -X_t & I \end{bmatrix},$$

hence,

$$T^{-1} = \begin{bmatrix} I & 0 \\ X_t & I \end{bmatrix},$$

and

$$T^{-1} A + RX = X A + T^{-1} R X.$$
follows statements.

Again, similar to Corollary 2.1, (2.1) can be obtained from ii) in Corollary 2.1 as follows:

\[
\| (SI - A_0)^{-1} \sqrt{\bar{X}} \|_\infty < 1.
\]

Remark 2.1: Since \( \beta \) in Corollary 2.1 can be chosen arbitrarily, a necessary and sufficient condition for the existence of a stabilizing solution \( X = X^T \geq 0 \) satisfying (2.1) can be obtained from ii) in Corollary 2.1 as follows:

\[
\| (SI - A_0)^{-1} \sqrt{\bar{X}} \|_\infty < 1.
\]
where $\gamma$ is any scalar such that

$$W_0 - \frac{XX}{\gamma} - \gamma \bar{A} \geq 0. \tag{3.2}$$

Proof (Theorem 3.1): By subtracting (3.1) from (1.4), we obtain

$$A_0(X - \bar{x}) + (X - \bar{x})A_0^T + \left( DA X + XDA^T + \frac{XX}{\gamma} + \gamma \bar{A} \right) = 0. \tag{3.3}$$

Using the inequality

$$\left( \frac{X}{\sqrt{\gamma}} + \sqrt{\gamma} \Delta A \right) \left( \frac{X}{\sqrt{\gamma}} + \sqrt{\gamma} \Delta A \right)^T \geq 0 \tag{3.4}$$

we can prove

$$\Delta A X + XDA^T + \frac{XX}{\gamma} + \gamma \bar{A} \geq 0. \tag{3.5}$$

Because $A_0$ is stable, and hence from (3.3), we have

$$X - \bar{x} \geq 0, \tag{3.6}$$

which completes our proof.

Q.E.D.

Remark 3.1: Note that $\gamma$ in (3.1), (3.2) is relatively easy to choose. For example, if we choose $\gamma$ so that

$$\bar{X} \leq \frac{X}{\sqrt{\gamma}} \tag{3.7}$$

then (3.2) is satisfied. (This is a sufficient condition only, but it involves information which is known a priori.)

Example 1: The nominal closed-loop system is

$$A_0 = \begin{bmatrix} -6.5000 & 1.0000 & 0 \\ 0 & -4.8333 & 0.6667 \\ 0 & 0.3333 & -5.1667 \end{bmatrix}$$

$$D_0 = \begin{bmatrix} 0.0148 & 0.2892 & 0.7524 \\ 0 & -0.2132 & 0.2294 \\ 1.4303 & -0.1419 & 0.0264 \end{bmatrix}$$

Suppose

$$\bar{A} = (0.1 \, A_0)(0.1 \, A_0)^T$$

then the uncertainty set of the system matrix is

$$\Omega = \{ \Delta A : \Delta A \Delta A^T \leq \bar{A} \}. \tag{4.1}$$

Using the algorithm presented in Section II-C, by choosing $\beta = 0.26$, we obtain $\bar{X}$; and using the algorithm presented in Theorem 3.1 with $\gamma = 0.26$, we obtain $\bar{X}$:

$$\bar{X} = \begin{bmatrix} 0.0597 & -0.0001 & 0.0002 \\ -0.0001 & 0.1115 & 0.0009 \\ 0.0002 & 0.0009 & 0.2258 \end{bmatrix}$$

$$\bar{X} = \begin{bmatrix} 0.0403 & 0.0001 & -0.0002 \\ 0.0001 & 0.0885 & -0.0009 \\ -0.0002 & -0.0009 & 0.1742 \end{bmatrix}$$

IV. DISCRETE-TIME CASE

Consider the perturbed discrete-time system

$$x_{k+1} = (A_0 + \Delta A)x_k + D_0 w_k \tag{4.1}$$

where $A_0$ is stable, $x_k \triangleq x(t_k)$, and $w_k \triangleq w(t_k)$ is white noise of unit covariance. We desire to compute upper and lower bounds on the state covariance $X$ of (4.1), which satisfies

$$X = (A_0 + \Delta A)x(A_0 + \Delta A)^T + W_0 \tag{4.2}$$

as long as $A_0 + \Delta A$ remains stable where $W_0 \triangleq D_0 D_0^T$ and $\Delta A$ describes the parameter perturbation.

Theorem 4.1: Suppose $A_0$ is stable and $(A_0 + \Delta A, D_0)$ is stabilizable for $\Delta A \in \Omega$:

$$\Omega \triangleq \{ \Delta A : \Delta A \Delta A^T \leq \bar{A} \}. \tag{4.3}$$

and there exists an $X \geq 0$ satisfying

$$X = A_0 X A_0^T + A_0 \bar{X} (I + \bar{X}^T / \gamma) \bar{X} A_0^T + (\beta + \gamma / \beta) \bar{A} + W_0 \tag{4.4}$$

for some $\gamma > 0$ and $\beta > \bar{\beta}(X)$; then $A_0 + \Delta A$ is asymptotically stable for all $\Delta A \in \Omega$ and

$$X \leq \bar{X} \quad \forall \Delta A \in \Omega \tag{4.5}$$

where $X$ satisfies (4.2).

Proof: Considering the following inequality

$$(\beta^{1/2} \Delta A - A_0 X / \beta^{1/2})(I - X / \beta^{1/2}) \beta^{1/2} \Delta A - A_0 X / \beta^{1/2} \geq 0 \tag{4.6}$$

we have

$$\beta \Delta A \Delta A^T + A_0 X (I - X / \beta) \bar{X} A_0^T / \beta + A_0 X^2 \bar{A}^T / \beta + \Delta A X^2 \bar{A}^T / \beta \geq A_0 X \Delta A^T + \Delta A X \Delta A^T. \tag{4.7}$$

Further, since $\Delta A \Delta A^T \leq \bar{A}$, we have

$$(\beta + \gamma / \beta) \bar{A} + A_0 X (I - X / \beta) \bar{X} A_0^T / \beta \geq A_0 X \Delta A^T + \Delta A X \Delta A^T. \tag{4.8}$$

Combing (4.6) and $1/ \beta$ times (4.8), we obtain

$$(\beta + \gamma / \beta) \Delta A \Delta A^T + A_0 X \left( I - \frac{X}{\beta} + \frac{X^2}{\gamma} \right) \bar{X} A_0^T / \beta \geq A_0 X \Delta A^T + \Delta A X \Delta A^T. \tag{4.9}$$

Now, rewrite (4.4) as follows:

$$\bar{X} = (A_0 + \Delta A) \bar{X} (A_0 + \Delta A)^T + W_0 + N \tag{4.10}$$

where

$$N \triangleq A_0 \bar{X} \left( I + \frac{X}{\beta} \right) \bar{X} A_0^T / \beta + (\beta + \gamma / \beta) \bar{A} \tag{4.11}$$

$$- (A_0 X \Delta A^T + \Delta A X \bar{A}^T + \Delta A X \Delta A^T). \tag{4.12}$$

$N$ is nonnegative definite because of (4.10). The matrix pair $(A_0 + \Delta A, D_0)$ is assumed to be stabilizable; hence, $(A_0 + \Delta A, W_0 + N)$ is
stabilizable [11]. This stabilizability, together with the fact that \( \dot{X} \geq 0 \), implies that \( A_0 + \Delta A \) is asymptotically stable. Further, the \( \dot{X} \) satisfying (4.4) can be written as

\[
\dot{X} = \sum_{i=0}^{\infty} A_0 [W_0 + (\beta + \gamma/\beta) \bar{A} + A_0 \bar{X} \left( I + \frac{X^2}{\gamma} \right) \bar{X} A_0^T] A_0'.
\]

(4.13)

Similarly, by rewriting (4.2) as

\[
X = A_0 X A_0^T + W_0 + A_0 X \Delta A^T + \Delta A X \Delta A^T + \Delta A X A_0^T
\]

(4.14)

its nonnegative solution can be obtained as

\[
X = \sum_{i=0}^{\infty} A_0 [W_0 + A_0 X \Delta A^T + \Delta A X \Delta A^T + \Delta A X A_0^T] A_0'.
\]

(4.15)

Because of (4.10), it is obvious that

\[
\dot{X} \geq X \quad \forall \Delta A \in \Omega.
\]

(4.16)

Q.E.D.

For the lower bound problem, we have the following theorem.

**Theorem 4.2:** Suppose \( A_0 + \Delta A \) is stable and \( (A_0 + \Delta A, D_0) \) is stabilizable for \( \Delta A \in \Omega \), and there exists an \( X \geq 0 \) and an \( \alpha > 0 \) satisfying both

\[
(1 + \alpha) \dot{X} = A_0 X A_0^T + W_0
\]

(4.17)

and

\[
\alpha X \geq (\beta + \gamma/\beta) \bar{A} + A_0 \bar{X} \left( I + \frac{X^2}{\gamma} \right) X A_0^T / \beta
\]

(4.18)

for some \( \beta > 0 \) and \( \gamma > 0 \); then

\[
X \geq \dot{X} \quad \forall \Delta A \in \Omega
\]

(4.19)

where \( \dot{X} \) satisfies (4.2).

**Proof:** Consider the inequality

\[
(\beta^{1/2} \Delta A + A_0 X / \beta^{1/2}) \left( I + \frac{X^2}{\beta} \right) (\beta^{1/2} \Delta A + A_0 X / \beta^{1/2})^T \geq 0;
\]

(4.20)

we have

\[
\beta \Delta A \Delta A^T + A_0 X \left( I + \frac{X^2}{\beta} \right) X A_0^T / \beta \geq - (\Delta A X A_0^T + \Delta A X A_0^T + A_0 X \Delta A^T). \quad (4.21)
\]

Combine (4.8) and (4.21), we obtain

\[
(\beta + \gamma/\beta) \Delta A \Delta A^T + A_0 X \left( I + \frac{X^2}{\beta} \right) X A_0^T / \beta \geq - (\Delta A X A_0^T + \Delta A X A_0^T + A_0 X \Delta A^T). \quad (4.22)
\]

Further, we have

\[
(\Delta A X A_0^T + \Delta A X A_0^T + A_0 X \Delta A^T) \geq - (\beta + \gamma/\beta) \bar{A} - A_0 \bar{X} \left( I + \frac{X^2}{\beta} \right) X A_0^T / \beta
\]

(4.23)

because \( \Delta A \Delta A^T \leq \bar{A} \) and \( X \leq \dot{X} \).

The \( \dot{X} \) satisfying (4.17) can be expressed as

\[
\dot{X} = \sum_{i=0}^{\infty} A_0 [W_0 - \alpha X] A_0'.
\]

(4.24)

Now, compare (4.15) and (4.24) by using inequalities (4.18) and (4.23); we have proven

\[
X \geq \dot{X} \quad \forall \Delta A \in \Omega.
\]

Q.E.D.

**Remark 4.1:** It is obvious from (4.2) that \( W_0 \) is also a lower bound. But we want to obtain a lower bound as big as possible, and hence it is desired to achieve our objective by using Theorem 4.2.

**Remark 4.2:** Both Theorems 4.1 and 4.2 require iterations to find the solution. The convergence of any such numerical schemes remains an open problem.

To illustrate the theory presented in this section, we give here an example.

**Example 4.1:** The nominal closed-loop system is described with

\[
A_0 = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.4 \end{bmatrix}, \quad D_0 = \begin{bmatrix} 0.4721 & 0 \\ 0 & 0.3162 \end{bmatrix}
\]

Suppose

\[
\bar{A} = \begin{bmatrix} 0.0026 & 0.0004 \\ 0.0004 & 0.0016 \end{bmatrix}, \quad \Delta A \Delta A^T \leq \bar{A}.
\]

By using Theorem 4.1 and choosing \( \beta = 2.9, \gamma = 0.8 \), and by using Theorem 4.2 and choosing \( \alpha = 0.08, \beta = 3.5, \gamma = 0.8 \), we obtain

\[
\dot{X} = \begin{bmatrix} 0.2916 & 0.0085 \\ 0.0085 & 0.1262 \end{bmatrix}, \quad X = \begin{bmatrix} 0.2429 & 0.0049 \\ 0.0049 & 0.1087 \end{bmatrix}.
\]

(4.25)

Note that the well-known lower bound \( W_0 \) satisfying

\[
\dot{X} \geq W_0 = D_0 D_0^T = \begin{bmatrix} 0.2000 & 0 \\ 0 & 0.1000 \end{bmatrix}
\]

is improved by the new bound (4.25).

**V. CONCLUSION**

The robustness of the state covariance (which might be assigned to the closed-loop system by using the covariance control theory) is analyzed. In the presence of plant parameter variations, both lower and upper bounds on the state covariance matrix are provided in terms of a specified set of parameter variations. Further work is required in the optimization of both the upper and the lower bounds of the perturbed state covariance for specified kinds of system perturbation by the design.

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