The computation applies to a dynamic load model with a general form of load dynamics but may be conveniently carried out on the load flow equations. The controls and parameters may appear anywhere in the load flow equations. The computation is simple, with the main computational burden being the computation of a left eigenvector at the bifurcation. However, two of the methods of computing the load power margin produce the required eigenvector as a by product. The computation has several applications to power system planning and security.

The computation applies to control or parameter changes to steer a general parameterized dynamical system away from a saddle node bifurcation if the formula (5) is used to calculate the normal vector \( n^* \).

**ACKNOWLEDGMENT**

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**REFERENCES**


which also guarantee specified $l_2$ performance constraints for stochastic inputs and $l_n$ performance constraints for deterministic inputs. This is not an analysis but a design method guaranteeing the specified robust performances. Also an upper bound on the covariance matrix is provided.

I. INTRODUCTION

There is much literature on the analysis of stability and performance robustness [1]-[4]. The limitation of these results is that one cannot know what robust performances can be guaranteed before the controller design. In other words, robust controller design techniques are needed. Practical controller designs should guarantee both stability and performance robustness. Output variances of a linear system represent the stochastic performance properties when the system is subject to stochastic disturbances. The output variance constraint (OVC) problem [5], [6] minimizes a quadratic function of the control vector subject to multiple inequality constraints on output variances. The OVC design algorithm has been applied to the “NASA Minimainst” at Langley Research Center (see [7]).

It is impossible to obtain an exact design model in practical engineering controller design problems. In the presence of modeling errors the inequality constraints of the closed-loop system must be satisfied. The published OVC controller may not be satisfied. The note is organized as follows. Section II presents an equation for the upper bound on the covariance matrix, and also provides the minimal solution of the equation for the upper bound on the state covariance matrix when $A_o$ and $D_o$ are asymptotically stable and $(A_o + D_o)$ is asymptotically stable. Using the minimal solution obtained in Section II, Section III discusses a robust discrete controller (1.2) design algorithm (called the OVC algorithm) which guarantees $l_2$ and $l_n$ performance requirements stated in (1.5) or (1.9). Section IV is an example to show some details of the controller design. Section V presents some conclusions.

II. THE UPPER BOUND COVARIANCE MATRIX AND ITS MINIMAL SOLUTION

For the closed-loop discrete system (1.3), suppose $A_o$ is asymptotically stable, then the nominal state covariance matrix is the unique positive semidefinite solution of the following Lyapunov equation:

\[ X_o = A_o X_o A_o^T + D_o W D_o^T. \]  
(2.1)

An upper bound on the covariance matrix of (1.3) has been given in [8]. But the equation that the upper bound in [8] satisfies is difficult to solve because of the nonlinear property of the equation. Theorem 1 gives an upper bound with the advantage of tractability, since the equation will be similar to the standard discrete Riccati equation.

**Theorem 1** Suppose $A_o$ is asymptotically stable and $(A_o + D_o + D_o)$ is stabilizable for any

\[ \Delta A + \Delta A^T \leq \bar{A}, \quad \Delta D \in \Omega_D, \quad \Omega_D \triangleq \{ \Delta D : (D_o + \Delta D)^T \leq W_o \}. \]  
(2.2)

\[ \Delta A \in \Omega_A, \quad \Omega_A \triangleq \{ \Delta A : \Delta A \Delta A^T \leq \bar{A} \}. \]  
(2.3)

Two classes of disturbances are considered in this note. First, $w(k)$ and $u(k)$ are zero mean white noises with covariance matrices $W_w$ and $V$. In the case of deterministic interpretation of the above problem, we consider the class of $l_2$ disturbances satisfying

\[ \|w(k)\|_2^2 \leq \sum_{k=0}^\infty \sum_{i=1}^{ny} \nu_i w_i(k) \leq \mu^2 < \infty. \]  
(1.5)

for any uncertainty matrices satisfying

\[ \Delta A \Delta A^T \leq \bar{A}, \quad (D_o + \Delta D) W (D_o + \Delta D)^T \leq W_o \]  
(1.6)

where $\bar{A}$ is the function of the estimation gain $F$ and feedback gain $G$, and $W_o$ is a given matrix bound such that (1.6) holds, and $W$ is the covariance matrix of the input white noise $w(.)$.

For deterministic interpretation of the above problem, we consider the class of $l_2$ disturbances satisfying

\[ \|w(.)\|_2^2 \leq \sum_{k=0}^\infty \sum_{i=1}^{ny} \nu_i w_i(.) \leq \mu^2, \]  
(1.7)

where $\mu$ is a given constant and matrix $W$ has the structure of (1.7). In this case, we require closed-loop performance in terms of inequality constraints ($\epsilon_j^2, i = 1, 2, \ldots$, $n_y$) on the output $u$ and covariance matrices

\[ \|y(.)\|_2^2 \leq \sum_{k=0}^\infty \sum_{i=1}^{ny} \nu_i y_i(.) \leq \epsilon_j^2 \]  
(1.9)

for any uncertainty matrices satisfying (1.6).
and there exists an \( \vec{x} \) satisfying

\[
\vec{x} = A_o \vec{x} + A_o (I - \vec{x} / \beta)^{-1} \vec{x} + 2 \beta \vec{A} + W_o \quad \text{(2.4)}
\]

for some \( \beta > 0 \), then \( A_o + \Delta A \) is asymptotically stable for all \( \Delta A \in \Omega_\delta \) and

\[
X \leq \vec{x}, \quad \forall \Delta A \in \Omega_\delta, \quad \Delta D \in \Omega_D \quad \text{(2.5)}
\]

where \( X \) satisfies

\[
X = (A_o + \Delta A)X(A_o + \Delta A)^T + (D_o + \Delta D)W(D_o + \Delta D)^T. \quad \text{(2.6)}
\]

The proof in the continuous case appears in [8]. The proof of Theorem 1 appears in [9]. Note that the upper bound presented in Theorem 1 is very much like a discrete version of the continuous time bounds used in [10], [11]. The discrete Riccati equation (2.4) given by Theorem 1 can be solved analytically. Now we are going to solve for the minimal positive definite solution of (2.4).

Consider the following discrete Riccati equation:

\[
\vec{x} = A \vec{x} + A \vec{x}(I - C \vec{x} / \beta)^{-1} C \vec{x} + BB^T \quad \text{(2.7)}
\]

where \( \beta > 0 \). Assume that \( A \) is invertible and let

\[
Z = \begin{bmatrix}
A - CC^T A^{-1} B B^T & \text{\( -CC^T A^{-1} \)}
A^{-1} A^{-1} B B^T & A^{-1}
\end{bmatrix} \quad \text{(2.8)}
\]

be a symplectic matrix. References [12] and [13] point out that the discrete Riccati equation (2.7) has a unique stabilizing solution if and only if \( Z \) in (2.8) has no eigenvalues on the unit circle and \( (A, C) \) is stabilizable. Moreover, the unique stabilizing solution is the minimal solution among all the positive definite matrices satisfying (2.7). The proof appears in [9]. The minimal property of the stabilizing solution is very important for solving the minimal upper bound covariance matrix \( \vec{x} \) satisfying (2.4). The numerical computation of the solution of (2.7) is presented in Lemma 1.

Lemma 1 [14]: Suppose \( Z \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} J \), where \( J \) is a square matrix and \( X_1 \) is invertible. Then \( \vec{x} = X_2 X_1^{-1} \) is a solution of (2.7) and matrices \( Z \) share the same eigenvalues.

Suppose that all the eigenvalues of \( J \) in Lemma 1 are in the unit circle. It has been shown that if \( (A, B) \) is stabilizable and \( Z \) has no eigenvalues on the unit circle, then \( X_1 \) is invertible (see [14]). In this case \( X_2 X_1^{-1} \) is the corresponding stabilizing, or minimal positive definite solution. Therefore, Lemma 1 actually provides a way to compute the minimal positive definite solution \( \vec{x} \). Related results can be found in [15] and [16].

Now, letting \( C = I, \ A = A_o \), and \( BB^T = 2 \beta \vec{A} + W_o \), the \( Z \) matrix in (2.8) becomes

\[
Z = \begin{bmatrix}
A_o^2 - A_o^{-1} (2 \beta \vec{A} + W_o) / \beta & A_o^{-1} \\
-2 \beta A_o^{-1} (2 \beta \vec{A} + W_o) & A_o^{-1}
\end{bmatrix}. \quad \text{(2.9)}
\]

Since \( A_o \), is asymptotically stable, if \( Z \) has no eigenvalues on the unit circle, the upper bound \( \vec{x} \) satisfying (2.4) can be solved by using Lemma 1. We also point out that the Riccati equation (2.7) with \( A_o \) has a stabilizing solution if and only if

\[
\| C (I - A_o)^{-1} B \|_\infty < 1. \quad \text{(2.10)}
\]
covariance matrix during iteration from the OUVC algorithm is in the form (1.2) and during iteration the minimal positive definite solution of the upper bound on the
Note that \( x \) in Step 7) is a function of \( \delta \) means that it is impossible to get a constant boundary matrix \( A \) during iteration.
However, the control gain \( G \) varies during the iteration, which means that \( \Delta A \Delta A^T \) depends on the feedback gain \( G \). Before the design, we do not know how large \( GG^T \) is in (1.3), which means that it is impossible to get a constant boundary matrix \( A \) during iteration. Therefore, \( A \) should be a function of \( G \).

The following lemma provides one way to choose the upper bound \( A \) of \( \Delta A \Delta A^T \).

\[ \Delta A \Delta A^T \leq \bar{A}. \] (3.13)

Since the estimation gain \( F \) depends on the white noise covariance matrices \( W_0 \) and \( V \), \( F \) will not change during the iteration. However, the control gain \( G \) varies during the iteration, which means that \( \Delta A \Delta A^T \) depends on the feedback gain \( G \). Before the design, we do not know how large \( GG^T \) is in (1.3), which means that it is impossible to get a constant boundary matrix \( A \) during iteration. Therefore, \( A \) should be a function of \( G \).

The following lemma provides one way to choose the upper bound \( A \) of \( \Delta A \Delta A^T \).

**Lemma 2:** The matrix \( \Delta A \Delta A^T \) can be bounded by the following matrix:

\[
\bar{A}(G) = \bar{A}(G) + \bar{A}_2
\] (3.14a)

where

\[
\bar{A}_2 = \begin{bmatrix}
1 + 1/\nu & \bar{A}_p & 0 \\
0 & 0 & 0 \\
0 & 0 & (1 + \nu)F\bar{M}_pF^T
\end{bmatrix}
\] (3.14b)

for any \( \nu > 0 \) and \( \bar{A}_p, \bar{A}_r \) and \( \bar{M}_p \) are defined in (1.1d)–(1.1e).

The proof appears in [9]. The matrix \( W_0 \) in (1.6) can be chosen as follows:

\[
W_0 = \begin{bmatrix}
I & 0 & 0 \\
0 & F & 0 \\
0 & 0 & F^T
\end{bmatrix}
\] (3.15)

By using the above algorithm and choosing \( \bar{A}, W_0 \) by (3.14) and (3.15), one can easily obtain a robust discrete time full-order dynamic controller.

**Lemma 3:** Considering the discrete system (1.3), suppose \( (A_d + \Delta A, D_p + \Delta D) \) is controllable and \( A_d + \Delta A \) is asymptotically stable for all \( \Delta A, \Delta D \) satisfying (2.2) and (2.3). Let \( \bar{A} = A_x X_c^T \) be the upper bound of the output covariance matrix of (1.3) when the input \( w(t) \) is zero mean white noise with

\[
E\{w(k)w^T(k)\} = \bar{W}
\]

\[
\|y(k)\|^2 = \sup_{k \geq 0} y^T(k)y(k) \leq \sigma(\bar{Y})\|w(\cdot)\|^2.
\] (3.16)

One can prove (3.16) in a similar manner to [17]. If the input deterministic disturbances satisfy (1.8), it is clear that for any \( \Delta A, \Delta D \) satisfying (2.2) and (2.3)

\[
\|y(\cdot)\|^2 \leq \mu^2 C_x X_c^T; \quad i = 1, 2, \ldots, n_y.
\] (3.17)

Hence, by selecting \( \bar{A} = A_x X_c^T \) the OUVC controllers will guarantee the output \( l_p \) performance (1.9) for any disturbances satisfying (1.6) and any \( \Delta A \) and \( \Delta D \) satisfying (2.2) and (2.3).

**Lemma 4:** If the OUVC algorithm converges, then it converges to a feasible solution, i.e., the closed-loop system (1.3) with the resulting controller will satisfy all constraints in the OUVC problem.

The proof appears in [9]. The next section is an example used to demonstrate the controller design procedure.

**IV. Example**

Given the following discrete system (1.1) with nominal system matrices:

\[
A_p = \begin{bmatrix}
9.9502e-01 & 9.9336e-02 & 3.6622e-02 \\
-9.9366e-02 & 9.8590e-01 & 6.2714e-02 \\
0 & 0 & 3.6788e-01
\end{bmatrix};
\]

\[
C_p = \begin{bmatrix}
1.0000e+00 & 5.0000e-01 & 0 \\
0 & 0 & 1.0000e+00 \\
1.0000e+00 & 1.0000e+00 & 0
\end{bmatrix};
\]

\[
B_p = \begin{bmatrix}
1.3170e-04 \\
3.6622e-03 \\
6.3212e-02
\end{bmatrix}; \quad D_p = \begin{bmatrix}
1.3170e-04 \\
3.6622e-03 \\
6.3212e-02
\end{bmatrix};
\]

\[
M_p = \begin{bmatrix}
1.0000e+00 & 1.0000e+00 & 0
\end{bmatrix}
\]
and the input white noise covariance matrices
\[ W_r = 1.0 \quad V = 0.1 \]
the uncertain matrices \( \Delta A_p, \Delta B_p, \Delta M_p, \) and \( \Delta D_p \) are structured uncertain matrices as follows:
\[
\Delta A_p = \begin{bmatrix}
0 & 0 & 0 \\
0.001 \epsilon_i & 0.000302 \epsilon_i & 0 \\
0 & 0 & 0 \\
0.005 \epsilon_i & 0 & 0
\end{bmatrix} \\
\Delta B_p = \begin{bmatrix}
0.01 \epsilon_i \epsilon_p \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} \\
\Delta D_p = \begin{bmatrix}
0.01 \epsilon_i \epsilon_p \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} \\
\Delta M_p = [0.05 \epsilon_i] 0 0
\]
(4.1)
where \( \epsilon_i \in [-1, 1], i = 1, 2, \ldots, 5. \) Then we can take the upper boundary matrices as follows:
\[
\bar{A}_p = \begin{bmatrix}
0 & 0 & 0 \\
0 & 1.01912e - 06 & 0 \\
0 & 0 & 0
\end{bmatrix} \\
\bar{B}_p = \begin{bmatrix}
4.8233e - 11 & 8.2535e - 10 \\
1.3412e - 09 & 2.3149e - 08 \\
3.9958e - 07 & 3.9958e - 07
\end{bmatrix} \\
\bar{M}_p = 0.0025
\]
The matrices \( W_r, \bar{A}_p, G, \) and \( \bar{A}_2 \) can be chosen by (3.15) and (3.14). For the matrix \( \bar{A}_2, \) we have a free parameter \( \nu \) [see (3.14)]. For this example, we chose \( \nu \) in such a way that \( \nu \) minimizes the trace of matrix \( \bar{A}_2. \) In this case
\[
\nu = \left[ \text{tr} \left( \bar{A}_p \right) / \text{tr} \left( F M_p F^T \right) \right]^{1/2}.
\]
(4.2)
Now we have described the system in the form of (1.1).

The design goal is to find a full-order dynamic controller (1.2) such that the output variances of the closed-loop system are subject to
\[
E_y^2(k) \leq 5.5000e - 03; \quad E_y^2(k) \leq 8.0000e - 03;
\]
for stochastic inputs. As the result of Lemma 4, the output \( l_n \) norms will satisfy
\[
\| y_n(\cdot) \|^2 \leq 5.5000e - 03 \mu^2; \quad \| y_n(\cdot) \|^2 \leq 8.0000e - 03 \mu^2;
\]
\[
\| y_n(\cdot) \|^2 \leq 1.0000e - 02 \mu^2
\]
(4.4)
for any deterministic input disturbance \( w(\cdot) \) satisfying (1.8) and any \( \Delta A, \Delta D \) satisfying (2.2) and (2.3). By using the OUV algorithm with \( R = I \) we designed three kinds of controllers for comparison.

**Controller 1**: Nominal design by setting \( \bar{A} = 0, W_c = DWD^T \) (standard OVC design).

**Controller 2**: Design with \( \Delta A_p, \Delta B_p \) presented in (4.1) but set \( \Delta M_p = 0 \) and \( \Delta D_p = 0 \) \( (\epsilon_i = 0) \) with the choice of \( W_e = DWD^T \) and \( \bar{A} \) as follows:
\[
\bar{A} = \bar{A}_p(G) + \bar{A}_2 = \begin{bmatrix}
0 & 0 & 0 \\
0.0104 & 0.0085 & 0.0076 \\
0.0102 & 0.0095 & 0.0054
\end{bmatrix} \\
0.0059 & 0.0051 & 0.0051 \\
0.0117 & 0.0096 & 0.0087 \\
0.0080 & 0.0062 & 0.0055 \\
0.0132 & 0.0102 & 0.0096 \\
0.0100 & 0.0080 & 0.0071 \\
0.0055 & 0.0056 & 0.0056 \\
0.0146 & 0.0131 & 0.0117
\]
(4.5)

**Controller 3**: Design with the presence of all \( \Delta A_p, \Delta B_p, \Delta M_p, \) and \( \Delta D_p \) in (4.1).

The three controller design results are compared in Table I. Note that Controller 1 is designed by the standard OVC algorithm and no uncertainty has been considered. Hence, there is no upper bound covariance matrix in Table I for Controller 1. We also evaluated all three closed-loop systems with six different types of uncertain matrices \( \Delta A_p, \Delta B_p, \Delta M_p, \) and \( \Delta D_p \) defined in (4.1) with the choice of \( \epsilon_i \) \( (i = 1, 2, \ldots, 5) \) listed in Table II. Note that in cases 3–6), the \( \epsilon_i \)'s do not belong to the given sets in (4.1). The main reason to evaluate those cases is to see the conservativeness of this design. The evaluated results appear in Table II [the \( x \) denotes a value violating (4.3)].

From Table II, the nominal design (Controller 1) has no robustness with respect to the parameter uncertainty (some output variances exceed the given bounds (4.3) when uncertainty is present). Controllers 2 and 3 are robust designs with respect to the uncertainty assumed in (\( \Delta A_p, \Delta B_p \)) and (\( \Delta A_p, \Delta B_p, \Delta M_p, \Delta D_p \)), respectively, as shown by (4.5) and (4.1). The closed-loop performances (output variances) are guaranteed with respect to the indicated uncertainty in (4.5) and (4.1), respectively. For Controller 2, when the closed-loop system is evaluated with the uncertainty in cases 1, 2, and 3) the output variances satisfy the requirements, and for cases 4, 5, and 6) they do not. For Controller 3, when the closed-loop system is evaluated, the output variances match the inequality (4.3), except case 6). But the design of Controller 2 guarantees the performance requirements only for case 1) and the design of Controller 3 guarantees the performance requirements for cases 1) and 2).

**Table I**

<table>
<thead>
<tr>
<th>Controller</th>
<th>Nominal system</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>0.0176</td>
<td>0.0169</td>
<td>0.0020</td>
<td>0.0127</td>
<td>0.0210</td>
<td>0.0697</td>
<td>1.0616</td>
</tr>
<tr>
<td>G</td>
<td>0.0055</td>
<td>0.0046</td>
<td>0.0046</td>
<td>0.0046</td>
<td>0.0055</td>
<td>0.0055</td>
<td>0.0055</td>
</tr>
<tr>
<td>Upper bound output variances</td>
<td>0.0088</td>
<td>0.0088</td>
<td>0.0088</td>
<td>0.0088</td>
<td>0.0088</td>
<td>0.0088</td>
<td>0.0088</td>
</tr>
<tr>
<td>Nominal input cost</td>
<td>5.1297e - 07</td>
<td>3.9538e - 06</td>
<td>1.2108e - 05</td>
<td>1.2108e - 05</td>
<td>1.2108e - 05</td>
<td>1.2108e - 05</td>
<td>1.2108e - 05</td>
</tr>
</tbody>
</table>

**Table II**

<table>
<thead>
<tr>
<th>Controller</th>
<th>Closed-loop output variances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal system</td>
<td>0.0055</td>
</tr>
<tr>
<td>Case 1</td>
<td>-1</td>
</tr>
<tr>
<td>Case 2</td>
<td>1</td>
</tr>
<tr>
<td>Case 3</td>
<td>-5</td>
</tr>
<tr>
<td>Case 4</td>
<td>5</td>
</tr>
<tr>
<td>Case 5</td>
<td>-10</td>
</tr>
<tr>
<td>Case 6</td>
<td>-10</td>
</tr>
</tbody>
</table>

\( \epsilon_1 = -\epsilon_2 = \epsilon_3 \quad \epsilon_4 = -\epsilon_5 \)
Table II shows that this design is a bit conservative. In order to get a less conservative robust controller, one may use smaller boundary matrices $A$ and $W_p$ than those required, but then the performances will not be guaranteed. Also the choice of the free parameter $\beta$ for the upper bound covariance matrix is very important. The choice of $\beta$ will effect the conservativeness of the upper bound covariance, which is directly related to the conservativeness of the controller design. For this example, we tried several $\beta$'s. For very big $\beta$ the solution of the upper bound covariance matrix will be very conservative, and for small $\beta$'s the matrix $\gamma$ in (2.8) will have eigenvalues on the unit circle, which means that the solution does not exist. We chose $\beta = 4$ because it gave the least conservative solution among the $\beta$'s we chose. One way to choose the “best” $\beta$ is to minimize the upper bound covariance. In summary, to reduce the conservativeness in the OUVVC design, the main problem is to reduce the conservativeness of the upper bound covariance solution.

From Table I, the estimation gain $F'$s for the three controllers are constant, which means that this type of robust control is independent of estimation. To compare “control efforts” (or system gains) we consider the nominal input cost defined by

$$E, u(k)'Ru(k) - tr(C'_pRC_p)X; \quad C_p = [0 \ G] \quad (4.6)$$

as one kind of controller gain criteria. Table I shows that the robust controllers are high gain controllers. The more uncertainty considered in the controller design, the higher the control gain, which is consistent with the continuous case treated in [18].

V. CONCLUSION

For discrete systems with uncertainty, this note presents several equations describing the upper bound covariance. One of these equations is a discrete Riccati type equation, and an analytical minimal positive definite solution of this equation is provided. With that method, one can obtain the upper bound covariance matrix easily. Based on the solution of the upper bound on the covariance and the OVC algorithm, this note provides a design method for discrete time dynamic controllers guaranteeing the output performances (output variances or $l_2$ norms), while guaranteeing robustness with respect to the input $l_2$ disturbance and parameter uncertainty. The resulting controller minimizes the input cost $E, u(k)'Ru(k)$ such that the output variances or $l_2$ norms are less than or equal to the given bounds. The example shows that the algorithm worked well and converged in 20 iterations with the error tolerance $1.0e \times 0.8$. Future work will address the convergence conditions of the OUVVC algorithm.

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Singularly Perturbed Zero Dynamics of Nonlinear Systems

A. Isidori, S. S. Sastry, P. V. Kokotovic, and C. I. Byrnes

Abstract—Stability properties of zero dynamics are among the crucial input/output properties of both linear and nonlinear systems. Unstable, or “nonminimum phase,” zero dynamics are a major obstacle to input/output linearization and high-gain designs. An analysis of the effects of regular perturbations in system equations on zero dynamics shows that whenever a perturbation decreases the system's relative degree, it manifests itself as a singular perturbation of zero dynamics. Conditions are given under which the zero dynamics evolve in two timescales characteristic of a standard singular perturbation form that allows a separate analysis of slow and fast parts of the zero dynamics.

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A. Isidori is with the Dipartimento di Informatica e Sistemi, Università di Roma, “La Sapienza,” 00184 Rome, Italy, and with the Department of Systems Sciences and Mathematics, Washington University, St. Louis, MO 63130.

S. S. Sastry is with the University of California, Berkeley, CA 94720.

P. V. Kokotovic is with the University of Illinois, Urbana-Champaign, IL 61801.

C. I. Byrnes is with Washington University, St. Louis, MO 63130. IEEE Log Number 9201954.