On Long-Range Dependence in High-Bitrate Wireless Residual Channels

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Abstract — **Wireless applications, protocols and simulators can significantly benefit from thorough understanding and accurate modeling of the medium access control (MAC) layer residual bit-errors at high bitrates. In this paper, we analyze and model the bit-errors encountered at the highest achievable 11 Mbps data rate of an 802.11b wireless local area network. We employ autocorrelation, aggregation and variance-time analyses to reveal *long-range dependence* in the bit-error process. This renders traditional models (e.g., simple Markov, Poisson) ineffective and therefore we employ a *multifractal wavelet model* (MWM) to accurately characterize the bit-error process. Our results outline that the MWM captures the long-range-dependent behavior of the bit-errors quite accurately. We illustrate that the MWM outperforms high-order Markov chains in both channel modeling and complexity.**

I. INTRODUCTION

Wireless networks suffer from frequent errors and losses due to their vulnerability to interference and transmission medium degradation. Real-time multimedia applications can inherently tolerate a certain level of errors. In addition, contemporary wireless multimedia applications are introducing enhanced error control and resilience features [1], [2]. Emerging wireless cross-layer protocols ignore the residual bit-errors[1] (i.e., errors that are not corrected by the physical layer) at the medium access control (MAC) layer in order to relay corrupted packets to the error-resilient applications [3], [4]. Analysis and modeling of these residual errors: (a) provide effective insights into the underlying characteristics of a wireless bit-error process; (b) facilitate design of wireless cross-layer protocols and real-time applications; (c) allow accurate simulations without having the actual network in place; (d) render real-time channel characterization and prediction for rate-adaptive applications.

The wide-spread deployment and high data rates of 802.11b local area networks (LANs) offer promising prospects for the support of multicast and on-demand multimedia distribution at very high bitrates. However, the robustness at the 802.11b physical layer decreases with an increase in bitrate and consequently more residual errors are observed at high data rates. While prior studies have modeled the bit-errors at 2 and 5.5 Mbps of an 802.11b LAN as stationary Markov chains [5]–[7], the error behavior at 11 Mbps differs significantly from the low data rates. Bit-errors observed at 11 Mbps exhibit non-stationarity and high correlation. Traditional stochastic models (e.g., simple Markov, Poisson) that assume stationarity and decaying correlation are therefore ineffective in characterizing the 11 Mbps residual error process.

In this paper, we provide analysis and modeling of residual bit-errors encountered at the highest achievable 11 Mbps data rate of an 802.11b wireless LAN. To the best of the authors' knowledge, this is the first attempt to characterize 802.11b bit-errors at 11 Mbps. We employ actual bit-error traces collected under realistic settings of an operational 802.11b LAN. In Section III we show that, unlike the decaying autocorrelations at 2 and 5.5 Mbps, the autocorrelation at 11 Mbps does not decay significantly with an increase in lag. Such high correlation is reminiscent of *long-range dependence* (LRD) [10] in the bit-error process. We further substantiate the LRD notion through *aggregation* and *variance-time* analyses. In view of the LRD nature of the bit-error process, and due in part to its highly non-stationary behavior, in Section 4 we employ the *multifractal wavelet model* (MWM) to characterize the random process [11]-[13]. To provide performance comparison with contemporary modeling paradigms, we also model the bit-error process using Markov chains of orders varying from 2 up to 12 (i.e., 2^2 up to 2^{12} states). We demonstrate that the MWM outperforms the Markov models in both complexity and channel approximation. Section V summarizes key conclusions of this work.

II. BACKGROUND

This section provides brief background that is required to understand the present work. Analytical formalisms of the presented material have been thoroughly explored in previous works and are referenced as required.

II.A Self-Similar Processes

Self-similar processes exhibit similar (statistical) behavior at different scales, that is, zooming into or out-of a sample path of the process gives a new process realization which is statistically similar to the original path. Thus, for a process $\mathrm{X}(t)$ self-similarity is analytically defined as

$$\mathrm{X}(t) \stackrel{d}{=} c^H \mathrm{X}(t/c), \tag{1}$$

where $\stackrel{d}{=}$ represents equivalence in finite-dimensional distribution, c is the scaling (compression/dilation) factor and H, known as the *Hurst parameter*, defines the regularity of a sample path. It is not possible to define a characteristic

[1] Zorzi *et al.* [8] introduced the term *residual errors* to refer to errors observed above the physical layer. In [6], we used the term *MAC-to-MAC* errors to refer to the same phenomenon.

scale for self-similar processes and therefore these processes are referred to as *scale-invariant*. In loose terms, *scaling* can be defined as a negative property of a time series, i.e., the absence of characteristics scales.

Scaling (or self-similarity) imposes very stringent constraints on a process. Relaxing the similarity requirement yields LRD processes which invoke a weaker notion of *second-order self-similarity*, i.e., zooming out-of the process' sample path will yield paths that are similar to the original in second-order statistics.

II.B Long-Range-Dependent Processes

A long-range-dependent process $\mathrm{X}(t)$ is a second-order self-similar process whose autocorrelation function, $\rho(\eta)$, exhibits the following properties: (a) $\rho(\eta)$ is strictly positive at all lags η, and (b) $\rho(\eta)$ decays so slowly with respect to η that it is non-summable, $\sum_\eta \rho(\eta) = \infty$. Thus, a long-range-dependent process depends heavily on previous samples which results in strong correlation even at large lags.

The sample variance of an aggregated (zoomed out) process plotted against the aggregation (averaging) level on a log-log plot should result in a straight line with a slope of $2H-2$, where H is the Hurst parameter. Long-range-dependent processes must have $H > 0.5$. For a more comprehensive representation of the process behavior at different times, instead of using a constant Hurst parameter, H can be expressed as a deterministic function of time, $H = h(t)$. This generalized definition encompasses the rich class of *multifractal processes*.

The *multifractal wavelet model* (MWM) was proposed in [11]–[13] to analyze and model LRD network data. In order to avoid repetition, we defer further discussion on the MWM to Section IV, where it is employed to model residual bit-errors.

II.C Residual Bit-Error Trace Collection

In our prior studies [5]–[7], we collected MAC layer residual bit-error traces in realistic settings of an operational 802.11b LAN. The traces were collected at 2, 5.5 and 11 Mbps. We employ the same traces in this paper. Interested readers are referred to [5]–[7] for data collection details.

III. Analysis of the 11 Mbps 802.11b Residual Bit-Error Process

In this section, we first reveal the LRD present in the autocorrelation of the 11 Mbps bit-error process. The LRD notion is then substantiated using aggregation and variance-time plots.

III.A Autocorrelation of the Bit-Error Process

Let $X(n_1)$ and $X(n_2)$ be two random variables derived from the bit-error random process $\mathrm{X}(t)$. We compute the *sample autocorrelation* as

$$\rho(\eta) = \frac{\mathrm{E}\{X(0)\,X(\eta)\} - \mathrm{E}\{X(0)\}\,\mathrm{E}\{X(\eta)\}}{\sigma_{X(0)}\sigma_{X(\eta)}}, \qquad (2)$$

where $\mathrm{E}\{.\}$ is the "sample" expectation, σ_X is the "sample" standard deviation, and η is the time lag.

The sample autocorrelations of the 2, 5.5 and 11 Mbps bit-error traces are shown in Figure 1. Clearly, the 2 and 5.5 Mbps traces have a decaying correlation and, therefore, a low-order memory-length can be easily identified for these processes. Markov chains can model such processes quite adequately as substantiated in [6] and [7], which model the 2 and 5.5 Mbps bit-error processes as stationary Markov chains. On the contrary, it can be seen that the 11 Mbps bit-error process has high correlation even at large lags. This is reminiscent of LRD and hence a low-order memory-length cannot be identified for the 11 Mbps bit-error process. Thus, Markov-based models will be ineffective in modeling of this long-range-dependent data.

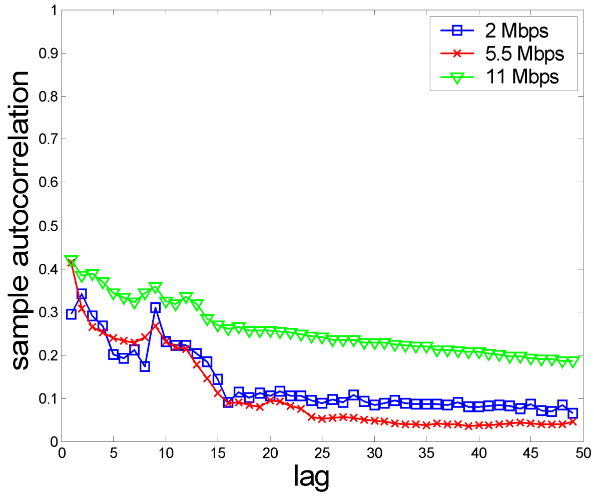

Figure 1: Sample autocorrelations of bit-error traces.

In the following section, we substantiate this preliminary notion of LRD by analyzing the 11 Mbps bit-error process in more detail. Once LRD is ascertained, a suitable model can be identified for the process. Henceforth we focus solely on the 11 Mbps data rate and the term "bit-error process" refers to the "11 Mbps residual bit-error process".

III.B Scaling Behavior of the Bit-Error Process

Since LRD processes typically demonstrate second-order self-similarity, zooming out from a sample path of the process should yield a path similar to the original in second-order statistics. In order to determine this characteristic in the bit-error process, we define an aggregate process such that

$$X^{(m)}[k] = \frac{1}{m}\sum_{i=(k-1)m+1}^{km} X[i], \qquad (3)$$

where m is the aggregation level defined such that the resulting "aggregate" sample path averages m points from the "actual" sample path. This averaging operation will impact the actual sample path in two ways: (a) the aggregate will smooth high variances in the sample path; (b) the aggregate will provide an on-average zoomed-out version of the actual sample path.

Figure 2 outlines three aggregate processes. The top figure is a process sample path $X^{(1)}[k]$ outlining the number of errors observed in each packet (packet transmission time=1 second). The second figure is a level-4 aggregate of the first sample path which depicts the average number of errors observed in 4 packets. Thus, the first point in this level-4 aggregate sample path is $X^{(4)}[1] = \frac{1}{4}\left(X^{(1)}[1] + X^{(1)}[2] + X^{(1)}[3] + X^{(1)}[4]\right)$. Similarly, aggregates at levels 8 and 16 are also shown in Figure 2.

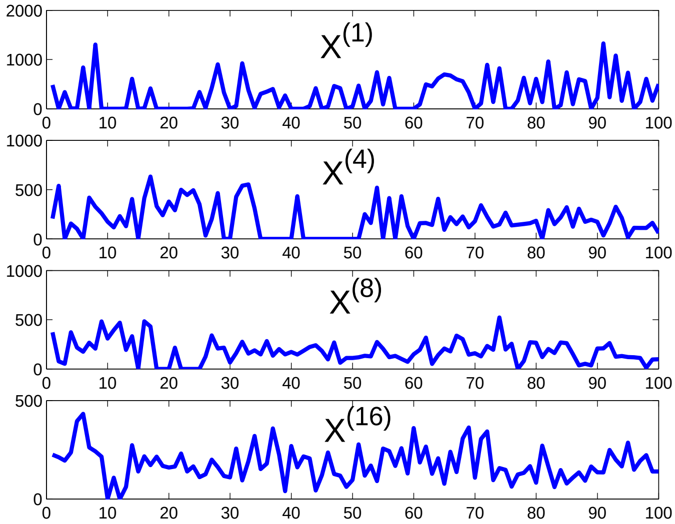

Figure 2: Distribution of errors at different time scales.

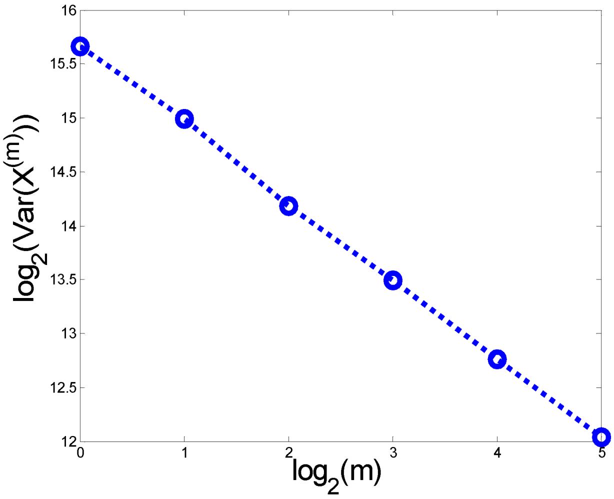

Figure 3: Variance-time plot of a sample path.

Each aggregate path is zooming out of the actual sample path and no statistically differentiating features are revealed by simple observation. Thus, it can be inferred that the decrease in variability with increased smoothing is very slow. This "slow-varying decay" is further highlighted by the analysis of second-order statistics in the following section.

III.C Second-Order Analysis

As mentioned previously, LRD processes exhibit second-order self-similarity. One method of examining second-order statistics is the *variance-time diagrams* [10]. The essence of these diagrams is to plot the logscale variance of the aggregate process as a function of the aggregation level. Second-order self-similarity is implied if the decay in the variance is strictly linear, that is, the change in variance is directly proportional to the aggregation (or scaling) level.

The variance-time plot of the bit-error process for different aggregation levels is given in Figure 3. Clearly, the variance has a linear decay with respect to the aggregation level. Further, the slope of the log-log plot is approximately -0.7, which corresponds to a Hurst parameter of $H \approx 0.65$. (Recall that process with $H > 0.5$ are long-range-dependent.) Thus, it can be strongly postulated that the process exhibits second-order self-similarity or long-range dependence.

IV. MODELING OF THE 11 MBPS 802.11B RESIDUAL BIT-ERROR PROCESS

In this section, we compare the performance of two models in characterizing the 11 Mbps bit-error process, namely the *multifractal Wavelet model* (MWM) and the *discrete-time Markov model*.

IV.A The MWM for Wireless Bit-Errors

The MWM has shown significant promise in modeling various long-range-dependent network phenomena [11]-[13]. The MWM relies on the premise that network data is inherently positive and generally spiky. Both these properties are clearly true for wireless bit-error data. Moreover, the non-stationarity and scaling properties of wireless bit-errors can be adequately characterized by wavelet-based analysis. Thus, the MWM achieves two basic objectives here: (a) provides a model for analysis and synthesis of bit-errors; and (b) captures the multiscale nature of the high-bitrate residual channel.

The MWM employs Haar wavelet family since the positivity constraint is greatly simplified if Haar wavelets are used. The scaling and wavelet coefficients are computed recursively as,

$$U_{j+1,2k} = \frac{1}{\sqrt{2}} \left(U_{j,k} + W_{j,k} \right) \tag{4}$$

and

$$U_{j+1,2k+1} = \frac{1}{\sqrt{2}} \left(U_{j,k} - W_{j,k} \right), \tag{5}$$

where $U_{j,k}$ and $W_{j,k}$ respectively represent the scaling and wavelet coefficients at time k and scale/level j.

Applying the positivity constraint yields [11]

$$W_{j,k} = A_{j,k} U_{j,k}, \tag{6}$$

where $A_{j,k}$ is a random variable defined over the interval$[-1, 1]$. In order to train the MWM to match the wireless bit-error traces, two random variables should be captured. The first random variable is the scaling coefficient at the coarsest scale U_{j_0,k_0}. The second set of random variables is the $A_{j,k}$'s at each level which in turn yield the wavelet coefficients $W_{j,k}$'s at that level. Once a general sense of probability distribution is ascertained for these random variables, the expectation maximization algorithm [14] can be used to fit that distribution to the actual dataset.

Parametric models are suggested for the modeling of U_{j_0,k_0}, while β and point-mass distributions have shown promise in approximating the $A_{j,k}$'s [12]. The training and synthesis algorithm is given in [11]. The complexity of synthesizing a length N trace using the MWM is $O(N)$, i.e., once trained the MWM has linear complexity. All results provided in this paper employ the β distribution and the MWM toolbox [15].

IV.B Performance in Capturing the Error Distribution

Number of errors in a packet was used to define a probability distribution which was in turn employed for training and

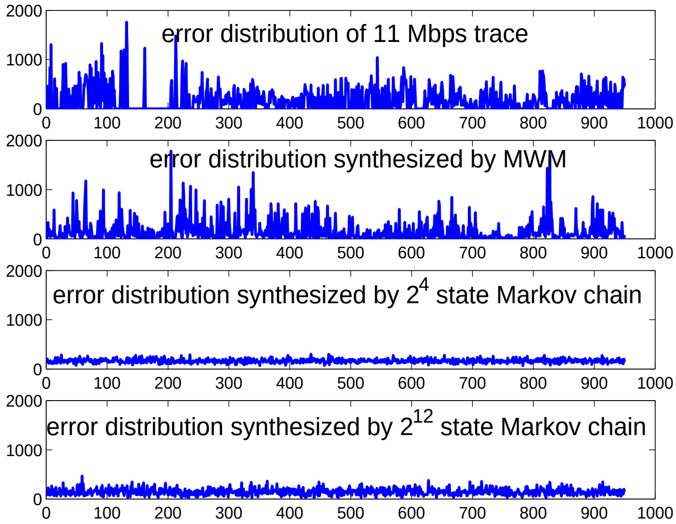

Figure 4: Actual and synthesized error distributions.

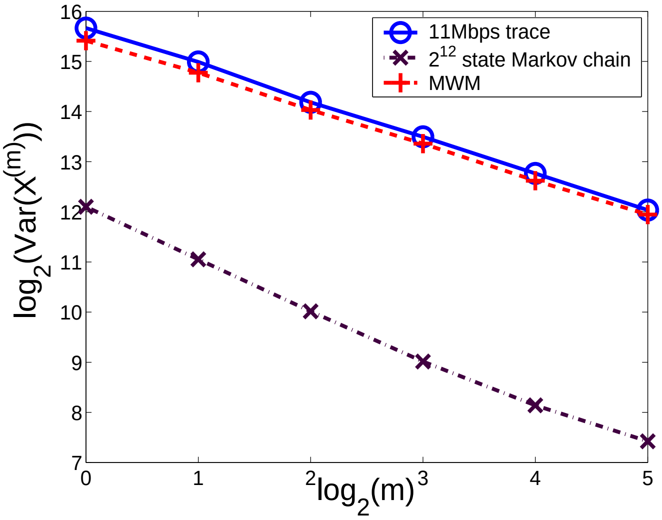

Figure 5: Comparison of variance-time plots.

synthesis. The actual trace-based distribution and the synthesized distributions using the MWM, the order-4 and the order-12 Markov chains are given in Figure 4.

It can be clearly observed that even high-order (exponential-complexity) Markov chains are not providing good estimates of the channel whereas the MWM is following the channel behavior quite closely. This result is congruent with our analysis which asserted that the process has LRD and therefore Markov-based models are inappropriate to capture its behavior. Moreover, and again in accordance with preceding discussions, the MWM renders an accurate and low-complexity model of the high-bitrate residual bit-error channel.

IV.C Performance in Capturing Second-Order Statistics

Since long-range-dependent processes are second-order self-similar, the variance characteristics of the two models (i.e., MWM and Markov chain) should be studied. Figure 5 provides a comparison between the variance-time plots of two synthesized distributions and the actual distribution. It is quite obvious from the variance-time plots that the exponential-complexity Markov chains are unable to match the second-order statistics of the process. On the other hand, the linear-complexity MWM captures the variance characteristics very accurately.

V. CONCLUSIONS

In this paper, long-range dependence in an 11 Mbps residual 802.11b channel was revealed. A multifractal wavelet model was proposed for the residual bit-error process. It was demonstrated that due to its long-range-dependent nature, traditional Markov chains are unable to model the bit-error process. However, the MWM accurately characterizes the non-stationarity and LRD characteristics of the bit-error process.

References

[1] ISO/IEC JTC 1/SC29/WG11 and ITU-T SG16 Q.6, “Draft ITU-T Recommendation and Final Draft International Standard of Joint Video Specification (ITU-T Rec. H.264 | ISO/IEC 14496-10 AVC),” Doc. JVT-G050, March 2003.

[2] ISO/IEC JTC 1/SC29/WG11, “Text of ISO/IEC 14496-2:2001 (Unifying N2502, N3301, N3056, and N3664,” Doc. N4350, July 2001.

[3] L. Larzon, M. Degermark, and S. Pink, “UDP Lite for Real Time Multimedia Applications,” *IEEE ICC*, June 1999.

[4] H. Zheng and J. Boyce, “An Improved UDP Protocol for Video Transmission Over Internet-to-Wireless Networks,” *IEEE Transactions on Multimedia*, (3)3, pp. 356–365, 2001.

[5] S. Khayam, S. Karande, H. Radha, and D. Loguinov, “Performance Analysis and Modeling of Errors and Losses over 802.11b LANs for High-Bitrate Real-time Multimedia,” *Signal Processing: Image Communication*, 18(7), pp. 575–595, 2003.

[6] S. Khayam and H. Radha, “Linear-Complexity Models for Wireless MAC-to-MAC Channels,” *ACM Wireless Networks Journal (WINET)*, 11(5), September 2005.

[7] S. Khayam and H. Radha, “Markov-based Modeling of Wireless Local Area Networks,” *ACM MSWiM*, September 2003.

[8] M. Zorzi and R. R. Rao, “On The Statistics Of Block Errors In Bursty Channel,” *IEEE Trans. On Vehicular Technology*, 46(2), pp. 445–455, 1997.

[9] A. Konrad, B. Y. Zhao, and A. D. Joseph, “A Markov-based Channel Model Algorithm for Wireless Networks,” *ACM Wireless Networks Journal*, 9(3), pp. 189–199, 2003.

[10] D. Cox, *Long-Range Dependence: A Review*, Statistics: An Appraisal, pp. 55–74, 1984.

[11] R. Riedi, M. Crouse, V. Ribeiro, and R. Baraniuk, “A Multifractal Wavelet Model with Application to Network Traffic,” *IEEE Trans. on Information Theory*, 45(3), pp. 992–1018, 1999.

[12] P. Arby, R. Baraniuk, P. Flandrin, R. Riedi, and D. Veitch, “Multiscale Nature of Network Traffic,” *IEEE Signal Processing Magazine*, 19(3), pp. 28–46, May 2002.

[13] V. Ribeiro, R. Riedi, and R. Baraniuk, “Wavelets and Multifractals for Network Traffic Modeling and Inference,” *IEEE ICASSP*, May 2001.

[14] A. P. Dempster, N. M. Laird, and D. B. Rubin, “Maximum-Likelihood from Incomplete Data via the EM Algorithm,” *Journal of Royal Statistics Society Series*, vol. 39, 1977.

[15] Multifractal Wavelet Model Toolbox, http://www-dsp.rice.edu/software/mwm.shtml.