ABSTRACT

The effect of buoyancy on the wake instability of a heated circular cylinder in a contra flow arrangement is investigated experimentally in the present study. A novel optical diagnostic technique, Molecular Tagging Velocimetry and Thermometry (MTV&T), was used for qualitative flow visualization and quantitative simultaneous measurements of velocity and temperature fields in the wake of the heated cylinder. The experiment was conducted in a water channel with the temperature and Reynolds number of the approaching flow being held constant at $T_w = 24^\circ C$ and $Re = \rho U_{\infty} D / \mu = 130$. The temperature of the heated cylinder varied between $24^\circ C$ (unheated cylinder) and $85^\circ C$, corresponding to a Richardson number (Ri) varying between zero (unheated) and unity. The heat transfer process around the heated cylinder changes from the forced convection regime to the mixed convection regime. With increasing Richardson number, significant modifications of the wake instability were revealed from both qualitative flow visualization images and quantitative simultaneous velocity and temperature fields. The effect of buoyancy on the wake instability of the heated cylinder was discussed in the terms of vortex shedding pattern, vortex shedding frequency, turbulent heat flux distribution, the wake closure length and averaged Nusselt number of the heated cylinder.

INTRODUCTION

An understanding of the flow around a bluff body is of great importance due to its fundamental nature as well as many related engineering applications. A circular cylinder is a most commonly studied bluff body. Despite of its simple shape, a circular cylinder generates a wake that is dynamically complex. By varying the Reynolds number, a variety of flow patterns and vortex shedding characteristics in the wake of a circular cylinder have already been observed and discussed in literature.

The wake behavior behind a heated circular cylinder is physically more complicated due to the buoyancy effect added to the viscous phenomena. The heat transfer from the surface of a heated cylinder to surrounding fluids can be either pure forced convection, free convection or mixed convection depending on the Richardson number ($Ri = \frac{Gr}{Re^2}$) of the case. In the forced convection ($Ri < 1$) regime, where the effect of buoyancy is neglected, the heat transfer is a function of Reynolds (Re) number and the Prandtl (Pr) number. In free convection ($Ri > 1$) regime, where the forced convection is negligible, the heat transfer is a function of the Grashof (Gr) number and the Prandtl (Pr = $\nu / \alpha$) number. In the mixed convection regime, both forced convection and free convection are important, and the heat transfer is a function of the Grashof number ($Gr = g\beta(T_w - T_\infty)D^3 / \nu^3$), the Reynolds Number (Re), the Prandtl number and the forced flow direction. Despite the fact of that mixed convection around bluff bodies is of great importance for various engineering applications (for example, electronics cooling, compact heat exchangers, etc.), the thermal effect (buoyancy effect) on the wake stability of a bluff body in mixed convection regime has received little attention compared to that in forced convection regime.

When a heated cylinder operates in mixed convection regime, buoyancy force plays an important role to the flow pattern in the wake of a heated cylinder. As shown in Fig. 1, for a horizontal heated cylinder, the forced flow approaching to the heated cylinder can be either horizontal, parallel (vertically upward) or contra (vertically down) depending on the angle between the buoyancy force acting on the fluid around the heated cylinder and the approaching flow direction.
Most of the earlier studies about heated cylinders in mixed convection regime were focused on the determination of the global effect of thermal induced flows on the heat transfer coefficients compared with those in forced convection regime. It turned out that the heat transfer coefficient was influenced considerably by the thermal induced flow when the Richardson number is larger than a critical value. It was also found that the value of the critical Richardson number changes with the angle between the buoyancy force and the approaching forced flow direction. Oosthuizen and Madan [1,2] studied a heated circular cylinder placed in a horizontal, parallel and contra flow of air as well as 135 degrees with respect to gravity force. They gave the following critical Richardson numbers, and suggested that above these critical Richardson numbers the heat convention would transit from forced convection to mixed convection.

- Parallel flow: $Ri > 0.10$
- Horizontal flow: $Ri > 0.53$
- 135 degrees: $Ri > 0.04$
- Contra flow: $Ri > 0.01$

More detailed studies about the effect of buoyancy on the vortex shedding in the wake of a heated cylinder in mixed convection regime were carried out slightly later, and some remarkable results were obtained. Noto and Matsumoto [3] studied the wake flow of a horizontal heated cylinder placed in a parallel flow, where the direction of the approaching forced flow is vertically upward. By increasing the heat input (increasing $Ri$), the Strouhal number of the vortex shedding was found to increase at first. However, above a critical Richardson number, the Strouhal number became zero abruptly and the vortex shedding is suppressed. The “Karman” vortex street at the downstream of the heated cylinder converts to two steady vortices in the near wake. A further increasing of the heat input (increasing $Ri$) causes the twin vortices to disappear, ending with a thermal plume at the downstream of the heated cylinder. The influences of buoyancy force on the vortex shedding frequency were further analyzed numerically by Chang and Sa [4]. The same results were also reported in the experimental study of Michaux-Leblond and Belorgey [5], and the dualism between buoyancy and viscous effects were considered to be responsible for the abrupt disappearance of the vortex shedding phenomenon for a heated cylinder exposed in a parallel flow.

More recently, Kieft et al. [6] and Steenhoven and Rindt [7] studied the effect of buoyancy on the wake instability of a heated cylinder exposed in a horizontal cross flow. In their studies, the direction of the approaching forced flows is horizontal, which is perpendicular to the direction of the buoyancy force acting on the fluid surrounding the heated cylinder. They found that the non-parallelism between the horizontal forced flow and the vertical buoyancy force caused the flow pattern to become asymmetrical with the strength of the vortices in the upper vortex row being different from those in the lower row. The strength difference of the shedding vortex increases with the increasing Richardson number. As a result, the shedding vortices are found to move slightly downward, which is a rather unexpected behaviors in considering the upward action of buoyancy force. Using an electrochemical tin precipitation method, Maas et al. [8] visualized the wake structures behind a heated circular cylinder placed in a horizontal cross flow. They found that the wake become three-dimensional when $Ri>0.3$. For small Richardson values ($Ri<1.0$), the warm fluid is initially collected in the coherent vortex structures shedding from the heated cylinder. Thermal plumes were found to originate from these vortex structures at further downstream. For larger Richardson values ($Ri>1.0$), the upward buoyancy force was found to prevent the formation of the coherent vortex structures and the wake structures were found to transit to be three-dimensional already at the rear end of the heated cylinder.

To the best of author’s knowledge, there is no literature available to report a detailed experimental study about the effect of buoyancy on the wake instability of a heated cylinder exposed in a contra flow, where the direction of the approaching forced flow is opposing to the direction of buoyancy force acting on the fluid around the heated cylinder. So far, only the numerical study of Chang and Sa [4] may be considered to involve this topic. Instead of studying a heated horizontal cylinder in a vertically downward forced flow, Chang and Sa [4] presented their numerical results (streaklines, streamlines and iso-thermal contours) of a cooled circular cylinder in a vertically upward forced air, where the direction of the approaching forced flow is reverse to the buoyancy force acting on the fluid around the cooled circular cylinder.

In the present study, the effect of buoyancy on the wake instability of a heated cylinder exposed in a contra flow is investigated experimentally. A novel optical diagnostic technique, Molecular Tagging Velocimetry and Thermometry (MTV&T), is used for qualitative flow visualization and quantitative simultaneous measurements of velocity and temperature fields in the wake of the heated cylinder. The effect of buoyancy on the wake instability of the heated cylinder is discussed in the terms of vortex shedding pattern, Strouhal number of vortex shedding ($St$), the distributions of turbulent heat flux (correlation terms between turbulent velocity and temperature), the wake closure length ($l_c$) and averaged Nusselt number (Nu) of the heated cylinder.
MOLECULAR TAGGING VELOCIMETRY AND THERMOMETRY TECHNIQUE

The Molecular Tagging Velocimetry and Thermometry (MTV&T) technique used in the present study is a novel molecule-based flow diagnostic technique. Instead of using particle tracers, MTV&T technique utilizes specially-designed molecules, which can be turned into long lifetime tracers upon excitation by photons of an appropriate wavelength, as tracers for simultaneous velocity and temperature measurements.

The specially-designed tracer molecules used in the present study are the supramolecules of phosphorescent triplex (1-BrNp-β-CD-ROH). The phosphorescent triplex is actually the mixture of three different chemicals, which are lumophore (indicated collectively by 1-BrNp), glucosyl-β-cyclodextrin (indicated collectively by Gβ-CD) and alcohols (indicated collectively by ROH). In the present study, we used a concentration of $2 \times 10^{-4}$ M for Gβ-CD, a saturated (approximately $1 \times 10^{-5}$ M) solution of 1-BrNp and a concentration of 0.06 M for the cyclohexanol (ROH), as suggested by Gendrich et al. [9].

Flow Velocity Distribution Measurement

The MTV&T technique can be thought as the molecular counterpart of the PIV technique for the flow velocity measurement. Typically a pulsed laser is used to “tag” the regions of interest, and those regions are interrogated at two successive times within the photoluminescence emission lifetime of the molecular tracers. The tagged molecules form a “glowing net” that convects, bends, and folds as the tracer molecules move with the working fluid. The Lagrangian displacement vectors of the tagged molecules over the prescribed time delay between the two interrogations provide the estimates of flow velocity vectors.

Figure 2 shows an example of the MTV measurement of a vortex ring impacting a solid wall at normal incidence [9]. The figure shows both the initially tagged regions and their subsequent evolution after a time delay, together with the resultant velocity vector field derived from the image pair using a spatial correlation method.

![Fig. 2. Typical MTV image pairs and the resultant velocity field. (a) grid image 1 μs after the laser pulse; (b) same grid imaged 8 ms later; (c) velocity field derived from (a) and (b).](image)

Flow Temperature Distribution Measurement

According to quantum theory [10], for a diluted solution and unsaturated laser excitation, the decay of phosphorescence emission intensity follows an exponential law, and can be expressed as:

$$I_p = I_0 \cdot \Phi \cdot e^{-\frac{t}{\tau}}$$  \hspace{1cm} (1)

where $I_0$ is the local incident laser intensity, $C$ is the concentration of the phosphorescence dye, $\varepsilon$ is the absorption coefficient of the phosphorescence dye, $\Phi$ is phosphorescence quantum yield, and $\tau$ is the lifetime of the phosphorescence emission, which refers to the time when the intensity drops to 37% (i.e. $1/e$) of the initial intensity.

For some phosphorescent dyes like the phosphorescent triplex (1-BrNp-Gβ-CD-ROH) used in the present study, the phosphorescence quantum efficiency (Φ) and the lifetime of the phosphorescence emission (τ) are temperature dependent. Therefore, the phosphorescence intensity may be considered, in principle, to depend only on temperature if the excitation laser intensity is uniform and the phosphorescent dye concentration remains constant in the measurement region.

By referring phosphorescent emission intensity to temperature directly, Thomson and Maynes [11] have demonstrated the possibility for fluid temperature measurement by measuring the phosphorescence intensity of the phosphorescent triplex (1-BrNp-Gβ-CD-ROH). It should be noted that since the phosphorescence intensity is also the function of incident laser intensity, therefore, the spatial and temporal variations of the incident laser intensity would have to be corrected for separately in order to extract quantitative temperature data from phosphorescence images by using the method suggested by Thomson and Maynes [11]. However, in practice, it is usually very difficult, if not impossible, to ensure a non-varying incident laser intensity distribution, especially for unsteady flows with index of refraction variations. This may cause significant error in the temperature measurement.

In order to overcome this problem, a novel ratiometric technique, named as lifetime-based thermometry technique, has been developed recently by the authors [12, 13], which achieves the quantitative temperature mapping of a fluid flow by taking advantage of the temperature dependence of the phosphorescence lifetime of the phosphorescent triplex molecules (1-BrNp-Gβ-CD-ROH).

Figure 3 shows the calibration curve of the phosphorescence lifetime of the phosphorescent triplex (1-BrNp-Gβ-CD-ROH) changing with temperature. It can be seen that the phosphorescence lifetime of the phosphorescent triplex (1-BrNp-Gβ-CD-ROH) varies significantly with increasing temperature. The relative temperature sensitivity of the phosphorescence lifetime ranges about 5.0% per degree at 20.0°C to about 20.0% per degree at 50.0°C. The temperature sensitivity is much higher than those of most commonly used LIF dyes (for example, the temperature sensitivity of Rhodamine B is about 2.0%).
In summary, the MTV&T technique achieve the simultaneous measurements of velocity and temperature in fluid flows by using a pulsed laser to “tag” the regions of interest. The tagged regions are interrogated at two successive times within the lifetime of the photoluminescence of the tracer molecules. The measured Lagrangian displacement vector of the “tagged” molecules between the two interrogations provides the estimate of velocity vector. The simultaneous temperature measurement is achieved by taking advantage of the temperature dependence of phosphorescence lifetime, which is estimated from the phosphorescence intensity ratio of the two interrogations. Further information about the technical basis, system setup, calibration procedure, and image processing algorithm of the MTV&T technique is available at [12, 13].

**EXPERIMENTAL SETUP AND FLOW CONDITIONS**

A schematic of the experimental setup used in the present study is shown in Fig. 4. The experiment was conducted in a small vertical water channel. A constant head tank with an overflow system was used to maintain a constant flow condition in the water channel during the experiment. The aqueous solution of the phosphorescent triplex (1-BrNp-Gβ-CD-ROH) in the constant head tank was filled from a reserve tank by an electric pump. Honeycomb and mesh structures were used at the inlet of the water channel to insure a uniform flow condition at the upstream of the test cylinder. A copper tube with outer diameter of 4.76mm (D=4.74mm) and inner diameter of 4.00mm was used as the test cylinder by an electric pump. Honeycomb and mesh structures were used at the inlet of the water channel to insure a uniform flow condition at the upstream of the test cylinder. A copper tube with outer diameter of 4.76mm (D=4.74mm) and inner diameter of 4.00mm was used as the test cylinder. A rod cartridge heater (Watlow Firerod™) with the outer diameter of 3.1mm was stuffed inside the copper cylinder to obtain the desire cylinder temperature. High thermal conductivity paste (OMEGATHERM 201) was pressed in to fill the gap between the cartridge heater and the inner wall of the test cylinder. Two J-type thermocouples were embedded in the gap at mid-span of the heated cylinder. The thermocouples were connected to a two-channel thermometer (Omega HH23), which has a resolution of ±0.1°C, to monitor the temperature of the test cylinder.

A Lambda-physik XeCl Excimer UV laser (wavelength \( \lambda=308 \text{ nm} \), energy 100 mJ/pulse, pulse width 20ns) with a set of optics was used to generate illuminating grids in the measurement region to achieve qualitative flow visualization and quantitative simultaneous measurements of velocity and temperature fields in the wake of the test cylinder. A 12-bit (1280 × 1024 pixels) gated intensified CCD camera (PCO DICAM-Pro) was used in the present study to conduct image recording. The camera was operated in the dual-frame mode, where two full-frame images of phosphorescence were acquired in quick succession from the same laser excitation pulse. The laser and the camera were synchronized using a digital delay generator (SRS DDG535), which controlled the timing of the laser illumination and the CCD camera data acquisition.

In the present study, the heated cylinder was placed horizontally in the middle of the water channel, and the approaching flow is vertically downward. Such arrangement makes the heated cylinder operates in a contra flow, i.e. the direction of the approaching flow opposes the direction of the buoyancy force acting on the fluid around the heated cylinder. The velocity of the approaching flow was about \( U_\infty = 0.0255 \text{m/s} \). The temperature of the approaching flow in the constant head tank was \( T_\infty = 24.0 \text{°C} \). The corresponding Reynolds number is \( Re=130 \). During the experiment, the temperature of the test cylinder was changed at several temperature levels, which varied form 24.0 °C (unheated cylinder) to 85.0 °C. The corresponding Richardson number (\( Ri \)) changes from 0.0 to 1.05. With increasing Richardson number, the heat transfer process around the heated cylinder changes from force convection regime to mixed convection regime.
EXPERIMENTAL RESULTS AND DISCUSSIONS

1. Flow Visualization and Simultaneous Velocity and Temperature Measurement Results

Figure 5(a) and 5(b) show a typical pair of phosphorescence images for the qualitative flow visualization and simultaneous quantitative MTV&T measurement in the wake of the heated cylinder when the temperature of the heated cylinder is $35^\circ$C (Ri=0.19). The resultant velocity vectors derived from the image pair by using a cross-correlation method is shown in Fig. 5(c).

In the phosphorescence images, well defined structures in the form of “dark clusters” can be seen clearly in the wake region of the heated cylinder. The “dark clusters” are actually the warmer fluid shedding periodically from the hot boundary layer around the heated cylinder. As described above, the phosphorescence intensity of the phosphorescent triplex (1-BrNp-Gjβ-CD-ROH) is temperature sensitive, the warmer phosphorescent triplex molecules from the hot boundary around the heated cylinder served as the tracers to visualize the alternative shedding of the “Karman” vortices in the wake of the heated cylinder. Since the phosphorescence intensity of the warmer phosphorescent triplex molecules decays faster than that in ambient fluid, the “dark clusters” are revealed more clearly in the second phosphorescence image than those in the first image.

In order to construct simultaneous temperature field from the phosphorescence images, small interrogation windows were chosen in the first phosphorescence image. The corresponding positions of the interrogation windows in the second phosphorescence image were determined based on the measured velocity vectors as shown in Fig. 5(c). The phosphorescence lifetime of the “tagged” phosphorescent triplex molecules contained in each interrogation window was calculated based on the intensity ratio of the interrogation windows in the first and the second phosphorescence images [12, 13]. Then, the simultaneous temperature field was constructed by using the calibration curve of phosphorescence lifetime versus temperature shown in Fig. 3. The resultant temperature distribution extracted from the phosphorescence image pair is shown in Fig. 5(d).

Corresponding to the “dark clusters” in the phosphorescence images, two sets of “warm blobs” at the two sides of the heated cylinder can be seen clearly in the temperature distribution, which are generated by the periodical shedding of the warmer fluids from the hot boundary layer around the heated cylinder. The “warm blobs” and the heated cylinder were connected by “braids” at the two sides of the heated cylinder. The “warm blobs” were found to grow in size as they traveled downstream. It should also be noted that even though the difference of the fluid temperature in the wake of the heated cylinder is quite small for this case (only about 2.0 °C), the shedding of the warmer fluid in the form of “warm blobs” can still be revealed very clearly in the simultaneous temperate distribution by using the present MTV&T technique.

Based on 360 frames of instantaneous measurement results, the ensemble-averaged velocity and temperature distributions in the wake of the heated cylinder were calculated, and the result are shown in Fig. 5(e) and Fig. 5(f). A recirculation zone at the downstream of the heated cylinder with a length of about 3 diameters is revealed clearly in the ensemble-averaged velocity field. The ensemble-averaged temperature field reveals that high-temperature regions exist at the two sides of the wake region, which are corresponding to the shedding paths of the “Karman” vortices and “warm blobs” revealed in the instantaneous measurement results.

Figure 6 shows the phosphorescence images and the corresponding MTV&T measurement results in the wake of the heated cylinder when the temperature of the heated cylinder increased to 53 °C (Ri = 0.50). Compared with the case of smaller Richardson number (Ri=0.19) described earlier, the shedding process of the “Karman” vortices and “warm bulbs” in the wake was found to change significantly due to the stronger buoyancy effect. Although the shedding of the “Karman” vortices and the “warm blobs” still occur alternatively at the two sides of the heated cylinder, the shedding process was found to be “delayed”, and the locations of the first appearance of the concentrated “Karman” vortices and the “warm blobs” was pushed farther downstream. The “braids”, which are the connection between the “warm blobs” and the heated cylinder, were found to become much longer and thicker. The ensemble-averaged velocity and temperature distributions revealed that the recirculation zone at the downstream of the heated cylinder become much wider and longer compared that of the Ri=0.19 case.

Figure 7 shows a pair of typical phosphorescence images and the resultant MTV&T measurement results in the wake of the heated cylinder when the temperature of the heated cylinder increased to 85.0 °C (Ri = 1.05). The flow pattern and the vortex shedding in the wake of the heated cylinder for this case were found to be very different from the cases described before. Rather than having alternative shedding of vortical structures or “warm blobs”, the shedding of the vortical structures or “warm blobs” were found to occur almost concurrently at the two sides of the heated cylinder. The sizes of these vortical structures were found to be smaller than that of the cases with smaller Richardson number, and they behaved more like “Kelvin-Helmholtz” type structures rather than “Karman” type vortical structures. The adjacent smaller vortical structures or “warm blobs” at the same side were found to merge together to form bigger vortical structures or larger “warm blobs” at further downstream. The merge process is quite similar to the “pairing process” of “Kelvin-Helmholtz” vortical structures. The merge processes of the smaller vortical structures and “warm blobs” were found to occur alternatively at the two side of the heated cylinder, which resulted in the alternative shedding of bigger vortical structures and larger “warm blobs” at far field.

The ensemble-averaged velocity and temperature distributions revealed that the wake region of the heated cylinder becomes even wider and longer at this Richardson number (Ri=1.05). The recirculation zone downstream of the heated cylinder became so long that its length was about 10 cylinder diameters.
a. first image (1ms after laser pulse)

b. second image (6ms after laser pulse)

c. instantaneous velocity field

d. simultaneous temperature field

e. averaged velocity (U, V)

f. averaged temperature(T)

Fig. 5 MTV&T measurement results ($T_0 = 35.0 \degree C$, Ri=0.19)
a. first image (1ms after laser pulse)
b. second image (6ms after laser pulse)

c. instantaneous velocity field
d. simultaneous temperature field

e. averaged velocity (U, V)
f. averaged temperature (T)

Fig. 6 MTV&T measurement results ($T_e = 53.0 \degree C$, $\text{Re}=0.50$)
Fig. 7 MTV&T measurement results ($T_e = 85.0 \degree C$, Ri=1.05)
2. Turbulent Heat Flux
Since the velocity and temperature fields were measured simultaneously by using the MTV&T technique, turbulent heat fluxes, which are the correlation terms between the velocity and temperature fluctuations, could be calculated based on the simultaneously measured velocity and temperature distributions. Figure 8 shows the distributions of the mean turbulent heat flux ($u_j' T'$) vectors in the wake of the heated cylinder. The distributions of the mean turbulent heat flux vectors indicate how the heat fluxes were propagated in the wake of the heated cylinder. The effect of buoyancy on the heat transportation process in the wake of the heated cylinder is also revealed clearly from the comparison of the turbulent heat flux vector distributions. As shown in Fig. 8, the structure of $u_j' T'$ in the near wake is dominated by two large counter-rotating vortex-like structures, which is almost symmetrical about the wake centerline. As the Richardson number increasing, the counter-rotating vortex-like structures were found to move farther downstream, which are corresponding to the delayed appearances of the “Karman” vortex structures and “warm blobs” revealed in the measured velocity and temperature distributions. Therefore, the structures are believed to be the result of the large entrainment of the cold free stream fluid by the shedding of “Karman” vortices in the wake behind the heated cylinder.

3. The Wake Closure Length ($l_c$)
Figure 9 shows the ensemble-averaged velocity profiles along the wake centerline at difference Richardson number levels. Since the flow condition of the approaching forced flow is kept in constant during the experiment, all the velocity profiles are found to collapse to one single curve at the upstream of the test cylinder as it is expected. Due to the effect of buoyancy, the velocity profiles were found to vary significantly at the downstream of the test cylinder.

As a typical behavior of the wake behind an unheated cylinder, the present MTV&T measurement result of unheated cylinder case (Ri=0.0) reveals that the streamwise velocity of the wake flow becomes negative at the near downstream of the test cylinder. The negative streamwise velocity reaches it peak value at the downstream location of $X/D \approx 2.2$, then the velocity deficit is found to recover gradually as the downstream distance increasing. The value of the streamwise velocity is found to change its sign and become positive again at the downstream locations of $X/D > 3.5$.

For the heated cylinder cases, the existence of sharp negative velocity peaks near the rear end of the heated cylinder indicates that there is a strong reversing flow zone near the rear end of the heated cylinder. In the reversing flow zone, the flow direction is upward. The upward reversing velocity was found to become stronger and stronger as the Richardson number increasing. It may be explained by that since the fluid around the heated cylinder was heated up, the warmer fluid near the trailing edge of the heated cylinder were pushed upwardly due to the effect of buoyancy force. As the temperature of the heated cylinder increasing (i.e. Richardson number increasing), the buoyancy force become stronger and stronger, which results in a stronger and stronger upward reversing flow near the rear end of the heated cylinder.

When the Richardson number is relatively small (Ri<0.31), the streamwise velocity far away from the heated cylinder ($X/D>1.5$) are found to behave very similarly as that in the wake of an unheated cylinder (Ri=0.0 case). The velocity deficits were found to recover gradually as the downstream distance increasing. However, when the Richardson number becoming big enough (Ri>0.5), the streamwise velocity profiles at downstream far away from the heated cylinder ($X/D>1.5$) were found to behave very different from that in the wake of an unheated cylinder (Ri=0.0 case). As shown in the ensemble-averaged velocity distributions shown in Fig. 6 and Fig. 7, a “dead flow” zone (in which the streamwise velocity being almost zero) was found in the wake of the heated cylinder.
Another strong recirculation zone exists at the downstream of the “dead flow” zone. As the Richardson number increasing, the strong recirculation zone was found to be pushed to farther downstream. The recovery rate of the streamwise velocity deficit was found to become slower and slower as the Richardson number increasing.

4. The Strouhal Number of the Vortex Shedding

Based on the time sequence of the instantaneous MTV&T measurement results, the shedding frequency of the “Karman” vortex structures in the wake of the heated cylinder can be obtained, and the result is given in Fig. 11. In the present study, the horizontal heated cylinder is placed in a vertically downward flow, i.e., in a contra flow. According to the results shown in Figure 11, the Strouhal number \( St = f D / U_\infty \) of the vortex shedding in the wake of the heated cylinder is found to decrease with the increasing Richardson number. The numerical results of Chang and Sa [4] for a cooled cylinder placed in a contra air flow at the condition of \( Re = 100 \) is also shown in the figure. It can be seen that both the present experimental data and the numerical simulation results of Chang and Sa [4] revealed the same decreasing trend of the Strouhal number of the vortex shedding with the increasing Richardson number.

5. Averaged Nusselt Number (\( Nu \))

In the present study, the voltage supplied to the cartridge heater and the strength of the electrical current in the circuit were measured during the experiment. Considering the power losses through end pieces of the cartridge heater, 90% of the total electrical power supplied to the heater is assumed to be dissipated by the approaching flow inside the water channel. The averaged Nusselt numbers \( \bar{Nu} \) of the heated cylinder at
different Richardson number levels can be calculated by using following equation:

\[ \frac{Nu}{hD} = \frac{q}{k_w} = \frac{A(T_w - T_\infty)}{k_w} = \frac{\pi D(T_w - T_\infty)}{k_w} \]  

(2)

where the \(q\) is heat flux dissipated by the approaching forced flow, \(L\) is the spanwise length of the cartridge heater inside the water channel, \(k_w\) is the thermal conductivity of the approaching flow, \(T_w\) and \(T_\infty\) are the temperature of the heated cylinder and temperature of the approaching flow respectively.

Figure 12 shows the experimental results of the present study. The predictions of two most commonly-used empirical models (Hilpert’s model [14] and Morgan’s model [15]) at the present experimental conditions are also given in the figure.

![Fig. 12. Nusselt number (Nu.) vs. Richardson Number (Ri).](image)

The experimental data of the present study shown in Fig. 12 reveal that the averaged Nusselt number of the heated cylinder decreases almost linearly with the increasing of the Richardson number when a heated cylinder placed in a contra flow. Compared with the present experimental data, the prediction of Hilpert’s model [14] is found to overestimate the averaged Nusselt number. The difference between the present experimental data and the prediction of Hilpert’s model [14] becomes bigger and bigger as the Richardson number increasing. Instead of predicting a decreasing trend of the averaged Nusselt number, the Hilpert’s model [14] predicts a increasing. Therefore, the difference between the prediction of Hilpert’s model [14] and the present experimental data becomes bigger and bigger as the Richardson number increasing.

With the contribution of the free convection being taken into account, Morgan’s model [15] predicts that the averaged Nusselt number of the heated cylinder decreases with the increasing Richardson number, which is consistent with the present experiment data qualitatively. When the Richardson number is relatively small (\(Ri<0.31\)), the prediction of Morgan’s model [15] were found to agree with the present experimental data reasonably well. The Morgan’s model was found to underestimate the averaged Nusselt number when the Richardson number becoming relative big (\(Ri>0.31\)). The similar result was also reported by Krauce and Tareiuk [16] in their study of the heat transfer process from a heated cylinder placed in a contra air flow.

**CONCLUSIONS**

The effect of buoyancy on the wake instability of a heated circular cylinder in a contra flow arrangement is investigated experimentally in the present study. A novel optical diagnostic technique, Molecular Tagging Velocimetry and Thermometry (MTV&T), was used for qualitative flow visualization and quantitative simultaneous measurements of velocity and temperature fields in the wake of the heated cylinder. The experiment was conducted in a water channel with the temperature and Reynolds number of the approaching forced flow being held constant at \(T_\infty=24^\circ\text{C}\) and \(Re=130\). The temperature of the heated cylinder varied between \(24^\circ\text{C}\) (unheated cylinder) and \(84^\circ\text{C}\), corresponding to a Richardson number \((Ri)\) varying between zero (unheated) and unity. The heat transfer process around the heated cylinder changes from the forced convection regime to the mixed convection regime.

Due to the effect of buoyancy force, the wake instability behind the heated circular cylinder was found to become much more complicated compared with that behind an unheated cylinder. With the increasing Richardson number, significant modifications of the wake instability were revealed from both qualitative flow visualization images and quantitative simultaneous velocity and temperature fields. Following facts were found as the results of the present experimental study:

1. When the Richardson number is relatively small (\(Ri<0.31\)), the flow pattern and the vortex shedding mechanism in the wake of the heated cylinder is found to be similar as those in the wake of an unheated cylinder.

2. When the Richardson number becoming big enough (\(Ri>0.31\)), the shedding of the “Karman” vortices in the wake of the heated cylinder is found to change significantly due to the strong buoyancy effect. Compared with that in the wake of an unheated cylinder, the “Karman” vortex shedding process was found to be delayed and occurred at much farther downstream.

3. As the Richardson number approaching unity (\(Ri>0.72\)), the effect of buoyancy become so strong that the alternative shedding of the “Karman” vortices was suppressed in the near field of the wake flow. Smaller vortices generated almost
concurrently were found to shed at the two sides of the heated cylinder. These smaller vortices behave more like “Kelvin-Hemholz”-typed vortex structures rather than “Karman”-typed vortices.

4. Within the Richardson number range investigated (0≤Ri≤1.05), the shedding frequency of the “Karman” vortex structures in the wake of the heated cylinder was found to decrease with the Richardson number increasing.

5. When the Richardson number being relatively small (Ri<0.31), the wake closure length (lC) of a heated cylinder is found to be shorter than that of an unheated cylinder. When the Richardson number becoming big enough (Ri>0.31), the wake closure length (lC) become longer and longer as the Richardson number increasing.

6. The average Nusselt number (Nu) of the heated cylinder was found to decrease almost linearly with the increasing Richardson number when the heated cylinder is placed in the contra flow.

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