Integral Equations in Computational Electromagnetics: Formulations, Properties and Isogeometric Analysis

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Abstract

Computational electromagnetics (CEM) provides numerical methods to simulate electromagnetic waves interacting with its environment. Boundary integral equation (BIE) based methods, that solve the governing Maxwell’s equations in the homogeneous or piecewise homogeneous medium, are both efficient and accurate, especially for scattering and radiation problems. The study on electromagnetic BIEs has been very active in the CEM research community; however, there are still many open problems that need to be addressed or further studied. A short list of them include (1) closed-form or quasi-analytical solutions to time-domain integral equations, (2) catastrophic cancellations at low frequencies, (3) ill-conditioning due to high mesh density, multi-scale discretization, and growing electrical size, and (4) lack of flexibility due to re-meshing when increasing number of forward numerical simulations are involved in the electromagnetic design process. This dissertation will address those several aspects of boundary integral equations in computational electromagnetics.

The first contribution of the dissertation is to construct quasi-analytical solutions to time-dependent boundary integral equations using a direct approach. Direct inverse Fourier transform of the time-harmonic solutions cannot be carried out stably due to the non-existence of the inverse Fourier transform of spherical Hankel functions. With the derived addition theorems for the time-domain Green’s function and dyadic Green’s functions, time-domain integral equations governing transient scattering problems of spherical objects are solved directly and stably for the first time. Besides, direct time-dependent solutions, together with the newly proposed time-domain dyadic Green’s functions, can enrich the time-domain spherical multipole theory.

The second contribution is to create a novel method of moments (MoM) framework to solve electromagnetic boundary integral equation on subdivision surfaces. The aim is to avoid the meshing and re-meshing stages to accelerate the design process when the geometry needs to be updated.
Two schemes to construct basis functions on the subdivision surface have been explored. One is to use the div-conforming basis function, and the other one is to create a rigorous iso-geometric approach based on the subdivision basis function with better smoothness properties. This new framework provides us better accuracy, more stability and high flexibility.

The third contribution is to propose new stable formulations to avoid catastrophic cancellations due to low-frequency breakdown or dense-mesh breakdown. Many of the conventional integral equations and their associated post-processing operations suffer from numerical catastrophic cancellations, which can lead to ill-conditioning of the linear systems or serious accuracy problems. Examples include low-frequency breakdown and dense mesh breakdown. Another instability may come from nontrivial null spaces of involving integral operators that might be related with spurious resonance or topology breakdown. This dissertation presents several sets of new boundary integral equations and studies their analytical properties. The first proposed formulation leads to the scalar boundary integral equations where only scalar unknowns are involved. Besides the requirements of gaining more stability and better conditioning in the resulting linear systems, multi-physics simulation is another driving force for new formulations. Scalar and vector potentials (rather than electromagnetic field) based formulation have been studied for this purpose.

Those new contributions focus on different stages of boundary integral equations in an almost independent manner, e.g. isogeometric analysis framework can be used to solve different boundary integral equations, and the time-dependent solutions to integral equations from different formulations can be achieved through the same methodology proposed.

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