Complex orthogonal decomposition (COD) is applied to an experimental beam to extract the dispersive wave properties using response measurements. The beam is made of steel and is rectangular with a constant cross section. One end of the beam is free and is hung by a soft elastic cord. An impulse is applied to the free end. The other end is buried in sand to absorb the wave as it travels from the impact site on the free-end; this effectivelly prevents reflections of the wave off the buried end and emulates a semi-infinite beam. COD is applied to the complex analytic displacement ensemble to obtain complex modal vectors and associated complex modal coordinates. The spatial whirl rates of nearly harmonic modal vectors are used to extract the modal wave numbers, and the temporal whirl rates of the modal coordinates are used to estimate the modal frequencies. The relationship between the frequencies and wave numbers are used to describe the dispersion relationships, which are compared favorably to those of the theoretical infinite Euler-Bernoulli beam. Further efforts were applied and a new methodology was developed.

A novel method called smooth complex orthogonal decomposition (SCOD) was applied to a simulated semi-infinite beam and to the above instrumented experimental beam. These measurements were converted into complex analytic displacements and velocities ensembles, which were used to compute two correlation matrices. These correlation matrices formed a complex generalized eigenvalue problem (GEVP) whose eigenvalues and eigenvectors led to the extractions of the frequencies and wavenumbers of the constituent waves of the traveling pulse. SCOD directly extracts the frequencies of the traveling waves from the eigenvalues. SCOD is able to extract the geometric relationship, phase velocity, and group velocity and agrees with analytical predictions. Applications of both methods were applied to a simulated infinite mass chain.

The dispersion relationship of a discrete chain of masses is extracted from numerically simulated data by applying complex modal decomposition. When an impulse excitation is applied to one end of a semi-infinite mass-spring chain, a wave is generated and propagates down the chain. This wave consists of various modes. The time record for the generated data is limited such that the wave reflection does not return to the “sensed” masses. For example, a 250-mass chain is simulated, and we consider (or sensed) the time record of the first 100 masses. The data collected from the numerical simulation consists of the displacements of each mass at each time step. This data is then used to extract complex modes using COD and SCOD. The extracted complex modes accommodate modal traveling waves. We then compute the frequencies and wave numbers from modal coordinates and mode shapes, respectively. The amplitudes and frequencies of the modes are also estimated using Rayleigh’s quotients. The COD extracted dispersion relationship matched the analytical prediction of the dispersion curve for the linear mass chain.