Current research on nonlinear structures sparks interest in building innovative smart materials, metamaterials, energy harvesters, and energy dissipative structures. Researchers have made significant advancements in developing materials that are engineered to have special properties. Nonlinearity plays an integral role in designing these types of materials. Understanding the dynamics of the structures that can transform from one stable configuration to another is of particular interest. Multi-stable structures can be used for various applications such as energy harvesting, energy dissipation, vibration absorption, vibration isolation, targeted energy transfer, bandgap design and metamaterials.

In this dissertation, we first explore the dynamics of a simple Duffing oscillator for its snap-through orbits around the separatrix. Then the bifurcations of equilibria are studied for a generalized two-degree-of-freedom (2-DOF) snap-through oscillator and its chaotic nature is shown using numerically computed fractal basin boundaries. These oscillators are called twinkling oscillators because they convert the low-frequency inputs into high-frequency oscillations. The twinkling oscillator has shown to exhibit rich transient and steady-state dynamics. The steady-state bifurcation analysis uncovered two unique bifurcations “star” and “eclipse” bifurcations, named due to their structures. An unpredictable steady-state equilibrium is attained when the system exhibits snap-through. The 2-DOF twinkler exhibits transient chaos in the snap-through regime. A fractal basin boundary study provides insight into the regions in the parameter space where the total energy level is predictable in an unsymmetric twinkler.
Due to its capacity to convert low frequency to high-frequency oscillations, the snap-through oscillators can be used to harvest energy from low-frequency vibration sources. This idea has led us to explore the energy harvesting capacity of twinkling oscillators. Using magnets and linear springs we built novel experimental twinkling oscillators (SDOF and 2-DOF) for energy harvesting using an inductor coil. When the magnets exhibit high-frequency oscillations through the inducting coil, a current is generated in the coil and this current is used to power a resistive load. This experiment shows promising results both for the SDOF and the 2-DOF twinkling energy generators by validating the frequency up-conversion and generating power from the low-frequency input oscillations. The experimental results show the twinkling phenomenon by converting a $0.1$ Hz input oscillation into $2.5$ Hz output oscillation, a 25 times frequency up-conversion.

For metamaterials design, it is important to study the wave dispersion properties in the material for channeling energy in a desired direction or to build frequency-selective materials. The second part of this dissertation focuses on the dispersive nature of the waves in one dimensional nonlinear chains with weak quadratic and cubic nonlinearities, and with strong snap-through nonlinearities. A limitation to the linear periodic structures is that the filtering properties depend only on the structural design and periodicity which implies that the dispersion characteristics are fixed unless the overall structure or the periodicity is altered. In nonlinear structures there are various design parameters that can be tuned to produce desirable properties. The motivation of the wave propagation analysis is to understand the quadratic and cubic nonlinearity effects on the wave propagation behavior in an uniform periodic chain. Here the dispersion properties are studied through a general perturbation approach for weakly nonlinear periodic media. In general, wave speed, cut-off frequencies, and wave-wave interaction characteristics are presented for a nonlinear chain where each element of the periodic structure has weak quadratic and cubic nonlinearities. The results show significant effect of quadratic nonlinearities in the dispersion characteristics of the waves in the chain.

The strong nonlinearity poses challenges when an infinite chain of snap-through elements is studied for wave propagation behavior. In our attempt to understand the complex wave dynamics in the snap-through chain, we adapted a continuum limit approximation of a dispersive soliton. This analysis resulted in an approximate Duffing equation with a positive linear stiffness term and negative cubic nonlinear term, whose solutions are well known. Using the harmonic balance technique we study the dispersion characteristics of the solitary waves in the nonlinear chain where each element exhibit snap-through bi-stable characteristics. This analysis show the presence of bandgaps depending on the initial and boundary conditions. In this dissertation, we have proposed various future works to advance the analytical study of wave-propagation dynamics in snap-through chains.

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