A thermodynamic cycle analysis is performed to compare the performance improvement of simple cycle (unrecuperated) and recuperated microturbines using a four-port wave rotor. The wave rotor consists of a drum with axial channels rotating at relatively low speed. It is an unsteady flow machine that utilizes shock waves to increase the pressure and temperature of the air entering the combustion chamber of a gas turbine. Utilizing the high-pressure gas leaving the combustion chamber, its energy is transferred directly to the air entering the combustion chamber. Thus, the wave rotor in a gas turbine engine can increase its overall pressure ratio and peak cycle temperature beyond the limits of turbomachinery. Wave rotor investigations have shown a significant potential for gains in both thermal efficiency and specific power, especially in smaller gas turbines, where the compressor pressure ratios are typically lower than those of larger machines.

The present investigation predicts an attractive performance enhancement by implementing a four-port wave-rotor in a microturbine engine. A computer program based on a thermodynamic approach is created to determine the thermodynamic properties of the gases in different states of the cycles. The results are used to calculate the theoretic performance (expressed by specific cycle work $w_{\text{net}}$, overall thermal efficiency $\eta$, and specific fuel consumption (SFC) ) of wave-rotor-topped and baseline engines. Furthermore, actual T-s diagrams of baseline and topped engines are created.

Based on possible design restrictions and preferences, two advantageous implementation cases are considered for the wave rotor into given baseline engines. Comparison of the theoretic performance parameters shows that for the simple cycle engine without recuperator, the commonly discussed Case A where the baseline compressor and turbine may remain unchanged, appears to be the most attractive. However, the combustor would work under higher pressure and with higher combustion end temperature, possibly requiring an enhanced structure and fuel injection system. For the recuperated engine, Case B where the baseline overall pressure ratio and turbine may remain unchanged shows a greater performance compared to that of implementation of Case A. Compared to Case A, Case B might reduce the cost of the compressor and turbine due to reduction of pressure ratios across them.

For the investigated simple cycle baseline engine it is shown that the recuperator is better for improving overall thermal efficiency and specific fuel consumption, while the wave-rotor-topping is mostly better for increasing the specific work output. Cost considerations may additionally favor the wave-rotor.
PERFORMANCE IMPROVEMENT OF RECUPERATED AND UNRECUPERATED MICROTURBINES USING WAVE ROTOR MACHINES

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ABSTRACT

A thermodynamic cycle analysis is performed to compare the performance improvement of simple cycle (unrecuperated) and recuperated microturbines using a four-port wave rotor. Based on possible design restrictions and preferences, two advantageous implementation cases are considered for the wave rotor into given baseline engines. Advantages and disadvantages are outlined. Comparison of the theoretical performance parameters shows that the greatest performance gain for the topped engine without recuperation is obtained if the topped engine operates with the same turbine inlet temperature and compressor pressure ratio as the baseline engine. For the recuperated engine, however, the case in which the topping cycle operates with the same turbine inlet temperature and same overall pressure ratio as the baseline engine results in the highest performance enhancement. Both simple cycle and recuperated engines benefit from the wave-rotor-topping, but the performance improvement is greater for the simple engine without recuperation.

INTRODUCTION

A growing market for distributed power generation and propulsion of small vehicles has motivated a strong interest in design of small gas turbine systems in the range of 30-300 kW. Known as microturbines, they are now widely used in the USA for distributed power generation, shaving peak loads, and providing backup power for critical needs. They propel small commercial aircraft, unmanned air vehicles (UAV), and terrestrial vehicles. Microturbines are often the preferred alternative to internal combustion engines, due to their higher power density and robustness. They also present several advantageous features such as compact size, simple operability, ease of installation, low maintenance, fuel flexibility, and low NOx emissions. However, compared with larger gas turbines, microturbines suffer from lower overall engine efficiency and output power, because of limited cycle pressure ratio and peak cycle temperature. For many applications improvement of their performance is desirable to enhance advantages over competing technologies. To achieve such improvements, current efforts are mainly focused on utilizing heat recovery devices, developing new high-strength, high-temperature materials for turbine blades, and improving the aerodynamic quality of turbomachinery components [1]. The aerodynamics of turbomachinery has already reached a very high level of component efficiencies up to around 90% [2]. Although improvement is possible, further huge enhancements seem to be unlikely. Geometries of microturbines make blade cooling very difficult. Hence, their lifetimes using materials typical of larger gas turbines are shorter [3]. Therefore, there is significant research toward developing advanced metallic alloys and ceramics for high-thermal-resistance turbine wheels used in microturbines[4-5].

Currently, recuperators play a key role in performance enhancement of microturbines. For example, experimental and theoretical research has shown that microturbines with pressure ratios of 3 to 5 without recuperation systems achieve only about 15% to 20% efficiency [6-8]. Utilizing conventional recuperators based on the use of existing materials can improve the overall efficiency of microturbines up to 30% [8-11]. An excellent example of a commercial microturbine with a recuperator is the Capstone 30 kW unit, with an efficiency of 26% using natural gas fuel [12].

Despite the attractive feature of the recuperator concept, a recuperator adds about 25-30 percent to the overall engine manufacturing cost, which is a challenge for commercialization of microturbines [13-15]. The current trend of the microturbine market is to reduce the investment cost. Therefore, alternative devices need to be considered to achieve higher performance at lower component costs. Topping a microturbine with a wave rotor device is an appropriate solution.

The wave rotor consists of a drum with axial channels rotating at relatively low speed. It is an unsteady flow machine that utilizes shock waves to increase the pressure and temperature of the air
entering the combustion chamber of a gas turbine. Utilizing the high-pressure gas leaving the combustion chamber, its energy is transferred directly to the air entering the combustion chamber. Furthermore, in a wave-rotor-topped cycle, the turbine inlet temperature can be equal to that of the baseline cycle, while the combustion takes place at a higher average temperature. Thus, the wave rotor in a gas turbine engine can increase its overall pressure ratio and peak cycle temperature beyond the limits of turbomachinery. Wave rotor investigations have shown a significant potential for gains in both thermal efficiency and specific power, especially in smaller gas turbines, where the compressor pressure ratios are typically lower than those of larger machines.

**Background**

Wave rotors as topping devices for gas turbine cycles have been studied extensively. Since the early 1960s, the General Electric Company (GE), General Power Corporation (GPC), and Rolls Royce were involved in the development of prototype wave rotors for propulsion applications [16, 17]. In the early 1980s, the US Defense Advanced Research Program Projects Agency (DARPA) and the US Navy sponsored programs to develop an understanding of wave rotor science and technology. Many developments were presented in the 1985 ONR/NAVAIR Wave Rotor Research and Technology Workshop [17].

Brown Boveri Company (BBC), later Asea Brown Boveri (ABB), of Switzerland also has a long history in -wave rotor technology. As reported by Meyer [18], the first successful wave rotor was tested by Claude Seippel of BBC in 1942 as a topping stage for a locomotive gas turbine engine [19]. ABB’s Comprex® pressure wave supercharger has been used commercially for passenger car and heavy diesel engines. The first wide application of the Comprex® in passenger cars has been in the Mazda 626 Capella since 1987 [20, 21]. Parallel to this development, ABB also began feasibility studies of utilizing the wave rotors as topping cycles for gas turbines. A conventional wave rotor incorporated into a gas turbine engine was build and tested from 1989 until 1991 [22]. The results showed that the efficiency and specific power of the topped engine were increased by 17% and 25%, respectively, compared with the baseline engine.

Interest in wave rotor technology has increased recently. Since the late 1980s, a sustained research program at NASA Glenn Research Center (GRC) collaborating with the US Army Research Laboratory (ARL) and Rolls-Royce Allison has aimed to develop and demonstrate the benefits of the wave rotor technology for future aircraft propulsion systems[23-30]. An excellent overview is provided by Welch [31].

The present work seeks to investigate the feasibility and potential of integrating a wave rotor into a microturbine, both simple cycle (without a recuperator) and the corresponding recuperated cycle, to demonstrate the performance improvement of such combined systems. Beside thermodynamic analyses of both baseline and topped engines, several mechanical design changes due to the proposed wave-rotor implementations into the baseline engines are discussed.

**WAVE-ROTOR TOPPING CYCLE**

Wave rotor geometry and details of its internal operation has been extensively explained in the references above and are not repeated here. Instead, two most promising implementations of a wave rotor into a baseline gas turbine with and without recuperation are discussed.

**Gas Turbine Without Recuperation**

In a conventional implementation of a wave rotor into a simple cycle gas turbine, the wave rotor is embedded between the compressor and turbine “parallel” to the combustion chamber. Figure 1 illustrates how a four-port wave rotor is used to top a simple gas turbine cycle. In the wave rotor channels, the hot gas leaving the combustion chamber compresses the air received from the compressor. After the additional compression of the air in the wave rotor, it is discharged into the combustion chamber. The burned gas expands during the compression of the air and is afterwards

![Figure 1: A schematic of a simple cycle gas turbine topped by a 4-port wave rotor](image-url)
scavenged toward the turbine. Then, the channels are re-connected to the compressor outlet, allowing fresh pre-compressed air to flow into the wave rotor channels and the cycle repeats again. Due to the pre-expansion in the wave rotor, the burned gas enters the turbine with a lower temperature than the combustor exit. However, the gas pressure is higher than the compressor exit pressure by the pressure gain obtained in the wave rotor. This is in contrast to the untopped engine, where - by the pressure loss occurring in the combustion chamber - the turbine inlet pressure is lower than the compressor discharge pressure.

There are several wave-rotor topping cycles that may be suited for a gas turbine application. Knowing about possible design restrictions and preferences when an existing gas turbine is to be enhanced, five different advantageous implementation cases for a wave rotor into given baseline engines have been extensively discussed by Akbari et al. [7, 32-34]. However, two of these topping cycles have been most commonly studied by other researchers [35-36]. These two cases, introduced as Cases A and B in this work, are compared with the thermodynamic cycle of the baseline engine in a schematic T-s diagram as shown in Fig. 2. Path 0-1b-4b-5b represents the baseline cycle shown and 0-1A-2A-3A-4A-5A with subscript i=A or B indicate the two wave-rotor-topped cycles.

In Case A, the pressure ratio of the compressor is kept unchanged, so the physical compressor of the baseline engine can also be used for the wave-rotor-enhanced engine, provided the mass flow is kept approximately the same. The pressure in the combustion chamber of the enhanced engine is increased by the compression ratio of the wave rotor. This may require modifications to the structure of the combustion chamber and to the fuel injection system. Every wave rotor considered in this work has zero shaft work. Therefore, the wave rotor compression work is equal to the wave rotor expansion work. Thus, in Fig. 2, the energy increase from point "1b" to "4b" in the baseline engine and from point "1A" to "4A" in the wave-rotor-topped engine is the same. This results in the same heat addition for both cycles, but in the enhanced cycle it takes place after the energy exchange in the wave rotor, hence starting at higher temperature. Thus, the combustion end temperature is higher than that of the baseline engine, possibly requiring additionally a thermal enhancement of the combustor structure. However, the output work of the topped engine is higher than that of the baseline engine due to the pressure gain across the wave rotor ($p_{t4A} > p_{t4b}$, where subscript "t" indicates total values). Therefore, the overall efficiency for the topped engine is higher than that of the baseline engine. The turbine of the topped engine might need to be adapted to efficiently utilize the obtained higher pressure ratio. The turbine inlet temperature, however, is the same as it is for the baseline engine. This case is the one most commonly discussed in references and, as will be shown later, this implementation case provides the highest overall efficiency and specific work and the lowest value of specific fuel consumption.

In Case B, the overall pressure ratio for the wave-rotor-enhanced engine is kept equal to that of the baseline engine, so that the combustor works at the same pressure. However, for the wave-rotor-topped engine, the heat addition in the combustor and the combustion end temperature are higher than those of the baseline engine. This may require some adaptation of the combustor, especially in the outlet region. The turbine and compressor work with a lower pressure ratio, with less challenging design requirements. This might reduce the cost of the compressor and turbine due to reduction of stages where multi-stage types (mostly axial) are used or due to reduction of the tip diameter where radial types (mostly single-stage) are used. With a smaller tip diameter the wheels can be manufactured more economically in shorter time from cheaper materials with less strength and on smaller machines. Beside an attractive performance enhancement, this case will be shown to provide the highest turbine outlet temperature, higher than those of Case A and the baseline engine ($T_{t4B} > T_{t4A} > T_{t4b}$). Therefore, this case is especially attractive for heat recovery by internal recuperation that enhances the performance even more.
Gas Turbine With Recuperation

Figure 3 shows a block diagram of a recuperated gas turbine topped with a four-port wave rotor. The wave rotor is placed after the compressor and before the recuperator. Compared to the baseline engine without the recuperator, the recuperated baseline engine here has lower turbine inlet pressure due to the pressure loss of the recuperator air-side. However, it has a higher turbine outlet pressure due to the pressure loss of the recuperator gas-side.

THERMODYNAMIC CALCULATIONS

To evaluate the performance enhancement of topping gas turbines with wave rotors, a computer program based on a thermodynamic approach is created to determine the thermodynamic properties of the gases in different states of the cycles. The results are used to calculate the theoretical performance (expressed by specific cycle work $\dot{w}_{ret}$, overall thermal efficiency $\eta$, and specific fuel consumption (SFC) ) of wave-rotor-topped and baseline engines. Furthermore, actual T-s diagrams of baseline and topped engines are created.

For thermodynamic calculations, a 30 kW microturbine engine produced by Capstone Turbine Corporation is considered here. The calculations are performed for both Cases A and B of cycles with and without a recuperation. The given data are compressor pressure ratio ($p_{t1}/p_{t2}=3.6$), turbine inlet temperature ($T_{turbine}=1116.5$ K), turbine isentropic efficiency ($\eta_T=0.84$), and pressure drop in the combustion chamber ($\Pi_{comb}=2$%). Incomplete combustion of the fuel is reflected by a combustor efficiency of $\eta_Q=98.0\%$. The gases are treated as ideal gases with constant values for specific heat coefficients ($C_{pair}=1.005$ kJ/kgK, $C_{pgas}=1.148$ kJ/kgK) and the ratio of specific heats ($\gamma_{air}=1.4$, $\gamma_{gas}=1.33$). Considering the data are valid for the unrecuperated engine and assuming that the air is entering the compressor at 300 K, the compressor isentropic and polytropic efficiencies ($\eta_C=79.6\%$ and $\eta_{PC}=82.9\%$), and the turbine polytropic efficiency ($\eta_{PT}=81.7\%$) are found. Considering the same “aerodynamic quality” of the turbine wheel, the polytropic efficiency is kept the same for the unrecuperated and recuperated cycles resulting in almost the same turbine isentropic efficiency for the recuperated cycle ($\eta_T=0.84$). For the recuperated cycle, the pressure losses across the air and gas side of the recuperator are assumed the same and equal to 0.98 ($\Pi_{recup-air}=\Pi_{recup-gas}=0.98$). The effectiveness of the recuperator is considered $\eta_{recup}=90\%$. This value has been selected based on typical recuperators. The major challenges in providing a recuperator with greater effectiveness are size and cost. For stationary applications, size and weight are not so critical, but for mobile applications due to such limitations recuperator effectiveness of less than 90% is common [10].

In the wave-rotor-topping cycle, it is assumed that the compressor inlet condition, turbine inlet temperature, compressor and turbine polytropic efficiencies remain unchanged and are the same as the baseline engines. According to previous wave rotor investigations [35-37], the wave rotor compression and expansion efficiencies are assumed with $\eta_{WC}=\eta_{WE}=0.83$. A wave rotor compression ratio of $PR_{w}=p_{t2}/p_{t1}=1.8$ appears to be conceivable for the envisioned application and is
chosen for the discussion of representative values. However, plots are shown for various wave rotor pressure ratios indicating its effect on the performance enhancement.

Analytical Approach

For the unrecuperated engine and the corresponding wave-rotor-topped cycle the analytical approach has been introduced by the authors in previous work [32] and is not stated here. Instead, a thermodynamic approach for the recuperated cycle with and without the wave rotor is presented. Complying with Fig. 3, the following steps are used to calculate the thermodynamic properties of the gases in different states of the topped cycle:

• Path 0-1 (compressor):
  At the compressor, inlet static thermodynamic properties are assumed to be equal to stagnation thermodynamic properties. Therefore, with the given compressor inlet temperature, the compressor outlet total temperature and pressure are calculated by:
  \[ T_{t1} = T_{t0} + T_{t0} \frac{\Pi_C \gamma_{air} - 1}{\eta_C \gamma_{air}} \]  
  \[ \frac{P_{t1}}{P_{t0}} = \frac{P_{t1}}{P_{t0}} = \Pi_C \]  
  where the compressor isentropic efficiency \((\eta_C)\) relates the compressor pressure ratio \((\Pi_C)\) to the compressor polytropic efficiency \((\eta_{PC})\) through:
  \( \eta_C = \frac{\Pi_C \gamma_{air} - 1}{\Pi_C \gamma_{air} \eta_{PC} - 1} \)  
  For Case A the compressor pressure ratio is equal to that of the baseline engine \((\Pi_C = 3.6)\). For Cases B its value is calculated by dividing the baseline compressor pressure ratio with the wave rotor compression ratio \(PR_w\). The compressor specific work can be obtained by:
  \[ w_C = C_{p_{air}} (T_{t1} - T_{t0}) = \frac{C_{p_{air}} T_{t0}}{\eta_C} (\Pi_C \gamma_{air} - 1) \]  

• Path 1-2 (compression in the wave rotor):
  The flow properties after the wave rotor compression process are obtained by:
  \[ T_{t2} = T_{t1} + \frac{T_{t1}}{\eta_{WC}} \left( PR_w \frac{\gamma_{air} - 1}{\gamma_{air}} - 1 \right) \]  
  \[ \frac{P_{t2}}{P_{t1}} = \frac{P_{t2}}{P_{t1}} = PR_w \cdot \Pi_C \]  
  where \(\eta_{WC}\) is the compression efficiency of the wave rotor.

• Path 2-2* (recuperator air-side):
  The recuperator effectiveness based on the actual and maximum heat transfers from the turbine exhaust gas to the air can be expressed as:
  \[ \eta_{recup} = \frac{T_{t2*} - T_{t2}}{T_{t5} - T_{t2}} \]  
  Using this equation, the temperature at the exit of the regenerator air-side \((T_{t2*})\) becomes:
  \[ T_{t2*} = T_{t2} + \eta_{recup} (T_{t5} - T_{t2}) \]  
  In this relationship, turbine outlet temperature \((T_{t5})\) is still an unknown parameter. More equations are required to find unknown values \(T_{t2*}\) and \(T_{t5}\). These additional equations will be derived later.
  The corresponding pressure at point 2* is found by:
  \[ \frac{P_{t2*}}{P_{t2}} = \Pi_{recup-air} \cdot \frac{P_{t2}}{P_{t0}} = \Pi_{recup-air} \cdot PR_w \cdot \Pi_C \]  
  where \(\Pi_{recup-air} = P_{t2*} / P_{t2}\) represents the pressure loss across the air side of the regenerator.

• Path 2*-3 (combustor):
  The value of fuel/air ratio \(f = \frac{m_f}{m_{air}}\) can be obtained by applying the energy equation to the combustion chamber as:
  \[ T_{t3} = T_{t2} + \frac{T_{t2}}{\eta_{Q}} (C_{p_f} T_{t3} - C_{p_{air}} T_{t2} - h_{PR} f) \]  
  where \(h_{PR}\) and \(\eta_{Q}\) are the heating value of the fuel and combustion efficiency, respectively. The value of 43000 kJ/kg for \(h_{PR}\) and 0.98 for \(\eta_{Q}\) will be used in all calculations here. For Cases A and B, \(T_{t3}\) is an unknown value, but \(T_{t4}\) is a known value and is equal to the baseline turbine inlet temperature. Therefore, \(f\) can not be obtained only by using Eq. (10). However, \(f\) can be alternatively expressed based on the turbine total inlet temperature \((T_{t4})\) and the compressor total exit temperature \((T_{t1})\). For this purpose, wave rotor compression and expansion specific works (per unit air mass flow) are defined respectively as follows:
\[ w_{WC} = C_{P_{air}} (T_{t2} - T_{t1}) \tag{11} \]

\[ w_{WE} = (1 + f) C_{P_{gas}} (T_{t3} - T_{t4}) \tag{12} \]

For simplicity of presentation, it is considered here that \( m_1 = m_2 = m_{air} \) and \( m_3 = m_4 = m_{air} + m_f \). (Generally, the mass flow rates \( m_1 \) and \( m_3 \) are not necessarily the same, and correspondingly mass flow rates \( m_2 \) and \( m_4 \) may not be equal either). Now, Eq. (10) can be expressed as:

\[ \eta_Q f h_{PR} = (1 + f) C_{P_{gas}} T_{t4} + (1 + f) C_{P_{gas}} (T_{t3} - T_{t4}) - C_{P_{air}} (T_{t2} - T_{t1}) - C_{P_{air}} (T_{t2} - T_{t1}) - C_{P_{air}} T_{t1} \]

where \( \eta_Q f h_{PR} \) represents the pressure ratio across the wave rotor. Equating the compression work to the expansion work leads to:

\[ \frac{C_{P_{air}} T_{t4}}{\eta_{WC}} (PR_{W}^{\frac{\gamma_{air}}{\gamma_{air}} - 1} - 1) = \]

\[ (1 + f) C_{P_{gas}} \eta_{WE} T_{t3} \left[ \frac{PO}{\eta_{comb} PR_{W}} \right]^{\frac{PR_{gas}^{\frac{\gamma_{gas}}{\gamma_{gas}} - 1}}{\frac{PR_{gas}}{\gamma_{gas}}} - 1} \tag{21} \]

Substituting \( T_{t3} \) from Eq. (17) in the above equation and some algebra gives:

\[ PO = \Pi_{comb} PR_{W} \left\{ \frac{A \frac{1}{\eta_{WE}} B T_{t1}}{(1 + A \frac{1}{\eta_{WC}} B T_{t1} - 1)} \right\} \tag{22} \]

where

\[ A = \frac{C_{P_{air}}}{(1 + f) C_{P_{gas}}} \tag{23} \]

\[ B = \frac{T_{t1}}{T_{t4}} \left[ PR_{W}^{\frac{\gamma_{gas} - 1}{\gamma_{gas}} - 1} \right] \tag{24} \]

Equation (22) is a modified version of the “wave-rotor characteristic” equation introduced in the literature [35]. By using Eq. (22), the turbine inlet total pressure is obtained by:

\[ \frac{p_{t4}}{p_0} = \Pi_{comb} \Pi_{recup-air} PR_{W} \Pi_{C} \tag{25} \]

• Part 4-5 (turbine):

The turbine total outlet pressure is calculated by:

\[ \frac{P_{t5}}{p_0} = \frac{1}{\Pi_{recup-gas}} \frac{P_{t5'}}{p_0} = \frac{1}{\Pi_{recup-gas}} \tag{26} \]

where \( \Pi_{recup-gas} = p_{t5'}/p_{t5} \) represents the pressure loss across the gas side of the regenerator. It is justified that the total pressure of the gas leaving the regenerator \( (p_{t5'}) \) is equal to the ambient pressure \( (p_0) \).
The turbine specific work (per unit air mass flow) is:

\[ w_T = (I + f) \frac{C_p \text{gas} (T_{14} - T_{i5})}{C_p \text{air}} \]  

(27)

This work can also be represented according to pressure ratio across the turbine as:

\[ w_T = \eta_T (I + f) \frac{C_p \text{gas} T_{i4}}{C_p \text{air} T_{i4}} \left[ 1 - \left( \frac{p_{i5}}{p_0} \right)^{\frac{\gamma_{\text{gas}} - 1}{\gamma_{\text{gas}}}} \right] \]  

(28)

where the isentropic turbine efficiency \( \eta_T \) can be obtained from the polytropic turbine efficiency \( \eta_{PT} \):

\[ \eta_T = \left( \frac{P_{i5}}{P_{i4}} \right)^{\frac{\gamma_{\text{gas}} - 1}{\gamma_{\text{gas}}}} \]  

(29)

By obtaining the turbine specific work, the turbine total exit temperature \( (T_{i5}) \) is given by:

\[ T_{i5} = T_{i4} - \eta_T T_{i4} \left[ 1 - \left( \frac{p_{i5}}{p_0} \right)^{\frac{\gamma_{\text{gas}} - 1}{\gamma_{\text{gas}}}} \right] \]  

(30)

Now, to find values of \( T_{i5}^* \), \( T_{i5} \), and \( f \), it is necessary to use Eq. (8), (16), and (30), along with Eq. (22), (25), and (26). The procedure is explained in the following.

Substituting Eq. (8) into (16) gives:

\[ f = \frac{C_p \text{gas} T_{i4} - \eta_{\text{recup}} C_p \text{air} (T_{i5} - T_{i2}) - C_p \text{air} T_{i1}}{\eta_q h_{\text{PR}} - C_p \text{gas} T_{i4}} \]  

(31)

or,

\[ T_{i5} = T_{i2} + \frac{C_p \text{gas} T_{i4} - f (\eta_q h_{\text{PR}} - C_p \text{gas} T_{i4}) - C_p \text{air} T_{i1}}{\eta_{\text{recup}} C_p \text{air}} \]  

(32)

Now, equating this equation with Eq. (30) results in:

\[ T_{i2} + \frac{C_p \text{gas} T_{i4} - f (\eta_q h_{\text{PR}} - C_p \text{gas} T_{i4}) - C_p \text{air} T_{i1}}{\eta_{\text{recup}} C_p \text{air}} \]

\[ = \eta_T T_{i4} \left[ 1 - \left( \frac{p_{i5}}{p_0} \right)^{\frac{\gamma_{\text{gas}} - 1}{\gamma_{\text{gas}}}} \right] \]

Using equations (25) and (26):

\[ T_{i2} + \frac{C_p \text{gas} T_{i4} - f (\eta_q h_{\text{PR}} - C_p \text{gas} T_{i4}) - C_p \text{air} T_{i1}}{\eta_{\text{recup}} C_p \text{air}} \]

\[ = T_{i4} - \eta_T T_{i4} \left[ 1 - \left( \frac{l}{P_{\text{recup}} - P_0} \right)^{\frac{1}{\gamma_{\text{gas}}}} \right] \]

(34)

where \( PO \) is a function of \( f \) and is obtained by Eq. (22). The above equation computes \( f \) as a function of other known cycle parameters.

- Part 5-5' (recuperator gas-side):

The temperature of the gas leaving the recuperator \( (T_{i5}^*) \) can be calculated by balancing the energy equation across the regenerator as follows:

\[ C_p \text{air} (T_{i5}^* - T_{i2}) = (1 + f) C_p \text{gas} (T_{i5} - T_{i5}^*) \]  

(35)

Therefore, \( T_{i5}^* \) is obtained by:

\[ T_{i5}^* = T_{i5} - \frac{C_p \text{air} (T_{i5}^* - T_{i2})}{C_p \text{gas}} \]  

(36)

Now, it is possible to calculate the engine performance parameters as follows. The net specific output work produced by the engine can be calculated by subtracting the turbine specific work from the compressor work:

\[ w_{net} = \eta_M w_T - w_C = \]

\[ \eta_M \eta_T (1 + f) C_p \text{gas} T_{i4} \left[ \frac{l}{p_{i4} / p_0} \right] \]

\[ \frac{C_p \text{air} T_{i0}}{\eta_C} \frac{\gamma_{\text{gas}} - 1}{\gamma_{\text{gas}} - 1} \]

(37)
where \( \eta_m \) is the shaft mechanical efficiency. In this study \( \eta_m = 100\% \) is assumed.

Since the thermal efficiency is defined as the net specific output work divided by the specific heat added to the cycle, it can be calculated by:

\[
\eta = \frac{w_{net}}{f h_{PR}} \tag{38}
\]

Finally, SFC can be obtained by:

\[
SFC = \frac{f}{w_{net}} \tag{39}
\]

RESULTS AND DISCUSSIONS

Implementing Case A to both unrecuperated and recuperated engines, Fig. 5 illustrates the variations of cycle thermal efficiency (dash dot), specific work (dashed), and specific fuel consumption (solid) with increasing wave rotor pressure ratio \( PR_W \). The plots visualize how the wave rotor effect develops from the baseline engine with \( PR_W = 1 \) until \( PR_W = 2 \), which might be a reasonable limit for the investigated application. However, if the wave rotor pressure ratio increases further, the trend indicate that the benefits diminish while technical challenges may increase. With a conceivable wave rotor pressure ratio of 1.8, the thermal efficiency of the unrecuperated engine is increased from 14.9% to 20.0%. For the recuperated engine, however, the thermal efficiency slightly increases from 24.8% to 26.6%. The specific work is increased from 128 kJ/kg to 171 kJ/kg for the unrecuperated engine and from 116 kJ/kg to 160 kJ/kg for the recuperated engine. The specific work of the recuperated cycle is slightly less than that of the simple cycle, due to the pressure losses across the recuperator. The thermal efficiency of the recuperated cycle remains almost unchanged when using the wave rotor, because the temperature difference between the air and the gas entering the recuperator of the topped engine \( T_{sat} - T_{st} \) is much less than that of the baseline engine \( T_{sat} - T_{st} \). This results in reduced recuperation effect, and hence more heat addition for the topped engine offsets the increased work output. This is in contrast to the unrecuperated engine where both the baseline and topped cycles have the same heat addition, as explained before. SFC of the unrecuperated engine is decreased from 0.156 kg/kN s to 0.116 kg/kN s and it slightly decrease from 0.094 kg/kN s to 0.087 kg/kN s for the recuperated engine. Again the increased heat addition for the recuperated topped cycle keeps SFC almost a constant value compared to the simple cycle, which is seen from Eq. (39).

A better picture about the performance improvement for both unrecuperated and recuperated engines is obtained by calculating the relative increases of thermal efficiency, specific work, and relative decreases of SFC shown in Fig. 6. For the unrecuperated engine, the relative increases of thermal efficiency and specific work (dash dot and dashed) are precisely the same as it is obvious from Eq. (38), where heat addition \( q = h_{PR} \) is the same for both topped and baseline.
engines. Figure 6 indicates an attractive relative improvement in thermal efficiency and specific work of about 33.9%, and a relative reduction of 25.3% in SFC (solid) for the unrecuperated engine using a wave rotor with pressure ratio of 1.8. However, for the recuperated cycle, specific work shows a significant improvement of about 37.6% and thermal efficiency and SFC are about 7% greater than those of the baseline engine.

Similar to Fig. 5, Fig. 7 shows the increases of cycle thermal efficiency (dash dot) and specific work (dashed), and the decrease of SFC (solid) upon implementing Case B to both unrecuperated and recuperated engines. For the unrecuperated engine, similar to Case A, both thermal efficiency and specific work are increasing function of $PR_W$ while SFC is decreased. The same trend is observed also for the recuperated engine. For a $PR_W = 1.8$, the thermal efficiency of the simple cycle is increased from 14.9% to 15.8%, and from 24.8% to 27.3% for the recuperated engine. Similarly, the specific work is increased from 128 kJ/kg to 150 kJ/kg for the simple cycle, and from

**Figure 6**: Relative values of thermal efficiency, specific work, and specific fuel consumption of the wave-rotor-topped engines versus wave rotor pressure ratio and overall pressure ratio (Case A consideration)

**Figure 7**: Thermal efficiency, specific work, and specific fuel consumption of the wave-rotor-topped engines versus wave rotor pressure ratio and overall pressure ratio (Case B consideration)
116 kJ/kg to 137 kJ/kg for the recuperated cycle. Finally, SFC of the simple cycle is decreased from 0.156 kg/kN·s to 0.147 kg/kN·s and from 0.094 kg/kN·s to 0.085 kg/kN·s for the recuperated cycle.

Using the results above, Fig. 8 is provided illustrating the relative increases of thermal efficiency, specific work, and relative decreases of SFC for both unrecuperated and recuperated topped engines. For the same $PR_W = 1.8$, wave-rotor-topping of the unrecuperated engine gives a relative increase of 5.9% and 17.1% for the thermal efficiency and specific work, respectively, and 5.6% reduction in SFC. For the recuperated engine, the performance improvement is greater with 9.7% and 18.0% increase of thermal efficiency and specific work, respectively, and 8.7% reduction in SFC.

Table 1 summarizes the obtained results and shows a comparison between performance improvements of the studied engines for implementations of both Cases A and B. It is seen from the table that the baseline engine with recuperation has much higher efficiency (about twice) and lower SFC than those of the unrecuperated engine due to the reduction of heat addition in the combustion process. Implementation of Case A into the baseline engine results in a

![Figure 8: Relative values of thermal efficiency, specific work, and specific fuel consumption of the wave-rotor-topped engines versus wave rotor pressure ratio and overall pressure ratio (Case B consideration)]
significant performance improvement of the unrecuperated engine, however, the thermal efficiency and SFC improvements of the recuperated engine are less than that of the specific work. In Case B, the topped cycle without the recuperator still benefits from the wave rotor even though the performance enhancement is less than that of Case A. However, the topped recuperated cycle has higher performance compared to the unrecuperated engine and its thermal efficiency and SFC gains are even more than those of Case A. The results clearly demonstrate the advantage of implementing Case A for unrecuperated engines and implementing Case B for recuperated engines. Among all topping cycles, the best relative performance improvement is obtained with Case A topping of the unrecuperated engine.

A comparison between the recuperated baseline engine and the topped simple cycle engine \( (PR_w=1.8) \) reveals that for the specific example, the recuperator enhances thermal efficiency and SFC better, while the wave-rotor is mostly better for enhancing the specific work output. Therefore, substituting a recuperator with a wave rotor may be guided by a preference for high power output and reduced unit cost, considering that a recuperator contributes about 25-30% to the unit cost and a wave rotor may be cheaper. Finally, topping a recuperated gas turbine with a wave rotor can increase the performance especially if the topping cycle operates with the same turbine inlet temperature and same overall pressure of the baseline engine (Case B implementation), which is preferable for the combustor and fuel injection design.

CONCLUSION

The present investigation predicts an attractive performance enhancement by implementing a four-port wave-rotor in a microturbine engine. Two different cases of implementing the wave rotor in both baseline engines with and without a recuperator are investigated. For the simple cycle engine without recuperator, the commonly discussed Case A where the baseline compressor and turbine may remain unchanged, appears to be the most attractive. However, the combustor would work under higher pressure and with higher combustion end temperature, possibly requiring an enhanced structure and fuel injection system. For the recuperated engine, Case B where the baseline overall pressure ratio and turbine may remain unchanged shows a greater performance compared to that of implementation of Case A. Compared to Case A, Case B might reduce the cost of the compressor and turbine due to reduction of pressure ratios across them. For the investigated simple cycle baseline engine it is shown that the recuperator is better for improving overall thermal efficiency and specific fuel consumption, while the wave-rotor-topping is mostly better for increasing the specific work output. Cost considerations may additionally favor the wave-rotor.

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