Department of Mechanical Engineering
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Solid and Structural Mechanics
Ph.D. Qualifying Examination

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One page of formulas and notes are allowed
All Questions are weighted equally.

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Q.1 (25) The beam AB shown below is fixed at both ends. In order to determine the deflection equation \(y(x)\) of the beam, the following questions must be answered.

(a) Draw a free-body diagram of the beam with all unknown reaction forces at the boundaries.
(b) Give the equilibrium equations.
(c) Based on a cantilevered beam, draw diagrams to exercise the principle of superposition required for finding the unknown reaction forces at the boundaries.
(d) Give the deformation equations (in terms of \(y\) and \(y'\)) required for identifying the unknowns.
(e) Express the deformation equations in terms of forces.
(f) Find the reaction forces, if the bending rigidity \(EI\) is constant through the length of the beam.
Q.2 (25) The strain rosette shown below has three arms. Any arm is 120° from the other two arms. If the strain measurements of the three arms are $\epsilon(1) = 100 \mu$, $\epsilon(2) = 200 \mu$ and $\epsilon(3) = 300 \mu$, find the strain components $\epsilon_{xx}$, $\epsilon_{yy}$ and $\gamma_{xy}$. Also find the principal strains and the maximum shear strain.
Q. 3 (25 pts) – Two forces are applied to the small post BD as shown (base D is fixed). Knowing that the vertical portion of the post has a cross section of 1.5 x 2.4 inches (see figure).

1. Calculate all forces, moments and torque acting at the cross section containing point H,
2. Assuming that the following normal and shearing stresses develop at point H.
   2.1 Calculate principal normal and shearing stresses.
   2.2 Calculate principal direction, \( \theta \).
Q.4 (25 pts) – The following loaded beam with a rectangular cross section (h=1 m and b=2 m) has a
Young’s modulus $E = 1.6 \times 10^6 Pa$

a. Write force and moment equilibrium equations for the beam.
b. Find the deflection at point D.