Ph.D. Qualifying Exam

Dynamic Systems and Control

Spring 2013

Professor Jongeun Choi
Professor George Zhu

Department of Mechanical Engineering
Michigan State University
(12-03-2012)

Open Book
Answer All Questions
All Questions Weight Equally

Time: 3.0 hours
1. Consider the following feedback control system, where $K_p > 0$ and $K_d > 0$ are two control parameters. Find $K_p$ and $K_d$ such that the step response of the closed loop system has 10% overshoot and 1.0 second 2% settling time.

\[
\frac{U(s)}{Y(s)} = \frac{K_p}{1 + \frac{K_d}{K_p} s} \frac{1}{s^2}
\]

**Sol:**
2. Consider the following closed loop system

\[ Y(s) = \frac{s + 1}{s^2 - 2s + 2} \]

- **a)** Find the closed loop characteristic equation and sketch the Root Locus for \( K > 0 \)
- **b)** Calculate the breakaway points on the Root Locus
- **c)** Calculate the stable range for \( K \) and oscillation frequency when the system is marginally stable.

**Sol:**
3. Consider a transfer function \( G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s+1} \).

   a. Obtain frequency responses to the following inputs
      i. \( u(t) = \sin(0.1t) \).
      ii. \( u(t) = \sin(10t) \).

   Hint: The frequency response of a system is defined as the steady-state response of the system to a sinusoidal input signal.

   Sol:
b. Draw the bode diagram of $G(s)$ with the aid of the frequency responses obtained in (a).

**Sol:**
4. Consider the following nonlinear system:
\[
\dot{x} = f(x,u) = 7x^3 + x + u^2 + u.
\]
a) Linearize this nonlinear system with respect to \( x_0 = 1 \). Assume that \( |u_0| \leq 2.5 \).

Hint: Perform coordinate transformation with \( x = x_0 + x \) and \( u = u_0 + u \) such that the linearized system can be written as
\[
\dot{x} = g(x, u) \tag{1}.
\]

b) Is \( g(x, u) \) in (1) a linear function in terms of \( x, u \)?

c) Is \( g(x = x_0, u = u_0) = g(x_0, u_0) \) a linear function in terms of \( x \) and \( u \)?

If it is not, provide a counter example to show that the superposition principle does not hold.

Sol: