Exam Number: ______________________

Department of Mechanical Engineering

Michigan State University

Mathematics
Ph.D. Qualifying Examination

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One Book, Closed Notes
All five questions are weighted equally
Question 6 is for extra credit

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Problem 1

Do the points $p_1=(4,-2,1)$, $p_2=(5,1,6)$, $p_3=(2,2,-5)$ and $p_4=(3,5,0)$ lie on a plane? If **YES**, what is the vector normal to that plane? If **NO**, what is the distance from p_4 to the plane formed by p_1 p_2 and p_3?

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Problem 2

Find the point (x,y,z) on the plane $x+2y+z=4$ that is closest to the point $p=(1,-1,0)$

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Problem 3

Solve the heat equation analytically.

$$\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le L, \ t > 0$$

subject to the boundary conditions

$$u(0,t) = 0 \text{ and } \left.\left(\frac{\partial u}{\partial x} + Ku\right)\right|_{x=L} = 0, \quad K > 0,$$

and the initial condition

$$u(x,0) = \sin(\pi x/L).$$

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Problem 4

Approximate the function $f(x,y,z)=3x^2y - 4xz + z^2 + 2y^2$ about the point $q=(1,1,0)$ using a 2[nd] order Taylor approximation. Then find all stationary points of the approximation and classify them (as either minima, maximum or saddle points).

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Problem 5

Solve the initial value problem by the Laplace transform

$$y'' + 2y' + y = e^{-t}, \qquad y(0) = -1, \;\; y'(0) = 1.$$

(Table of Laplace transforms provided. See last page)

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Problem 6 (Extra Credit)

A box contains 10 screws, three of which are defective. Two screws are drawn at random. Find (a) the probability that none of the two screws is defective, (b) the probability that one of the two screws is defective, and (c) the probability that two of the two screws are defective. (Considering both sampling with and without replacement).

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TABLE 7.1 Laplace Transform Pairs

$f(t)$	$F(s) = \mathcal{L}\{f(t)\}$	Condition on s
1. 1	$1/s$	$s > 0$
2. t	$1/s^2$	$s > 0$
3. t^n $(n = 1, 2, \dots)$	$n!/s^{n+1}$	$s > 0$
4. t^a $(a > -1)$	$\Gamma(a+1)/s^{a+1}$	$s > a$
5. e^{at}	$1/(s-a)$	$s > a$
6. $t^n e^{at}$ $(n = 1, 2, \dots)$	$n!/(s-a)^{n+1}$	$s > a$
7. $H(t-a)$	e^{-as}/s	$s \geq a$
8. $\delta(t-a)$	e^{-as}	$s > 0, a > 0$
9. $\sin at$	$a/(s^2+a^2)$	$s > 0$
10. $\cos at$	$s/(s^2+a^2)$	$s > 0$
11. $t \sin at$	$2as/(s^2+a^2)^2$	$s > 0$
12. $t \cos at$	$(s^2-a^2)/(s^2+a^2)^2$	$s > 0$
13. $e^{at} \sin bt$	$b/[(s-a)^2+b^2]$	$s > a$
14. $e^{at} \cos bt$	$(s-a)/[(s-a)^2+b^2]$	$s > a$
15. $\frac{1}{2a^3} \sin at - \frac{1}{2a^2} t \cos at$	$1/(s^2+a^2)^2$	$s > 0$
16. $\frac{1}{2a} \sin at + \frac{1}{2} t \cos at$	$s^2/(s^2+a^2)^2$	$s > 0$
17. $1 - \cos at$	$a^2/[s(s^2+a^2)]$	$s > 0$
18. $at - \sin at$	$a^3/[s^2(s^2+a^2)]$	$s > 0$
19. $\sinh at$	$a/(s^2-a^2)$	$s > \|a\|$
20. $\cosh at$	$s/(s^2-a^2)$	$s > \|a\|$
21. $\frac{1}{2a^3} \sinh at + \frac{1}{2a^2} t \cosh at$	$1/(s^2-a^2)^2$	$s > \|a\|$
22. $\frac{1}{2a} t \sinh at$	$s/(s^2-a^2)^2$	$s > \|a\|$
23. $\frac{1}{2a} \sinh at + \frac{1}{2} t \cosh at$	$s^2/(s^2-a^2)^2$	$s > \|a\|$
24. $\sinh at - \sin at$	$2a^3/(s^4-a^4)$	$s > \|a\|$
25. $\cosh at - \cos at$	$2a^2 s/(s^4-a^4)$	$s > \|a\|$