Student Code Number:_____________

Ph.D. Qualifying Exam

Dynamics and Vibrations

January, 2008

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Directions: **Work all four problems.**
Note that the problems are EVENLY WEIGHTED.
You may use two books and two pages of notes for reference.

3. The mass of a system may be changed to improve the vibration isolation characteristics. Such isolation systems often occur when mounting heavy compressors on factory floors. This is illustrated in Figure 3a, with an idealized model of the system being shown in Figure 3b. In this case the soil provides the stiffness of the isolation system (damping may be ignored) and the design problem becomes that of choosing the value of the mass, *M*, of the concrete isolation block.
Assume the stiffness of the soil, $k = 2.0 \times 10^7$ N/m, the mass of the compressor, $m = 1000$ kg, and that the compressor runs at a constant speed of 1800rpm, find the value of *M* such that the isolation system reduces the transmitted force by 75%.

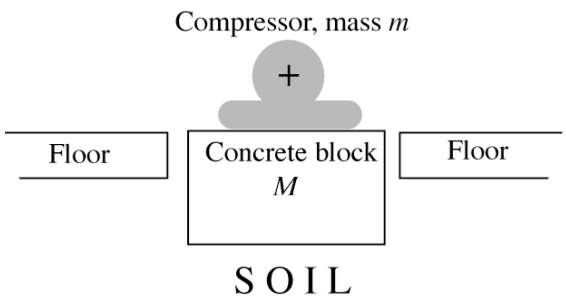

Figure 3a

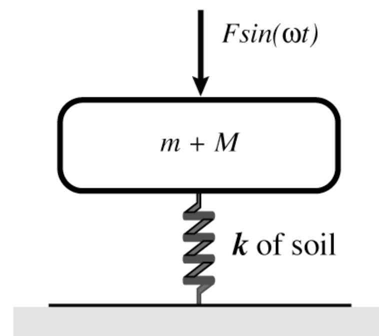

Figure 3b

4. The equations of motion of a 3DOF system are:

$$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\ddot{\mathbf{x}} + \begin{pmatrix} 0.0800 & -0.0100 & 0 \\ -0.0100 & 0.040 & -0.0100 \\ 0 & -0.0100 & 0.0200 \end{pmatrix}\dot{\mathbf{x}} + \begin{pmatrix} 4 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}\mathbf{x} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}\sin\Omega t$$

The transformation $\mathbf{x} = \mathbf{P}\mathbf{p}$, where $\mathbf{P} = \begin{pmatrix} -0.1581 & 0.4472 & 0.1581 \\ -0.5000 & 0.000 & -0.5000 \\ -0.6325 & -0.4472 & 0.6325 \end{pmatrix}$, uncouples the equations as follows,

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\ddot{\mathbf{p}} + \begin{pmatrix} 0.0121 & 0 & 0 \\ 0 & 0.020 & 0 \\ 0 & 0 & 0.0279 \end{pmatrix}\dot{\mathbf{p}} + \begin{pmatrix} 0.2094 & 0 & 0 \\ 0 & 1.00 & 0 \\ 0 & 0 & 1.7906 \end{pmatrix}\mathbf{p} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}\sin\Omega t$$

(a) If $\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$, find the values $\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$. If you are also told that Ω=0.46, clearly outline how you would find the steady-state solutions of the displacements **x.** (i.e., $x_1(t)$, $x_2(t)$, and $x_3(t)$ after any transient motion has decayed to zero).

(b) Would it be reasonable to reduce the order of the system (i.e., only include one or two modes) to obtain a good approximation to the answer to part (a)? Clearly explain why or why not and give numerical values to substantiate your claim.

(c) If the forcing values were now changed to $\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, how many resonant peaks would you see in a plot of $|x_1|$ versus Ω? Explain. Would this be the same for the remaining two coordinates, $|x_2|$ and $|x_3|$? Explain.