Student ID ____________________

Department of Mechanical Engineering
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Ph.D. Qualifying Exam in Fluid Mechanics

- Closed book, but one sheet (8.5" x 11", front and back) of your own notes with equations permitted.
- Some basic equations are provided on an attached information sheet.
- Answer all questions.
- All questions have the same weighting.

Exam prepared by

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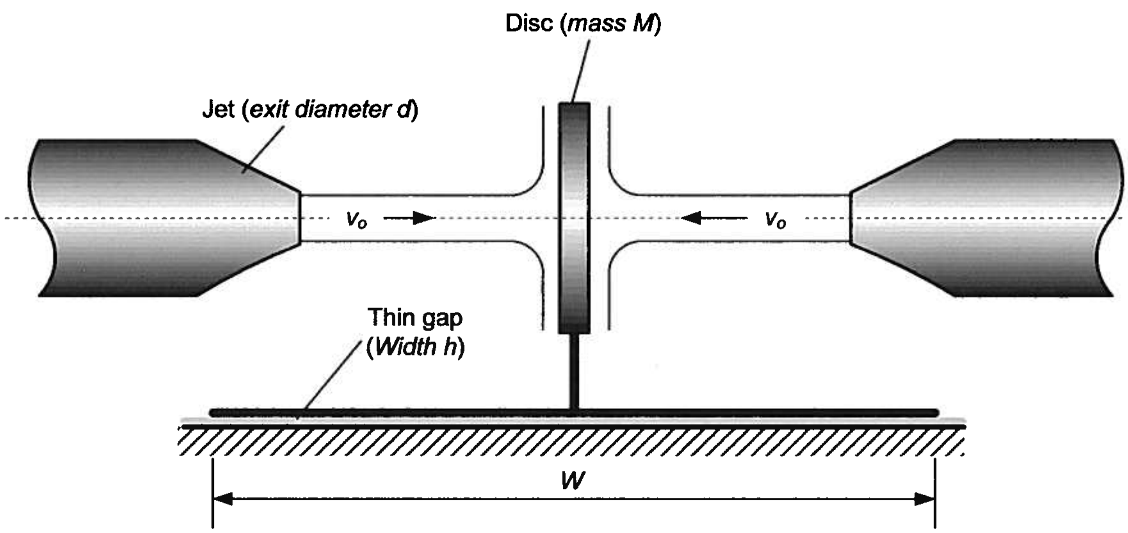
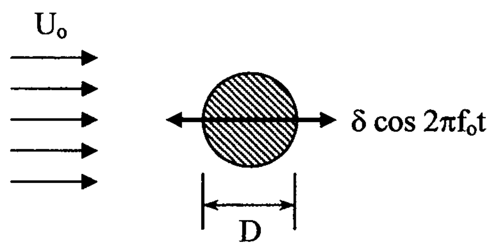
Problem 1:

A disc with mass M is initially in equilibrium under the action of two identical but opposing water (density ρ) jets as shown in the figure below. The disc is supported on a square base with a side length of W and negligible mass. The base is free to move sideways on top of a thin oil film with viscosity μ and thickness h. The disc is suddenly given a velocity equal to the jet velocity (i.e. v_o) to the right. Obtain: (1) an expression for the resulting displacement of the disc versus time; and (2) the maximum displacement. Neglect any mass of liquid adhering to the disc/base and the air drag force. Also, consider the flow to be steady, assume that the motion takes place only along the horizontal direction and ignore the oil-film behavior near the edge of the base.



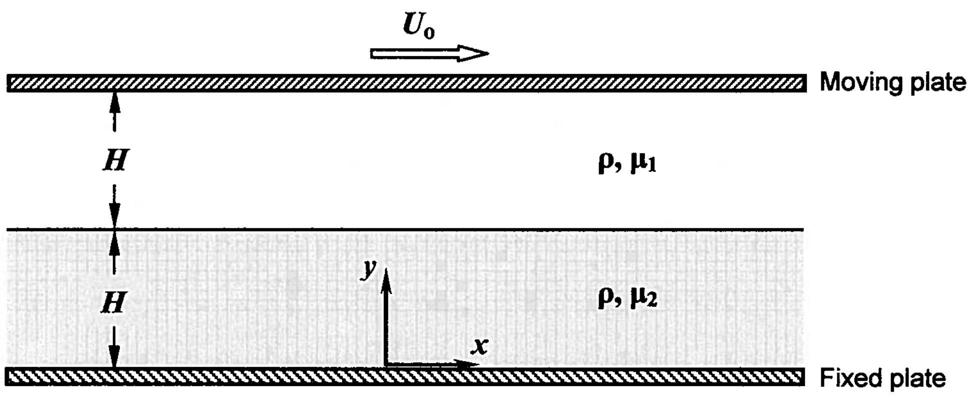
Problem 2:

It is desired to build a micro device in which a 10 μm-diameter cylinder oscilates at a frequency of 100 kHz with/against an air flow of uniform velocity (U_o) of 10 m/s . The amplitude of oscillation (δ) is anticipated to be 1 μm. For the purposes of designing the device, it is desired to estimate the unsteady flow force acting on the cylinder (which may be assumed infinitely long). Because of the difficulty and cost in fabrication of the micro device as well as the difficulty in measuring forces acting on such a small object, an engineer proposes to conduct a scale-up test in a water channel, in which the cylinder diameter is 5 mm. Employ a procedure of your choice to obtain the appropriate non-dimensional parameters for this problem (show details of your work). What should the values of U_o, δ and f_o be for the test? (you may take the kinematic viscosity and density of water as ten and thousand times that of air, respectively)



Problem 3:

Two superposed layers of immiscible liquid of equal density ρ but different viscosity μ, each of constant uniform thickness H, are contained between two infinite, horizontal, parallel plates. The bottom plate is fixed and the upper plate moves with a constant velocity U_o. The fluid motion is caused entirely by the movement of the upper plate, i.e. there is no pressure gradient in the flow direction. The flow can be considered laminar, two-dimensional, steady, and parallel to the plates. Use the coordinate system indicated below.



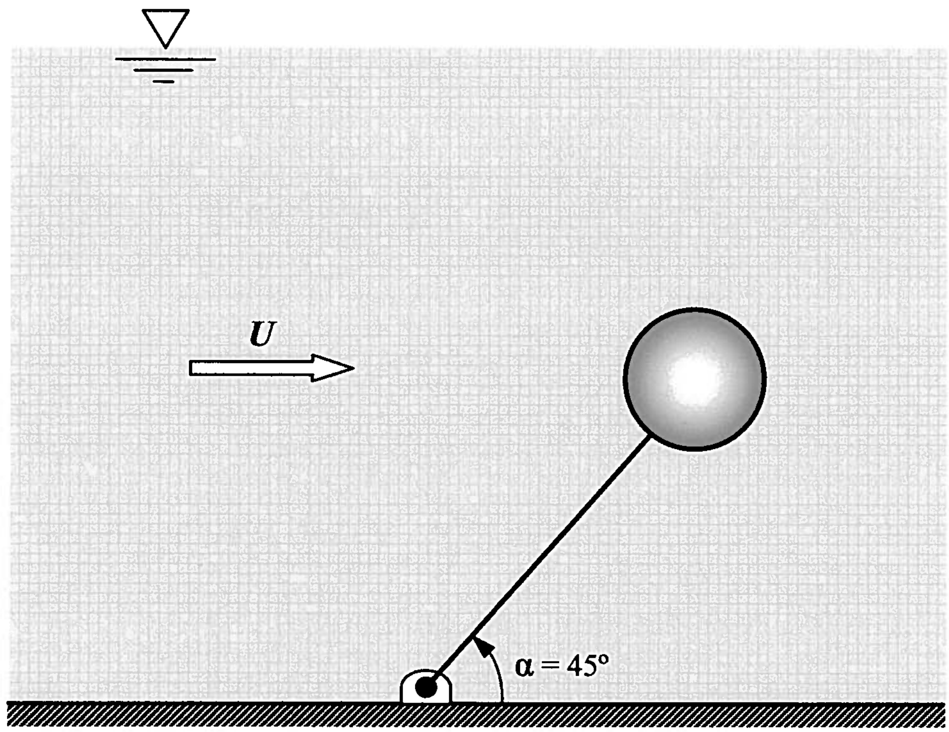
(a) Reduce the equations of motion for this problem and derive the differential equation satisfied by the velocity profiles $u_1(y)$ and $u_2(y)$ in the two fluids. You must explain all your steps, i.e. what are the consequences of the continuity equation, and what happens to the various terms in the Navier-Stokes equations. Clearly indicate the boundary conditions for this problem.

(b) Derive the shape of the velocity profiles $u_1(y)$ and $u_2(y)$ in the two liquid layers.

(c) Derive the velocity U_i at the interface in terms of the plate velocity U_o and the viscosity ratio $M \equiv \mu_2/\mu_1$. For the special case of $M = 100$, determine the interface velocity and sketch the velocity profiles in the two liquid layers.

Problem 4:

A 0.1-m diameter cork ball (specific gravity SG = 0.2) is tied to the bottom of a river as shown below. Calculate the speed *U* of the river current. You may neglect the weight of the cable and the drag on it. For water, $\rho = 999\ kg/m^3$, and $\mu = 0.00112\ kg/(m.s)$.



Integral mass conservation equation:

$$0 = \frac{\partial}{\partial t}\int_{CV}\rho d\forall + \int_{CS}\rho\vec{V}.d\vec{A}$$

Integral momentum equation:

$$\vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV}\vec{V}\rho d\forall + \int_{CS}\vec{V}\rho\vec{V}.d\vec{A}$$

Integral angular momentum equation:

$$\sum(\vec{r}\times\vec{F}) = \frac{\partial}{\partial t}\int_{CV}(\vec{r}\times\vec{V})\rho d\forall + \int_{CS}(\vec{r}\times\vec{V})\rho\vec{V}\cdot d\vec{A}$$

The Equation of Continuity

Rectangular Coordinates (x, y, z):

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Cylindrical Coordinates (r, θ, z):

$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Momentum Equations for Incompressible Newtonian Fluid with Constant Viscosity (μ)

Rectangular Coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] - \frac{\partial p}{\partial x} + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] - \frac{\partial p}{\partial y} + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z$$

Cylindrical Coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right] - \frac{\partial p}{\partial r} + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta}\right] - \frac{1}{r}\frac{\partial p}{\partial\theta} + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z$$

Table C-3 Components of the Stress Tensor for Newtonian Fluids

Rectangular Coordinates (x, y, z)	Cylindrical Coordinates (r, θ, z)
$\tau_{xx} = \mu\left[2\frac{\partial v_x}{\partial x} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{rr} = \mu\left[2\frac{\partial v_r}{\partial r} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$
$\tau_{yy} = \mu\left[2\frac{\partial v_y}{\partial y} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{\theta\theta} = \mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$
$\tau_{zz} = \mu\left[2\frac{\partial v_z}{\partial z} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{zz} = \mu\left[2\frac{\partial v_z}{\partial z} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$
$\tau_{xy} = \tau_{yx} = \mu\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]$	$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$
$\tau_{yz} = \tau_{zy} = \mu\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right]$	$\tau_{\theta z} = \tau_{z\theta} = \mu\left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right]$
$\tau_{zx} = \tau_{xz} = \mu\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right]$	$\tau_{zr} = \tau_{rz} = \mu\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]$
$(\nabla \cdot \mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$	$(\nabla \cdot \mathbf{v}) = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$

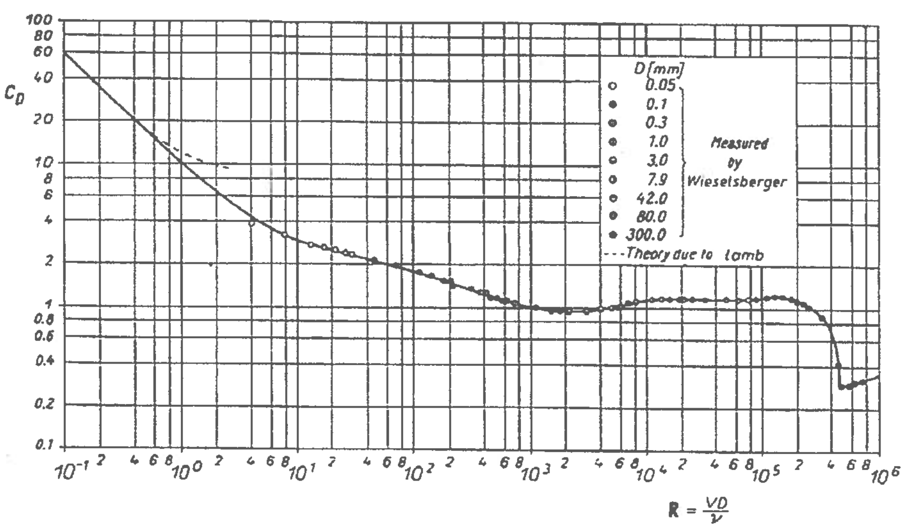


Fig. 1.4. Drag coefficient for circular cylinders as a function of the Reynolds number

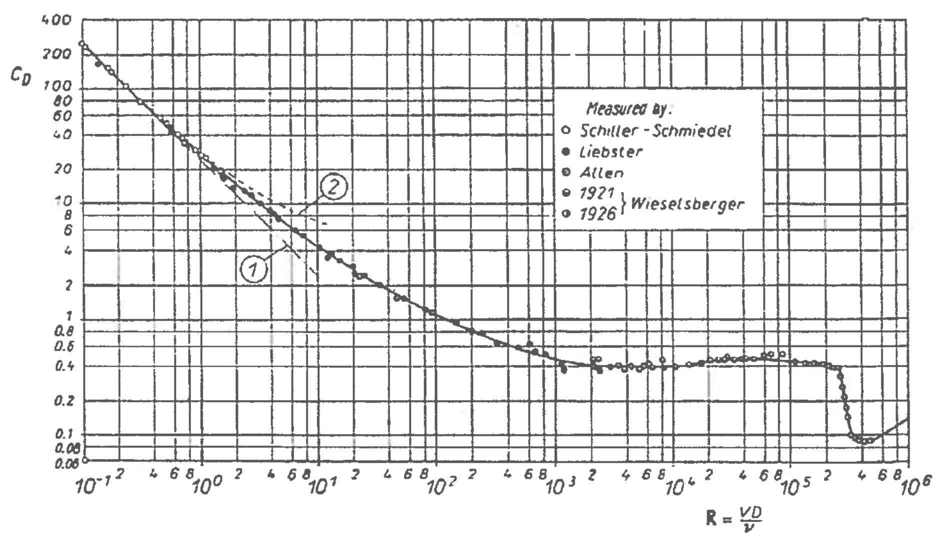


Fig. 1.5. Drag coefficient for spheres as a function of the Reynolds number
Curve (1): Stokes's theory, eqn. (6.10); curve (2): Oseen's theory, eqn. (6.13)

Curve (1), Stokes' theory: $C_D = 24/\mathbf{R}$