Student ID ____________________

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Ph.D. Qualifying Exam in Fluid Mechanics

- Closed book, but one sheet (8.5" x 11", front and back) of your own notes with equations permitted.
- Some basic equations are provided on an attached information sheet.
- Answer all questions.
- All questions have the same weighting.

Exam prepared by

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Spring 2009

Problem 1:

The thrust from an airplane propeller is a function of the following variables:
V - speed of the airplane
D - diameter of the propeller
ρ - air density
μ - air viscosity
c - speed of sound in air
ω - rotational speed of the propeller

1. Find the dimensionless groups that characterize the thrust and give them names where appropriate.

2. Suppose a one-quarter scale model of the propeller is to be tested in an air flow and that effects of compressibility can be neglected. What values of V and ω should be chosen for these tests?

3. Suppose effects of compressibility could not be ignored. Could full similarity still be achieved in an air flow for the one-quarter scale model? Explain why or why not.

Problem 2:

A Newtonian fluid flows steadily in the x-direction between two parallel plates a distance h apart under the influence of a pressure gradient dp/dx, although no body forces are present. The flow is of unit depth, the lower plate at $y=0$ is stationary and the upper one at $y=h$ moves at velocity U in the x direction.

1. Give the simplest form of the x-momentum equation that describes this flow.

2. Find an expression for the velocity $u(y)$

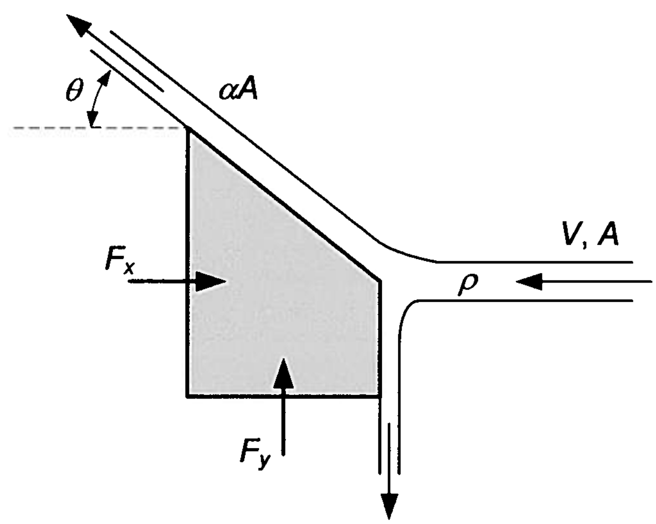
3. Find an expression for the average velocity across this flow.

4. At what value of dp/dx is the average velocity zero?

Problem 3:

A jet of liquid of density ρ and area A strikes a block and splits into two jets, as in the figure below. Assume the same uniform velocity V for all three jets. The upper jet exits at an angle θ and area αA. The lower jet is turned 90° downward. Neglecting block and fluid weight as well as viscous effects:

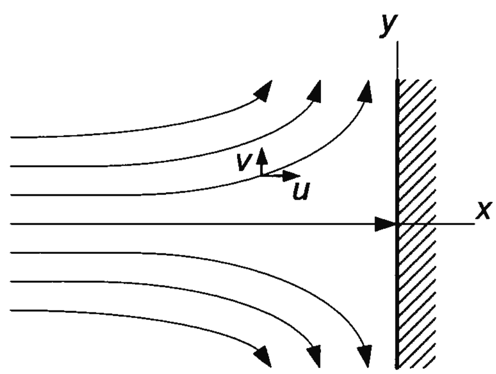
a) Derive a formula for the forces (F_x, F_y) required to support the block.
b) Show that $F_y = 0$ only if $\alpha \geq 0.5$.
c) Find the values of α and θ for which both F_x and F_y are zero. What is the block shape corresponding to these values?



Problem 4:

The inviscid, incompressible flow in the vicinity of a stagnation point (see Figure below) is approximated by $u = -10x$ and $v = 10y$. If the pressure at the origin is p_o, find an expression for the pressure neglecting gravity effects:

a) Along the negative x-axis
b) Along the positive y-axis
c) Would the procedure for obtaining the pressure in parts a and b above be different if the flow was viscous? If so, then describe the procedure for viscous flow (but don't actually solve for the pressure).



Integral mass conservation equation:

$$0 = \frac{\partial}{\partial t} \int_{CV} \rho d\forall + \int_{CS} \rho \vec{V} . d\vec{A}$$

Integral momentum equation:

$$\vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t} \int_{CV} \vec{V} \rho d\forall + \int_{CS} \vec{V} \rho \vec{V} . d\vec{A}$$

Integral angular momentum equation:

$$\sum (\vec{r} \times \vec{F}) = \frac{\partial}{\partial t} \int_{CV} (\vec{r} \times \vec{V}) \rho d\forall + \int_{CS} (\vec{r} \times \vec{V}) \rho \vec{V} \cdot d\vec{A}$$

The Equation of Continuity

Rectangular Coordinates (x, y, z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Cylindrical Coordinates (r, θ, z):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Momentum Equations for Incompressible Newtonian Fluid with Constant Viscosity (μ)

Rectangular Coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}\right) = \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] - \frac{\partial p}{\partial x} + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}\right) = \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] - \frac{\partial p}{\partial y} + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z$$

Cylindrical Coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r} \frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta}\right] - \frac{\partial p}{\partial r} + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r} \frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta}\right] - \frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z$$

Table C-3 Components of the Stress Tensor for Newtonian Fluids

Rectangular Coordinates (x, y, z)	Cylindrical Coordinates (r, θ, z)
$\tau_{xx} = \mu\left[2\frac{\partial v_x}{\partial x} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{rr} = \mu\left[2\frac{\partial v_r}{\partial r} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$
$\tau_{yy} = \mu\left[2\frac{\partial v_y}{\partial y} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{\theta\theta} = \mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$
$\tau_{zz} = \mu\left[2\frac{\partial v_z}{\partial z} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$	$\tau_{zz} = \mu\left[2\frac{\partial v_z}{\partial z} - \frac{2}{3}(\nabla \cdot \mathbf{v})\right]$
$\tau_{xy} = \tau_{yx} = \mu\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]$	$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$
$\tau_{yz} = \tau_{zy} = \mu\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right]$	$\tau_{\theta z} = \tau_{z\theta} = \mu\left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right]$
$\tau_{zx} = \tau_{xz} = \mu\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right]$	$\tau_{zr} = \tau_{rz} = \mu\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]$
$(\nabla \cdot \mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$	$(\nabla \cdot \mathbf{v}) = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$

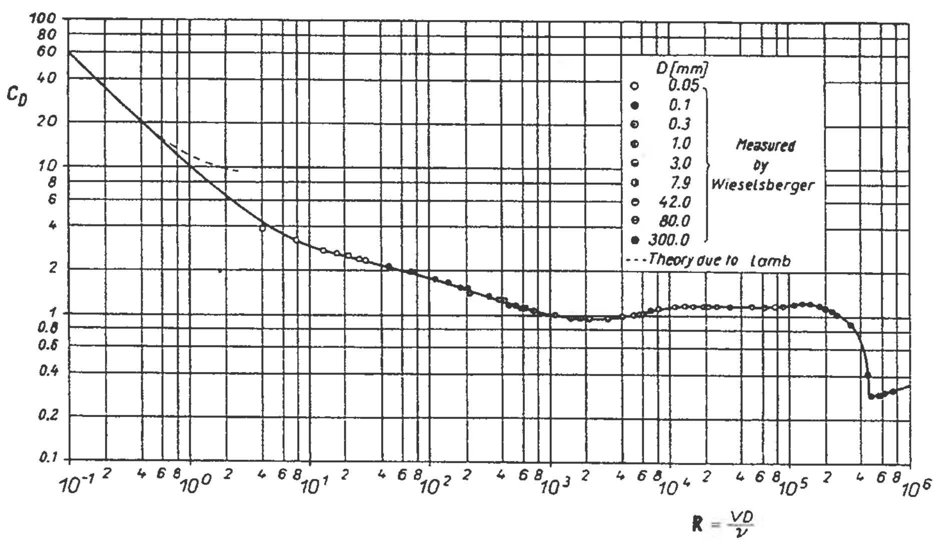
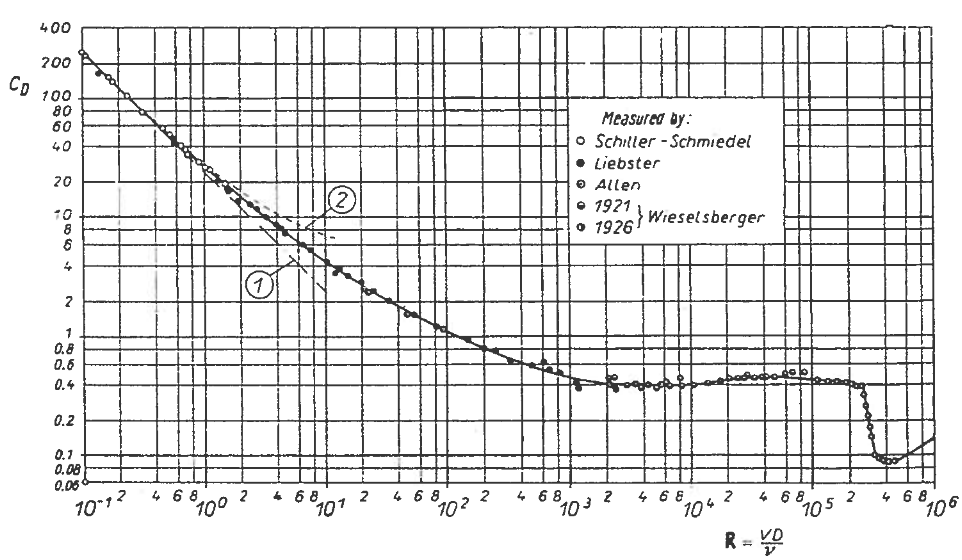


Fig. 1.4. Drag coefficient for circular cylinders as a function of the Reynolds number



Curve (1), Stokes' theory: $C_D = 24/\mathbf{R}$

Fig. 1.5. Drag coefficient for spheres as a function of the Reynolds number
Curve (1): Stokes's theory, eqn. (6.10); curve (2): Oseen's theory, eqn. (6.13)