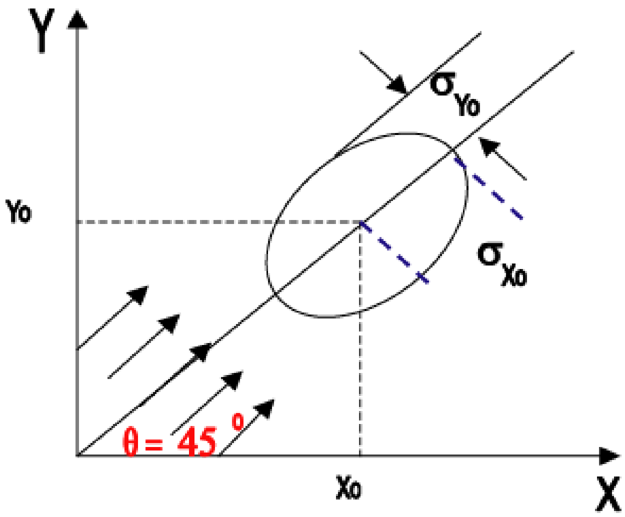
Advection and Anisotropic Dispersion of a Gaussian Spill in Uniform Flow at an Angle with Grid Orientation

An analytical solution for the migration of a gaussian plume in uniform flow was presented by Baetsle in 1969. A comparison of a special case, when there is an orientation in the uniform flow with the horizontal axes, is presented below (Figure 1).



(moving with the plume center)

Figure 1. The initial concentration is gaussian plume centered at (x_0 and y_0) with an initial standard deviation of σ_{x0} and σ_{y0}

ANALYTICAL SOLUTION

The analytical solution for the concentration distribution of the plume in space and time is given by equation 1:

$$C(x,y,t) = \frac{C_m \sigma_{x_0} \sigma_{y_0}}{\sigma_x \sigma_y} e^{-\frac{(X-x_0)^2}{2\sigma^2 x} - \frac{(Y-y_0)^2}{2\sigma^2 y}} \qquad (1)$$

Where

$$\sigma_x^2 = \sigma_{xo}^2 + 2D_L t \qquad (2)$$

$$\sigma_y^2 = \sigma_{yo}^2 + 2D_T t \qquad (3)$$

$$X = x' - \bar{v}_x t \qquad (4)$$

$$Y = y' - \bar{v}_y t \qquad (5)$$

$$x' = x\cos(\theta) + y\sin(\theta) \qquad (6)$$

$$y' = y\cos(\theta) - x\sin(\theta) \qquad (7)$$

and

C_m is the initial concentration of the gaussian plum[ML^{-3}]

σ_{x_0} initial standard deviation at x direction

σ_{y_0}	initial standard deviation at y direction
x_0, y_0	is the central coordinate of the gaussian plume[L]
D_L	is the longitudinal dispersion coefficient [l^2T^{-1}]
D_T	is the transverse dispersion coefficient [l^2T^{-1}]
$\overline{v_x}$, $\overline{v_y}$	are the velocity of uniform flow in x and y direction [LT^{-1}]
θ	is the angle of uniform flow with the horizontal axis

IGW NUMERICAL SOLUTION

IGW is utilized to solve the flow problem for the given situation presented below.

Given physical parameters:

σ_{x_0} =50	$\overline{v_x}$ =10 m/day
σ_{y_0} =35	$\overline{v_y}$ =1 m/day
D_L=10 m^2/day	θ=45 degree
D_T =1 m^2/day	C_m=100 ppm

Given Numerical Parameters:

$\Delta x = 25$ m	Spacing between the cells in the x-direction
$\Delta y = 25$ m	Spacing between the cells in the y-direction
$\Delta t = 0.5$-day	Time step

Analytical Solution versus IGW

IGW applies an improved method for approximating the general numerical scheme. The IGW solution is presented below and can be compared with the exact solution found in Figure 2. The results obtained from IGW are physically realistic with no oscillations and no negative concentrations.

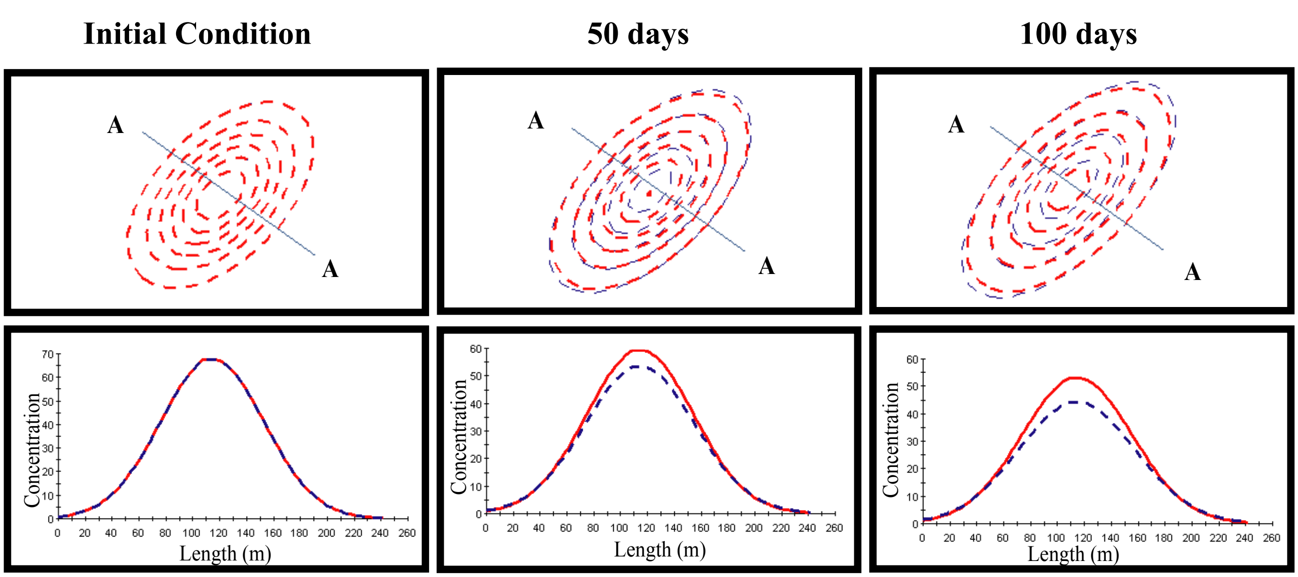


Figure 2. Comparison between the improved solution using IGW and the exact solution. (Red tracer is the exact solution and the blue tracer is the IGW solution).

Analytical Solution versus the Traditional Method (MT3D and RT3D)

The results obtained from the traditional finite difference method, which is used in some popular packages such as MT3D and RT3D, are also compared with those found using the exact solution. This comparison is displayed below (Figure 4). The results obtained from the traditional finite difference method show unphysical oscillations and negative concentrations.

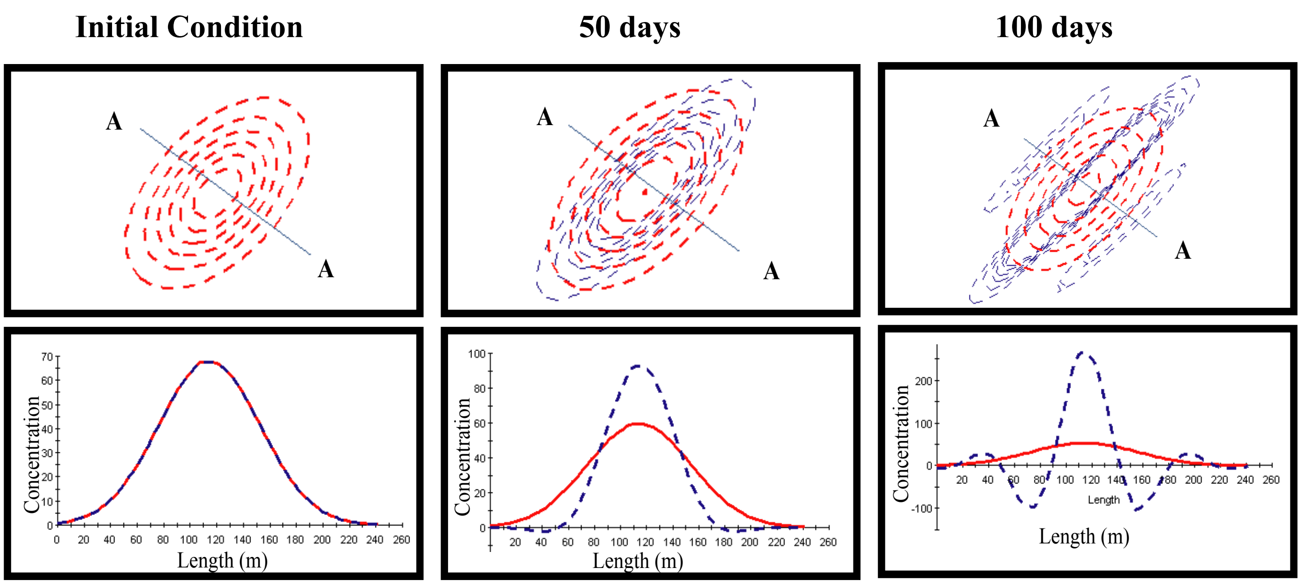


Figure 3. Comparison between the traditional method and the exact solution (red tracer is the exact solution and the blue tracer is the traditional method solution).

Direct Comparison

A direct comparison between the exact, IGW and the traditional solution is illustrated in figure 4. The results indicate that the IGW solution and the analytical solution provide very similar solutions, however, the traditional method provides significantly different and incorrect results.

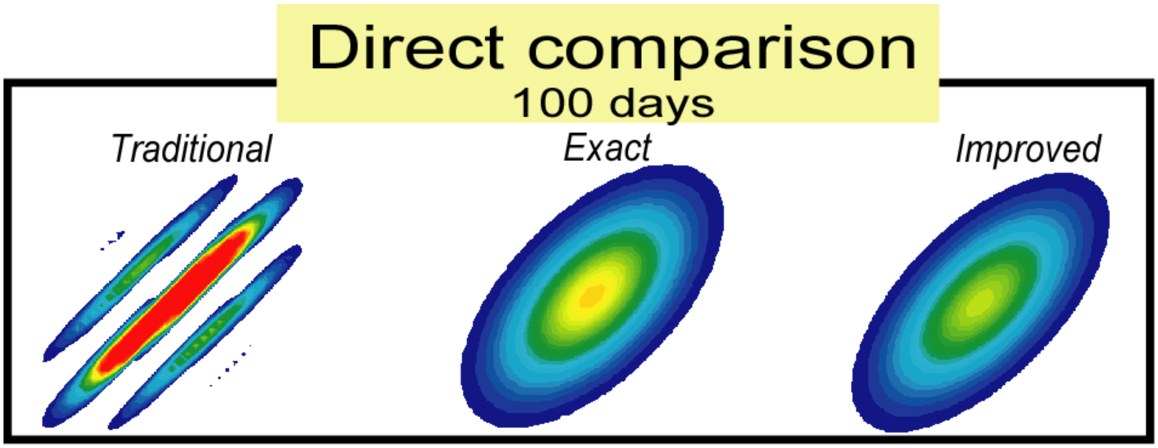


Figure 4. Direct comparison for concentration contours between the Analytical Solution, IGW, and the Traditional solution.