Advection and Anisotropic Dispersion of a Gaussian Spill in Uniform Flow

Baetsle presented an analytical solution for the migration of a gaussian plume in uniform flow in 1969 (See Figure 1).

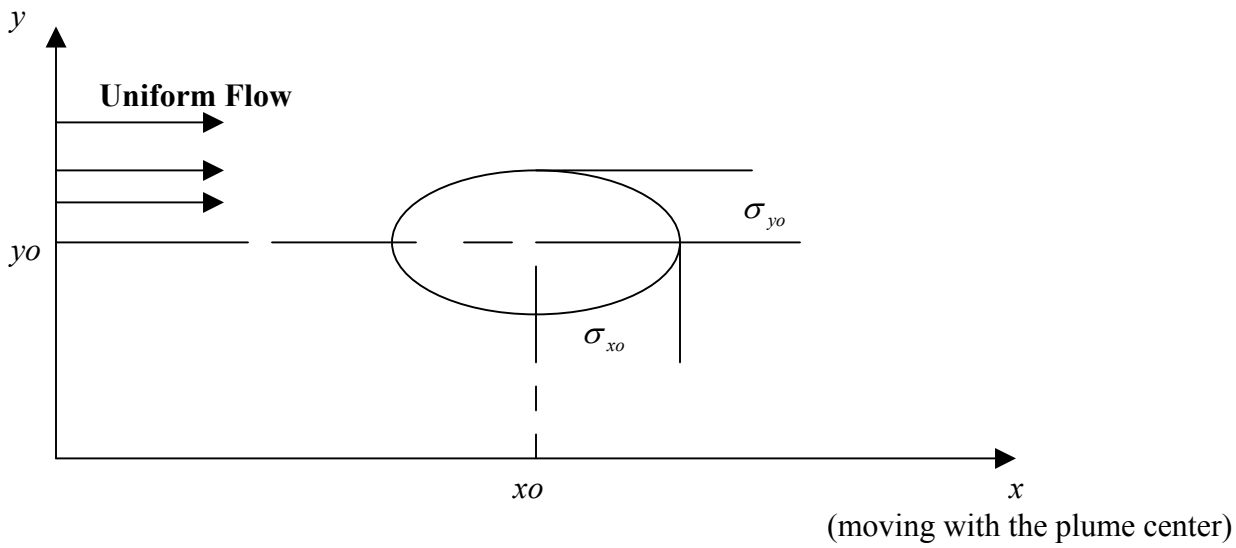

Figure 1. The initial concentration is gaussian plume centered at (x_0 and y_0) with an initial standard deviation of σ_{x0} and σ_{y0}

ANALYTICAL SOLUTION

The analytical solution for the concentration distribution of the plume in space and time is given by equation 1:

$$C(x,y,t) = \frac{C_m \sigma_{x_0} \sigma_{y_0}}{\sigma_x \sigma_y} e^{-\frac{(X-x_0)^2}{2\sigma^2 x} - \frac{(Y-y_0)^2}{2\sigma^2 y}} \quad (1)$$

Where

$$\sigma_x^2 = \sigma_{xo}^2 + 2D_L t \quad (2)$$

$$\sigma_y^2 = \sigma_{yo}^2 + 2D_T t \quad (3)$$

$$X = x - \bar{v}_x t \quad (4)$$

$$Y = y \quad (5)$$

and

C_m is the initial concentration of the gaussian plum[ML^{-3}]

σ_{x_0} initial standard deviation at x direction

σ_{y_0} initial standard deviation at y direction

x_0, y_0 is the central coordinate of the gaussian plume[L]

D_L is the longitudinal dispersion coefficient [l^2T^{-1}]

D_T is the transverse dispersion coefficient [l^2T^{-1}]

$\overline{v_x}$ is the velocity of uniform flow in x direction [LT^{-1}]

IGW NUMERICAL SOLUTION

IGW is utilized to solve the transport problem for the situation presented below.

Given physical parameters:

σ_{x_0} =50	$\overline{v_x}$ =1 m/day
σ_{y_0} =35	C_m =100 ppm
D_L =10 m^2/day	D_T =1 m^2/day
x_0 = 500 m	y_0 = 375 m

Given Numerical Parameters:

$\Delta x = 10$ m	Spacing between the cells in the x-direction
$\Delta y = 10$ m	Spacing between the cells in the y-direction
$\Delta t = 0.5$-day	Time step

Analytical Solution versus IGW

The IGW solution is presented below and can be compared with the exact solution found in Figure 2. The results obtained from IGW are analogous with those obtained by the analytical solution.

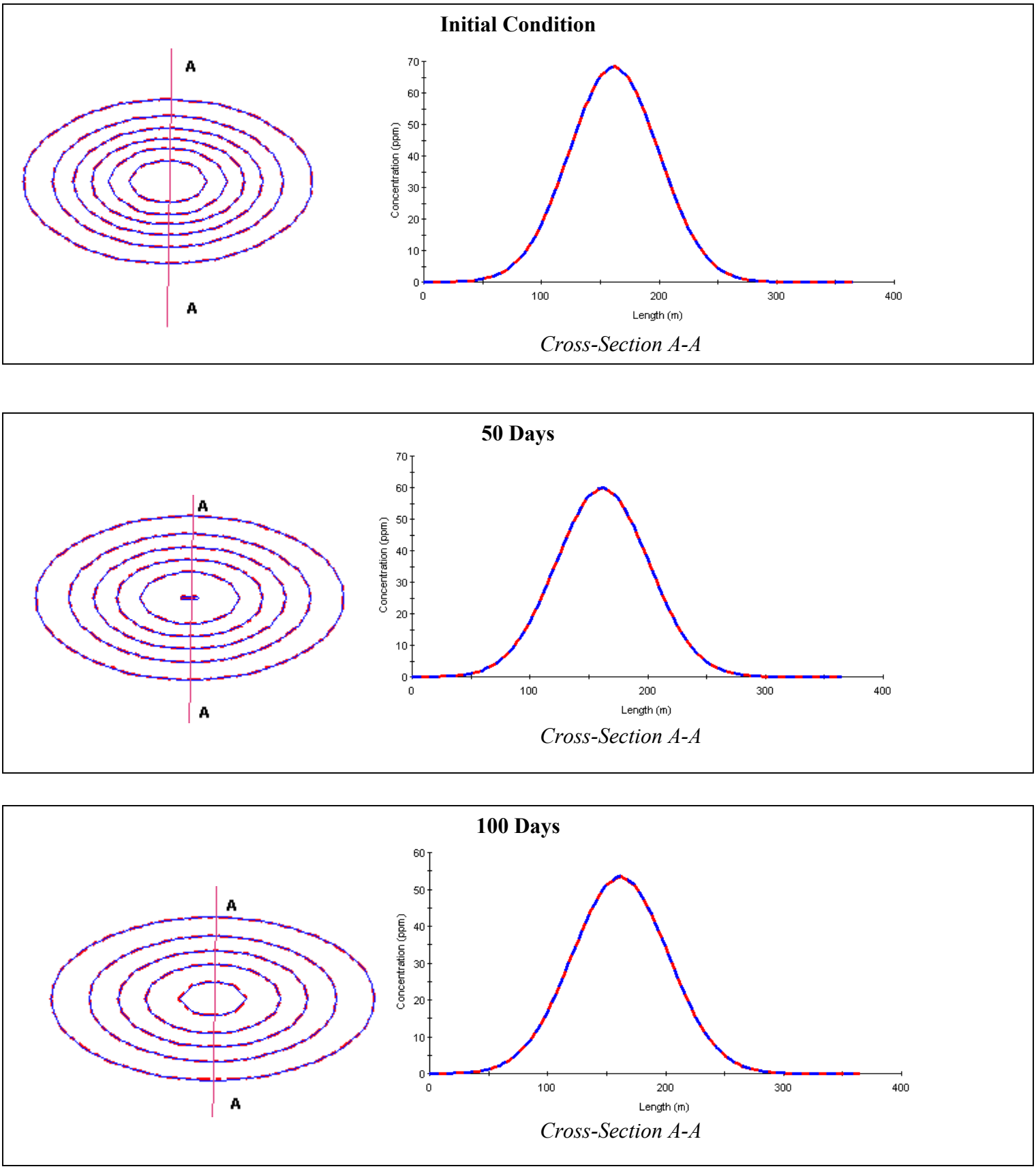

Figure2. Comparison between IGW and the *exact solution.*