

IGW Solution Vs. Exact Solution
Flow in an Infinite, Uniform, Anisotropic, and Confined Aquifer
Under Constant Pumping

Exact Solution

The exact solution to flow in an infinite, anisotropic, and confined aquifer under constant pumping is (USGS Water-Supply Paper 2308, 1987):

$$h_0 - h(x, y, t) = \frac{Q\sqrt{p}}{4\pi T} W(u) \quad (1)$$

Where

$$u = \frac{(x^2 + py^2)S}{4Tt} \quad (2)$$

$$W(u) = \int_u^\infty \frac{e^{-u} du}{u} \quad (3)$$

and

$x, y =$ rectilinear coordinates relative to the pumping well, [m]

$p = \frac{K_x}{K_y}$ is the ratio of anisotropy

$S =$ is the aquifer storage coefficient, [-]

$T =$ is the aquifer transmissivity, [L^2/T]

$t =$ is the time, [T]

$h_0 =$ is the initial head, [L]

$Q =$ is the constant flow rate abstracted from the well, [L^3/T]

$W(u) =$ is the Well function

IGW Numerical Solution

IGW is applied to solve the flow problem given the following assumptions:

Physical parameters:

$$Q = 1000 \text{ m}^3/\text{day}$$

$$h_0 = 20 \text{ m}$$

$$S = 0.0002$$

$$t = 0.00175 \text{ days} = 151.37 \text{ sec}$$

$$T = 1000 \text{ m}^2/\text{day} = K_x \cdot \text{Thickness} = 50 \text{ m/day} \cdot 20 \text{ m}$$

$$p = 10$$

Numerical Parameters:

$$\Delta x = 10 \text{ m}$$

$$\Delta y = 10 \text{ m} \quad \Delta t = 1.036 \text{ sec}$$

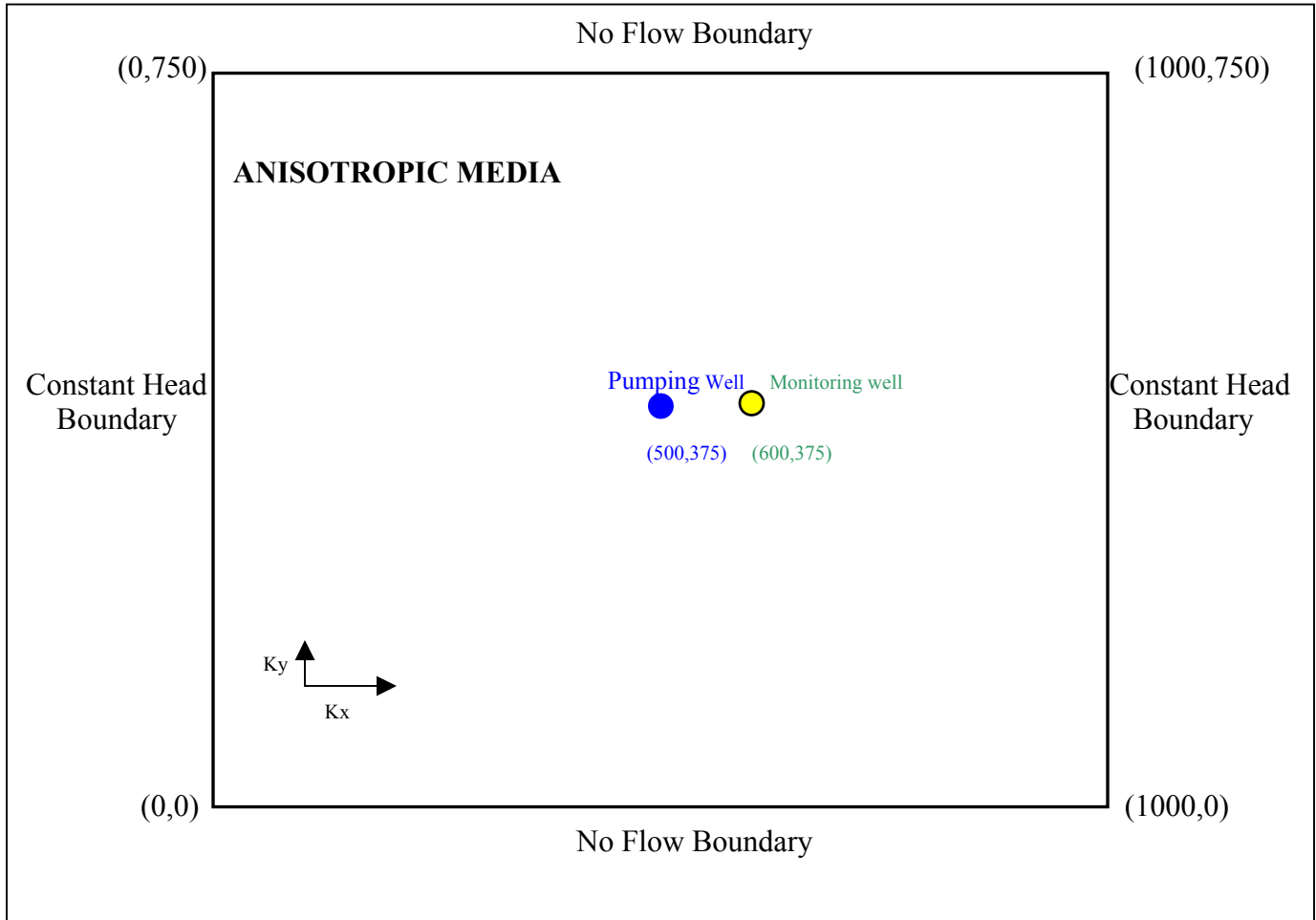
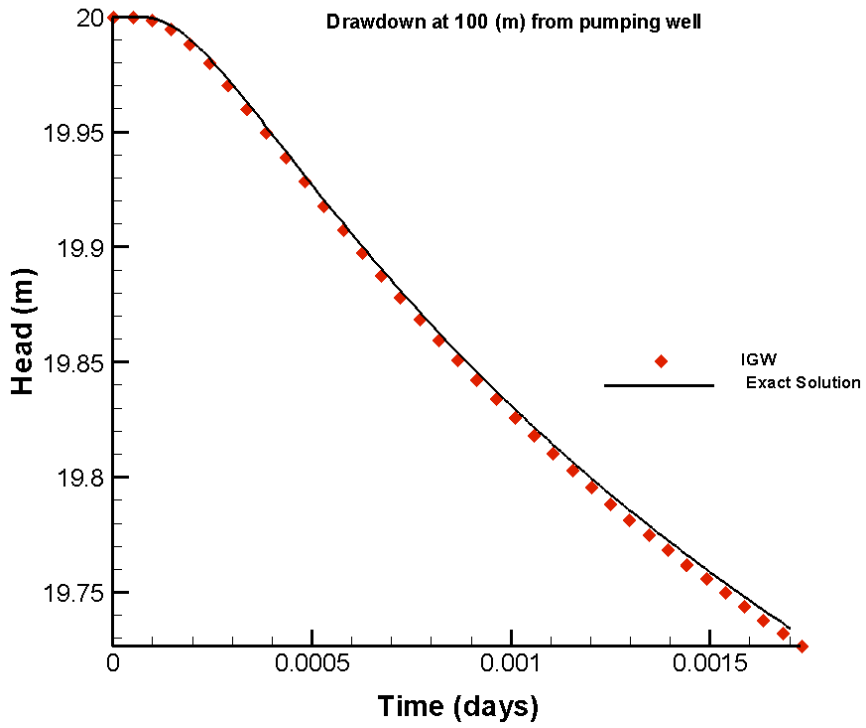


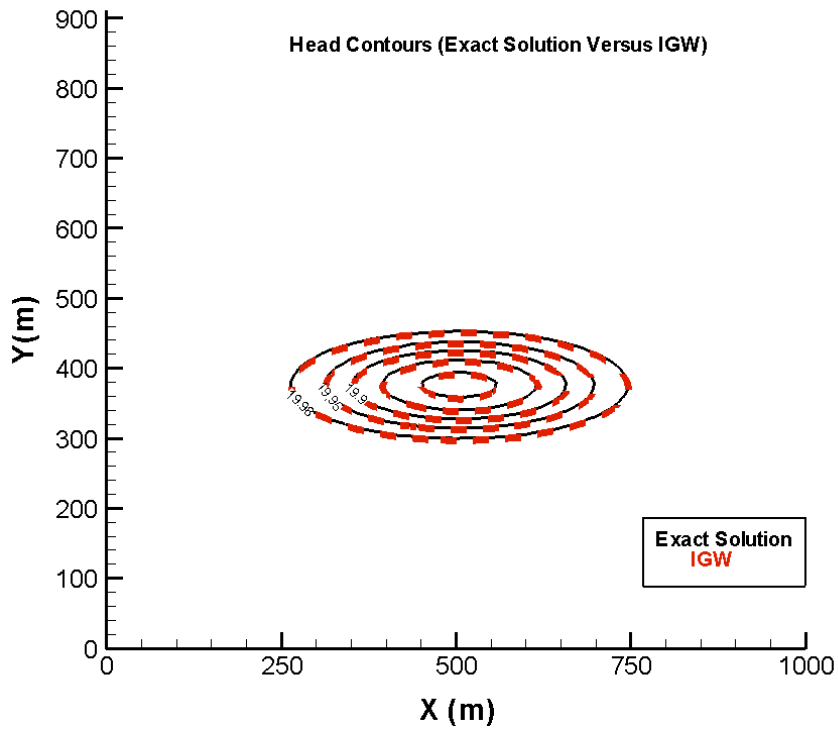
Fig 2. Plan view of IGW model set up for comparison to the exact solution

Analytical Solution versus IGW

The IGW solutions are presented and compared with the exact solution in Figures 3 and 4.



[Fig 3. Comparison of the Analytical solution and IGW solution at a location 100 meters from the pumping well.](#)



[Fig 4. Comparison of the Analytical solution with IGW solution at 151.37 seconds](#)

The numerical solution is graphically indistinguishable from the exact solution until the drawdown influence begin to reach the boundaries.

The cone of depression is also plotted in 3D in Figure 5.

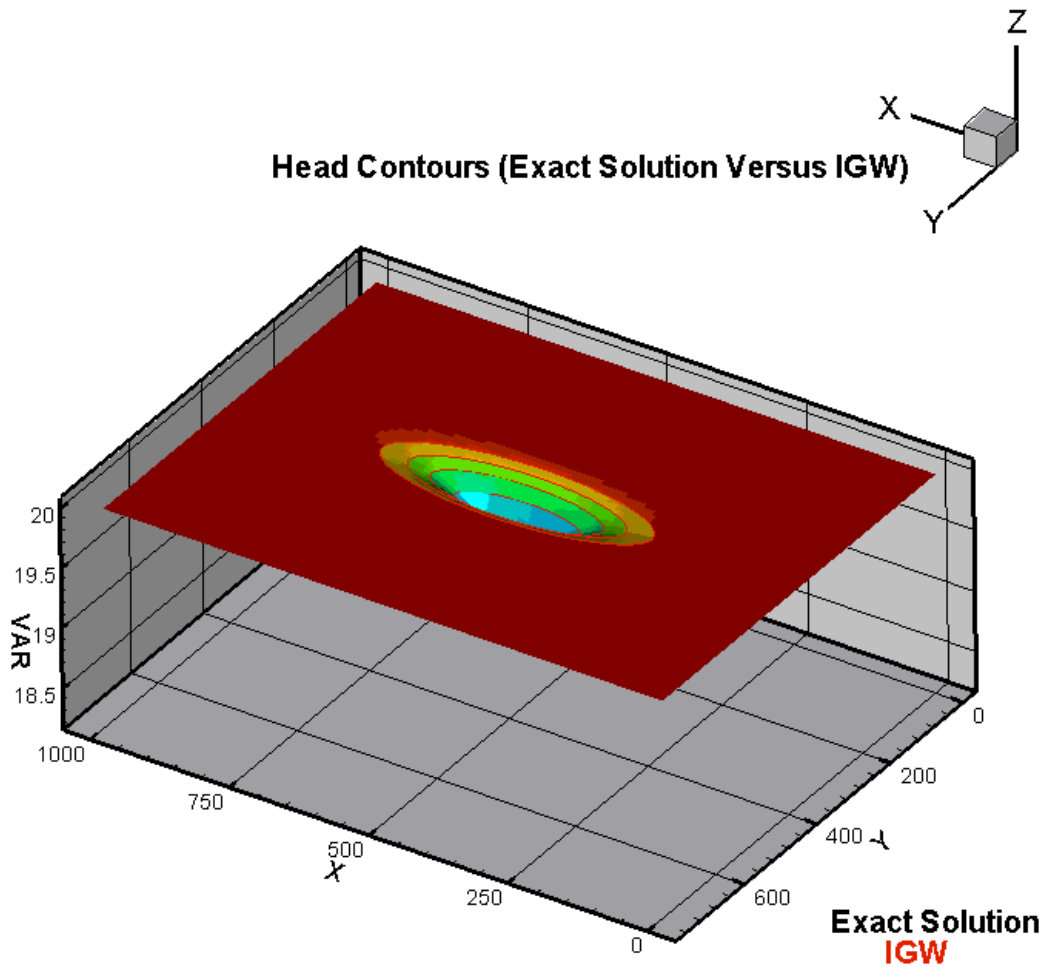


Figure 5. The Cone of Depression in an Uniform Anisotropic Aquifer