Shyh-Leh Chen
Associate Professor,
Department of Mechanical Engineering,
National Chung-Cheng University,
Chia-Yi 621, Taiwan

Steven W. Shaw
Fellow ASME and Professor,
Department of Mechanical Engineering,
Michigan State University,
East Lansing, MI 48824

Hassan K. Khalil
Professor,
Department of Electrical Engineering,
Michigan State University,
East Lansing, MI 48824

Armin W. Troesch
Professor,
Department of Naval Architecture
and Marine Engineering,
The University of Michigan,
Ann Arbor, MI 48109

Robust Stabilization of Large Amplitude Ship Rolling in Beam Seas

The dynamics and control of a strongly nonlinear 3-DOF model for ship motion are investigated. The model describes the roll, sway, and heave motions occurring in a vertical plane when the vessel is subjected to beam seas. The ship is installed with active antiroll tanks as a means of preventing large amplitude roll motions. A robust state feedback controller for the pumps is designed that can handle model uncertainties, which arise primarily from unknown hydrodynamic loads. The approach for the controller design is a combination of sliding mode control and composite control for singularly perturbed systems, with the help of the backstepping technique. It is shown that this design can effectively control roll motions of large amplitude, including capsize prevention. Numerical simulation results for an existing fishing vessel, the twice-capsized Patti-B, are used to verify the analysis. [S0022-0434(00)02701-5]

1 Introduction

Ship roll stabilization is an important issue for comfort in passenger vessels, cargo integrity in cargo vessels, and targeting in military vessels. In addition, the matter of safety against capsize in extreme seas is of utmost importance for all vessels. Attempts at controlling or reducing ship rolling motions have a long history dating back to the late nineteenth century, and several methodologies have been proposed. A historical account of this subject is given in Bennett [1]. Passive methods appeared first, including bilge keels, antiroll tanks, moving weights, and gyroscopic methods (see [2] for examples). Following the development of feedback control theory, active methods began to emerge, many of which were inspired by or modified from the passive ones, including fin stabilizers, active tanks, controlled moving weights, and active gyroscopic methods (see [3] for examples).

As control theory has progressed and ship dynamic models have improved, new control strategies have been brought to bear on this problem. Examples include a recently reported controlled-wing actuator (similar to fin stabilizers) with an adaptive controller based on gain scheduling and neural networks [4], and a rudder-roll stabilization system that has been incorporated with optimal control [5], adaptive control and gain scheduling [6]. A good collection of recent developments on the topic of control of sea-going vehicles, such as autopilots and ship positioning, is provided in the book by Fossen [5].

In this paper, roll stabilization for a strongly nonlinear multi-DOF model for ship motion is investigated. The model has 3-DOF, including roll, sway, and heave motions occurring in a vertical plane, under the action of beam seas (that is, waves that encounter the vessel directly broadside). The vessel is assumed to be at anchor or under low speed for work and hence has negligible forward speed. The objective of this study is to design a stabilizing feedback controller that takes into account model uncertainties. Robustness is a major consideration since it is virtually impossible to develop accurate models for large amplitude ship motions, due to the difficulties involved in solving the associated free-surface hydrodynamic problem.

Because of these uncertainties, the best one can achieve is ultimate boundedness of the motion. This is sufficient for present purposes, as there will be a significant reduction in the amplitude of rolling motions and the control will prevent the ship from capsizing under severe sea conditions. Without control a vessel is more susceptible to the possibility of capsize under large amplitude seas [7,8].

Of the possible actuation methods, the gyroscopic method and moving weight schemes are impractical, while the fin stabilizer and rudder-roll systems are not effective at low vessel speeds. Therefore, antiroll tanks and pumps are employed as actuators in order to dynamically change the horizontal position of the vessels center of gravity (CG) in such a way that the roll motions are reduced. However, the position of the CG cannot be shifted instantaneously, and therefore the control scheme will involve a dynamic state feedback controller.

The control system considered has three time scales, and can be cast in a singularly perturbed form. Our approach for the robust controller design is based on a smooth version of sliding mode control, which handles the uncertainties, together with the backstepping method and the idea of composite control for singularly perturbed systems [9].

The paper is organized as follows. The ship model is very briefly described in Section 2. The design of the robust state feedback controller is outlined in Section 3. In Section 4, simulation results for an example vessel are presented and some practical issues regarding the control effort are discussed. Conclusions are drawn in Section 5, where some directions for further investigation are also provided.

2 The Ship Model

In this section the ship dynamic state model is stated and its general structure is discussed. The nondimensional state equation

Contributed by the Dynamic Systems and Control Division for publication in the JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL. Manuscript received by the Dynamic Systems and Control Division February 24, 1997. Associate Technical Editor: Woong-Chul Yang.

model which represents the roll, sway, and heave motions of a vessel traveling in regular beam seas has been obtained in previous work by the authors [10]. In terms of variables and functions defined below, and under suitable nondimensionalization and rescaling, this model is of the form

$$\dot{x}_1 = x_2, \tag{1}$$

$$\dot{x}_2 = f_{11}(x_1) + f_{12}(x_1)x_3 + \Delta f(x_1, x_3) + \epsilon(g_1 + \Delta g_1) \times (x_1, x_2, x_3, y, z_1, z_2, \tau), \tag{2}$$

$$\dot{x}_3 = 0,$$

$$\dot{y} = \epsilon(g_2 + \Delta g_2)(x_1, x_2, y, z_2, \tau), \tag{3}$$

$$\epsilon \dot{z}_1 = z_2, \tag{4}$$

$$\epsilon \dot{z}_2 = -a z_1 - b z_2 + \epsilon(g_3 + \Delta g_3)(x_1, x_2, x_3, y, z_1, z_2, \tau), \tag{5}$$

where x_1, x_2, y, z_1, and z_2 represent, respectively, the roll angle, roll velocity, sway velocity, heave displacement (relative to the water surface), and heave velocity; x_3 is the horizontal position of the CG, which is simply a constant in the open-loop system; and f_{ij} and g_i are known functions that approximately model the effects of wind, hydrostatic, and hydrodynamic forces.

These state equations are derived in a wave-fixed coordinate system. The nonlinear effects of hydrostatics and inertia have been accounted for, but an essentially linear hydrodynamics model is employed. The only nonlinear hydrodynamic effect accounted for is quadratic roll damping. Note also that the time variable has been rescaled using the unbiased[1] roll natural frequency, which is assumed to be small compared to the heave natural frequency. Vessel sway does not have a ''stiffness'' effect, and hence its dynamics are first order in nature.

There are two sources of model uncertainties, one from hydrostatics and the other from hydrodynamics. The functions f_{ij}'s in the state equation represent the contributions from hydrostatic forces. For a given hull shape, these functions can be obtained in an integral form, but quite often they cannot be expressed in a closed form in terms of the roll angle. However, in most cases, polynomials can well approximate them in a best-fit sense. It should be noted that if better functional fits for f_{ij}'s are available, they can be easily employed in place of the cubic polynomials used in the present study. The discrepancy between the actual and approximate righting moment (i.e., $f_{11}(x_1) + f_{12}(x_1)x_3$) is represented by the uncertainty function $\Delta f(x_1, x_3)$. For the other hydrostatic functions, the differences are contained in the functions Δg_i's.

On the other hand, the significant model uncertainties arising from the hydrodynamics are represented in part by the uncertainty functions Δg_i's and in part by the unknown positive constants a and b in Eq. (5). All these uncertainty functions are assumed to be continuously differentiable in their arguments.

As one can see from the state equations, the system is a three time-scaled, singularly perturbed system in which the heave motion is the fastest, the roll motion is next, and the sway motion is the slowest. The small perturbation parameter ϵ is the order of the ratio of the unbiased roll natural frequency to the heave natural frequency. The unperturbed system with $\epsilon = 0$ corresponds to the condition in calm water with no damping and no wind. In this case there exists a two-dimensional invariant manifold containing roll motions uncoupled from heave and sway, and it possesses three equilibrium states—a center at the origin, corresponding to the upright position, and two saddles representing the angles of vanishing stability; see Fig. 1. In the unbiased case the two saddle points are connected by a heteroclinic cycle whose interior is referred to as the unperturbed *safe region*, which is denoted by the shaded region in Fig. 1. The controller will essentially keep the vessel from escaping the safe area in the invariant roll manifold of the perturbed system, and it will account for the coupling to heave and sway dynamics.

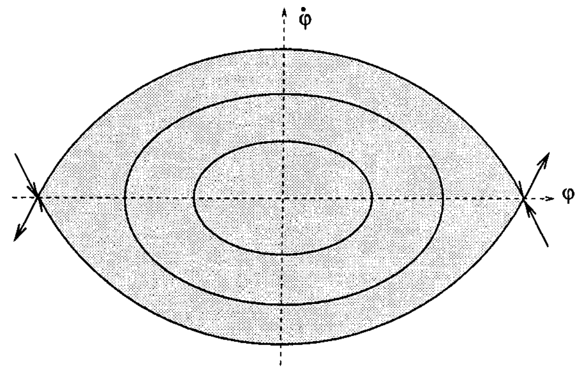

Fig. 1 The unperturbed system in the roll manifold

3 Design of a Robust Stabilizing Controller

In this section, a robust state feedback controller will be designed using the method of active antiroll tanks. The antiroll tanks, as shown in Fig. 2, consist of two tanks connected at the bottom with one on the port side of the vessel and the other on the starboard side. The fluid in the tanks can be moved from one side to the other through the connection tubes, and, in this way, the CG of the vessel can be controlled.

Assume that the flow rate of the fluid between the tanks can be directly controlled by actuators, such as pumps, added to the connection tubes. When equipped with such antiroll tanks, the third state equation is given by:

$$\dot{x}_3 = u, \tag{6}$$

where u is proportional to the flow rate and serves as the control input.

Due to space limitations, the fluid weight in the tanks is usually less than 5 percent of the vessel displacement. This implies that in order to shift the CG by 1 unit, the CG of the fluid must be moved by at least 20 units. Hence, x_3 is limited by available space. On the other hand, the flow rate (the control effort) also has practical limitations. These limitations on x_3 and u will be taken into account when designing the controller.

Before starting the controller design, a specific statement of the associated mathematical problem is given. Let S_0 be the unperturbed, unbiased safe region in the (x_1, x_2) invariant manifold,

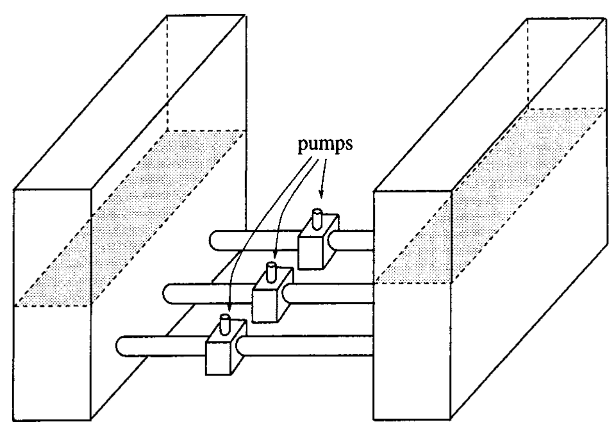

Fig. 2 The active antiroll tanks

[1]When a ship's righting arm is an odd function of the roll angle, it is called an unbiased ship. Physically, this means that its equilibrium state in calm water is the upright vertical position. For the present system, an unbiased ship can be interpreted as one whose CG is located in the symmetry plane of its hull. Otherwise, the ship is called biased.

i.e., the one enclosed by the heteroclinic cycle. Let S_1 be some compact set containing S_0 in the same manifold. Then the domain of interest is defined by

$$D=\{(x_1,x_2,x_3,y,z_1,z_2)|(x_1,x_2)\in S_1,|x_3|\leqslant L_x,|y|\leqslant L_y,\|(z_1,z_2)\|\leqslant L_z\}, \quad (7)$$

where $\|\cdot\|$ denotes the Euclidean 2-norm, and L_x, L_y, and L_z are positive constants.

Our goal is to design a feedback law

$$u=\psi(x_1,x_2,x_3,y,z_1,z_2) \quad (8)$$

such that for any initial condition in D,

(i) All state variables are bounded for $\tau\geqslant 0$;
(ii) $(x_1(\tau),x_2(\tau))$ asymptotically approaches a small neighborhood of the origin as $\tau\to\infty$.

In other words, for the ship initially in the safe region, the roll motions are to be reduced as much as possible and, at the same time, bounded motions of the other degrees of freedom are to be maintained. It will be shown below that the desired feedback function can be chosen to depend only on x_1, x_2, and x_3. That is, partial state feedback is sufficient to achieve the goal. This is due to the large damping in heave and the essentially inconsequential nature of sway.

The full control system given by Eqs. (1)–(6) is a singularly perturbed system. Therefore, it is natural to design the controller via the approach of composite control [9,11]. The composite control is a sum of two components, the *slow control* and the *fast control*. The former is designed on the slow manifold to satisfy the desired requirement. The fast control, on the other hand, is designed to guarantee that the slow manifold is attractive. In the following analysis, it is first assumed that the slowly varying variable y is bounded for all $\tau\geqslant 0$ and then this assumption is investigated at the final stage of the design.

The design of the slow control is started by restricting the dynamics to the slow manifold which, to leading order, is simply $z_1=z_2=0$. The slow system is thus given by

$$\dot{x}_1=x_2, \quad (9)$$

$$\dot{x}_2=f_{11}(x_1)+f_{12}(x_1)x_3+\Delta_1(x_1,x_2,x_3,y,\tau), \quad (10)$$

$$\dot{x}_3=u, \quad (11)$$

where

$$\Delta_1=\Delta f(x_1,x_3)+\epsilon(g_1+\Delta g_1)(x_1,x_2,x_3,y,0,0,\tau). \quad (12)$$

The controller for this system will constitute the slow control for the full system.

It is clear that the uncertainties in the slow dynamical system do not satisfy the *matching condition* (Khalil [9]). In other words, the uncertainties and the control input enter the state equations at different points. As a consequence, most robust control methods cannot be applied without incorporating the backstepping technique [9,12]. In what follows, the slow control is designed by a smooth version of sliding mode control with the help of the backstepping technique.

As the first step in the backstepping procedure, assume for the moment that x_3 is a direct control input, i.e., that the CG can be altered instantaneously. Since $f_{12}(x_1)$ is basically a normalized inertia term, it is always positive within the angles of vanishing stability. Hence, the uncertain term Δ_1 will now satisfy the matching condition by treating x_3 as the control input. The problem then is to design a smooth feedback law $x_3=\psi_x(x_1,x_2)$ such that the 2D system in Eqs. (9) and (10) is ultimately bounded. Note that the smoothness requirement is due to the use of backstepping.

This 2D control problem appears to be well suited for the method of sliding mode control. Other methods like Lyapunov redesign and adaptive control are also possible choices. However, it is easier to obtain a simple smooth feedback law by employing a smooth version of sliding mode control. The idea of sliding mode control is to design a *sliding manifold*, $x_2=s(x_1)$, such that the dynamics on this manifold, given by $\dot{x}_1=s(x_1)$, will be asymptotically stable. The sliding mode control thus consists of two parts. One part is used to bring the system onto the sliding manifold in finite time—this is called the *switching control* and is denoted by ψ_s. The other part is used to maintain the situation afterwards, which is called the *equivalent control* and is denoted by ψ_{eq}.

The equivalent control is designed first. The sliding manifold will be taken as the linear form $s(x_1)=-\beta x_1$, $\beta>0$, resulting in an asymptotically stable reduced system, $\dot{x}_1=-\beta x_1$, on the sliding manifold. Let $\sigma_1(x_1,x_2)=x_2-s(x_1)=\beta x_1+x_2$ so that the sliding manifold is represented by $\sigma_1(x_1,x_2)=0$. Then, maintaining the system on $\sigma_1=0$, once it is there, is equivalent to maintaining $\dot{\sigma}_1=0$, which leads to

$$\beta x_2+f_{11}(x_1)+f_{12}(x_1)x_3=0,$$

in the absence of uncertainty. This is to be done by ψ_{eq}, yielding

$$\psi_{\mathrm{eq}}(x_1,x_2)=-\frac{f_{11}(x_1)+\beta x_2}{f_{12}(x_1)}. \quad (13)$$

Upon applying $x_3=\psi_x(x_1,x_2)=\psi_{\mathrm{eq}}(x_1,x_2)+\psi_s(x_1,x_2)$ with $\psi_{\mathrm{eq}}(x_1,x_2)$ given by Eq. (13), the $\dot{\sigma}_1$ equation becomes

$$\dot{\sigma}_1=v+\Delta_1\left(x_1,x_2,\psi_{\mathrm{eq}}+\frac{v}{f_{12}(x_1)},y,\tau\right), \quad (14)$$

where $\psi_s=v/f_{12}(x_1)$. The task now is to choose v to force σ_1 toward the manifold $\sigma_1=0$ in the presence of the uncertainty. To this end, it is assumed that there exist constants $\rho_1\geqslant 0$ and $0\leqslant k<1$ such that

$$\left|\Delta_1\left(x_1,x_2,\psi_{\mathrm{eq}}+\frac{v}{f_{12}(x_1)},y,\tau\right)\right|\leqslant\rho_1+k|v|, \quad (15)$$

within the domain of interest. The positive constant ρ_1 represents an upper bound on the uncertainty and is not necessarily small.

With inequality (15), a Lyapunov analysis using the candidate function $V_\sigma=\frac{1}{2}\sigma_1^2$ suggests that a smooth version of switching control

$$v=-\frac{\lambda_1+\rho_1}{(1-k)\tanh(1)}\tanh\left(\frac{\sigma_1}{\epsilon_1}\right), \quad \epsilon_1>0, \quad (16)$$

where ϵ_1 is the thickness of the boundary layer near the sliding manifold, will satisfy the requirement. The smoothness requirement is due to the backstepping method. While asymptotic stability is guaranteed by the conventional discontinuous feedback law, only ultimate boundedness can be achieved by the smooth control equation (16). This can be shown by a Lyapunov analysis.

Next, consider the 3D system given by Eqs. (9)–(11). With the above preliminary analysis, the backstepping method proceeds by applying sliding mode control again, with the sliding manifold now given by $\sigma_2(x_1,x_2,x_3)=x_3-\psi_x(x_1,x_2)=0$. In other words, on the sliding manifold, the foregoing desired results will hold. The time derivative of σ_2 with respect to the 3D system takes the form

$$\dot{\sigma}_2=f_{13}(x_1,x_2,x_3)+u+\Delta_2(x_1,x_2,x_3,y,\tau). \quad (17)$$

With the assumption that within the domain of interest

$$|\Delta_2(x_1,x_2,x_3,y,\tau)|\leqslant\rho_2, \quad \rho_2\geqslant 0, \quad (18)$$

and applying the same design procedure as in the previous 2D system, the following slow control is obtained,

$$u=\frac{\partial\psi_x}{\partial x_1}x_2+\frac{\partial\psi_x}{\partial x_2}(f_{11}(x_1)+f_{12}(x_1)x_3)-\frac{\lambda_2+\rho_2}{\tanh(1)}\tanh\left(\frac{\sigma_2}{\epsilon_2}\right). \quad (19)$$

Given the slow control, the next step in the design of a composite controller is to obtain a fast control to ensure the attractiveness of the slow manifold. However, in light of the asymptotically stable linear part in the fast dynamics (Eqs. (4) and (5)), feedback control of the fast dynamics is not necessary. On the other hand, one can see from the previous analysis that the attractiveness of the slow manifold is not crucial as long as z_1 and z_2 remain bounded. This is because the fast variables only show up in the perturbation terms. Therefore, it is expected that the heave damping will naturally bound the motions. Indeed, a Lyapunov analysis using a quadratic Lyapunov function in z can confirm this point provided that within the domain of interest, the following holds:

$$|(g_3+\Delta g_3)(x_1,x_2,x_3,y,z_1,z_2,\tau)|\leqslant l_1, \quad l_1\geqslant 0. \tag{20}$$

The analysis to this point has been predicated on the boundedness of the sway velocity, y. The validity of this assumption is now investigated. It should be physically correct since the little energy fed into the sway direction through coupling from heave and roll is easily absorbed by the sway damping. Like the heave damping, the sway damping plays an important role in limiting the sway velocity.

Assume that the only y-dependent term in the uncertainty Δg_2 is $\Delta\delta_{22}y$ and that the actual sway damping is $\delta_{22}-\Delta\delta_{22}\geqslant\hat{\delta}_{22}>0$. This is a reasonable assumption in view of the expression for g_2. Now, the sway equation is rewritten as

$$\dot{y}=\epsilon[-(\delta_{22}-\Delta\delta_{22})y+(\hat{g}_2+\Delta\hat{g}_2)(x_1,x_2,z_2,\tau)],$$

where $\hat{g}_2(x_1,x_2,z_2,\tau)$ and $\Delta\hat{g}_2(x_1,x_2,z_2,\tau)$ are self-evident. By the continuity of $\Delta\hat{g}_2$, there exists $\hat{L}>0$, independent of y, such that

$$|(\hat{g}_2+\Delta\hat{g}_2)(x_1,x_2,z_2,\tau)|\leqslant\hat{L}, \tag{21}$$

within the domain of interest. A Lyapunov analysis using $V_y=\frac{1}{2}y^2$ is then used to verify the boundedness of y.

The design of a robust stabilizing controller for the full vessel system has been decomposed into several simpler control problems. In each subsystem, it is easy to verify that the design indeed works. The question thus arises: Will it work for the full system? There are some coupling terms between subsystems, and for the design to be valid for the full system, these terms must be well behaved in the sense that they do not destroy the established analysis. Generally, they are required to satisfy some smallness conditions. For the current system, as one can see from the state equations, the coupling terms are not dominant, indicating that the design should work for the full system, as is shown below. Indeed, in addition to the inequalities satisfied by the uncertainties and perturbations, an inequality must be imposed on the coupling term between the slow and fast systems. Specifically, within the domain of interest,

$$|\hat{g}_1(x_1,x_2,x_3,y,z_1,z_2,\tau)|\leqslant l_2, \quad l_2\geqslant 0, \tag{22}$$

since

$$\hat{g}_1=(g_1+\Delta g_1)(x_1,x_2,x_3,y,z_1,z_2,\tau)-(g_1+\Delta g_1)(x_1,x_2,x_3,y,0,0,\tau)$$

is a continuous function in its arguments.

The foregoing analysis is now summarized as the main theorem. The proof is based on Lyapunov analysis and is omitted here for brevity. Recall that the compact set $D\subset\Re^6$ given by Eq. (7) is the domain of interest. Also, let

$$D_0=\{(x_1,x_2,x_3,y,z_1,z_2)|(x_1,x_2)\in S_0,|x_3|\leqslant L_x,|y|\leqslant L_y,\|(z_1,z_2)\|\leqslant L_z\}$$

be the stabilization region.

Main Theorem. Consider the vessel control system given by Eqs. (1)–(6). Suppose that within the domain of interest D, the perturbations and uncertainties satisfy the inequalities (15), (18), (20), and (21), and the coupling term satisfies inequality (22). Then for λ_1, λ_2, and β large enough and ϵ_1, ϵ_2, and ϵ sufficiently small, the partial state feedback controller given by Eq. (19) will stabilize the vessel system in the sense that for any initial condition in D_0,

(i) x_1, x_2, and x_3 are ultimately bounded with bounds depending on ϵ_1 and ϵ_2,
(ii) z_1 and z_2 are ultimately bounded with bounds depending on ϵ,
(iii) $|y(\tau)|\leqslant L_y$, $\forall\tau\geqslant 0$.

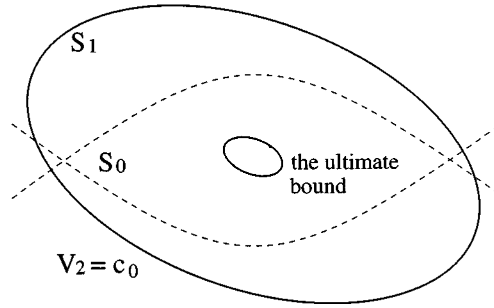

Fig. 3 The domain of interest and the ultimate bound

Remarks. (i) All the bounds on the perturbations, uncertainties, and coupling terms in the inequalities (15), (18), (20), (21), and (22) can be obtained from the fact that these functions are continuous on the compact domain of interest. (ii) The perturbations and uncertainties depend on the wave amplitude. Hence, the upper bounds should be chosen to include the worst sea condition expected to be encountered. (ii) There exists a positive constant c_0 such that $S_0\subset\{V_2\leqslant c_0\}$, which can be used to serve as S_1. This is depicted in Fig. 3.

4 Simulation Results and Discussion

In this section, numerical simulations for a fishing vessel, the twice-capsized clam-dredge *Patti-B* [7,8] are carried out to examine the performance of the controller. A detailed system description and parameter values for the *Patti-B* can be found in [7,10]. Of interest here is the comparison of the vessel response in open loop and closed loop configurations under severe sea conditions. Some issues regarding the control effort are also addressed.

For a given vessel, a simple procedure based on the main results can be followed to obtain a robust stabilizing controller. The procedure includes the following 6 steps: (1) Determine β. (2) Choose the domain of interest D. (3) Estimate ρ_1 and k in inequality (15). (4) Determine λ_1 and ϵ_1 for $\psi_x(x_1,x_2)$. (5) Estimate ρ_2 in inequality (18). (6) Determine λ_2 and ϵ_2. Step 1 amounts to determining the sliding plane for the 2D roll system and the stability on the sliding plane. It will also affect the level of control effort required. This step precedes step 2 due to the fact that the choice of D involves S_1, which depends on β.

Ideally, the controller should meet the following requirements: (*a*) it can stabilize a large set of initial conditions; (*b*) it is effective under severe sea states; (*c*) it does not require large control efforts. Requirement (*a*) is equivalent to enlarging D as much as possible, which will increase the estimates for ρ_1, ρ_2, and k. Requirement (*b*) also leads to large values for ρ_1, ρ_2, and k. Then a choice of large values for the parameters λ_1, λ_2, and β is needed, as indicated in the previous analysis. In other words, both requirements need sufficiently large control effort. However, in reality the control effort cannot be arbitrarily large, as mentioned in the beginning. Therefore, there is a tradeoff between the ideal requirements and practical limitations in choosing the domain of interest D and the design parameters λ_1, λ_2, and β. A feasible approach is to choose D as small as possible such that it still includes most of the safe region. Moreover, the design parameters can be tuned according to the sea conditions, where larger values are used in bad conditions.

It is also important to point out that the controller design is conservative, typical for a Lyapunov-based design. The main purpose of the analysis is to ensure that the controller design will work. Although it can also provide some estimates on the ultimate bounds of states and lower bounds for design parameters, quite often the controller works better than predicted. This is why these bounds were not explicitly calculated in the analysis. Hence, one can be a bit generous when choosing design parameters, as is seen below.

For a chosen β, take c_0 to minimize the range of $S_1=\{V_2 \leqslant c_0\}$ that contains S_0 in its interior. It can be shown that

$$c_0=\frac{\beta^2}{3.144(1+\beta^2)} \tag{23}$$

is the value needed. In the following numerical results, the domain of interest D is chosen with S_1 determined from Eq. (23), $L_x=2.0$, $L_y=\hat{L}/\hat{\delta}_{22}$ (where $\hat{\delta}_{22}$ is the bound on the sway damping), and $L_z=2.0$.

Three different configurations for the *Patti-B* are considered for comparison. The first is the open-loop system for an unbiased ship. The second is a closed-loop system with the linear partial state feedback law,

$$u=k_1x_1+k_2x_2+k_3x_3. \tag{24}$$

The linear controller is designed based on the linearized model of the slow system using pole placement method. The third is the closed-loop system with the nonlinear partial state feedback controller designed herein.

In the following simulations, the linear feedback gains in Eq. (24) are chosen to be,

$$k_1=0, \quad k_2=10, \quad k_3=-6,$$

which assign the closed-loop poles of the linearized slow system to -1, -2, and -3. These gains were chosen to yield the same order of control effort as the nonlinear controller. The design parameters for the nonlinear feedback system are taken to be

$$\beta=0.1, \quad \lambda_1=0.005, \quad \lambda_2=0.01, \quad \epsilon_1=0.1, \quad \epsilon_2=0.1$$

and the uncertainty bounds are estimated as

$$\rho_1=0.6, \quad \rho_2=0.6, \quad k=0.4.$$

Throughout this example the position of the CG (x_3) and the flow rate (u) were tracked over a wide variety of conditions, and the design parameters for the nonlinear controller were selected such that the performance specifications were met while certain limits on these quantities were not exceeded.

The sea condition is set at a wave amplitude of 5 m and a wave frequency of 0.6 rad/s. These conditions can capsize the uncontrolled vessel even when it is given initial conditions near the calm water stable operating point. In addition to wave excitation, some of the following simulations also include parameter variations to demonstrate the robustness of the proposed nonlinear controller. A 5 percent variation of hydrostatic parameters (including ship geometry) and as large as 25 percent variations of hydrodynamic parameters (including wave amplitude and frequency) are assumed to be present in the system.

Figure 4 shows the state trajectories (projected on the (x_1, x_2) plane) for the nominal system under each of the three configurations with the following initial conditions:

IC1: $x_1=0.55$, $x_2=0$, $x_3=0.1$, $y=1$, $z_1=1$, $z_2=1$, denoted by ‘‘x’’;

IC2: $x_1=0$, $x_2=0.4$, $x_3=0.1$, $y=1$, $z_1=1$, $z_2=1$, denoted by ‘‘o,’’

which are near the boundary of the calm-water safe region. Figure 5 depicts the corresponding trajectories with parameter variations. From Fig. 4, it is seen that the uncontrolled and linearly controlled vessels with IC1 immediately capsize for the nominal model, while the nonlinear controller demonstrates good stabilization in this case, as it will for any initial conditions inside the safe region. Although the linear controller can stabilize the initial condition IC2 for the nominal model, it fails when the parameter variation is included, as demonstrated in Fig. 5. On the other hand, although roll reduction performance is not as good as those without parameters variation, the nonlinear controller still works for both initial conditions under the parameter variation.

Without parameter variations, the peak control effort using nonlinear feedback control is $u_{\max}=1.45$ for IC1 and $u_{\max}=2.24$ for IC2, and for the linear feedback control that value is $u_{\max}=3.40$ for IC2. With parameter variations under nonlinear control, $u_{\max}=3.88$ for IC1 and $u_{\max}=8.05$ for IC2. These peak control efforts usually occur during the initial transient period and settle down to smaller levels soon after. The peak control effort required for the cases with parameter variations is somewhat larger, as expected. It can be reduced by tuning down the control parameters, however, at the expense of roll reduction performance.

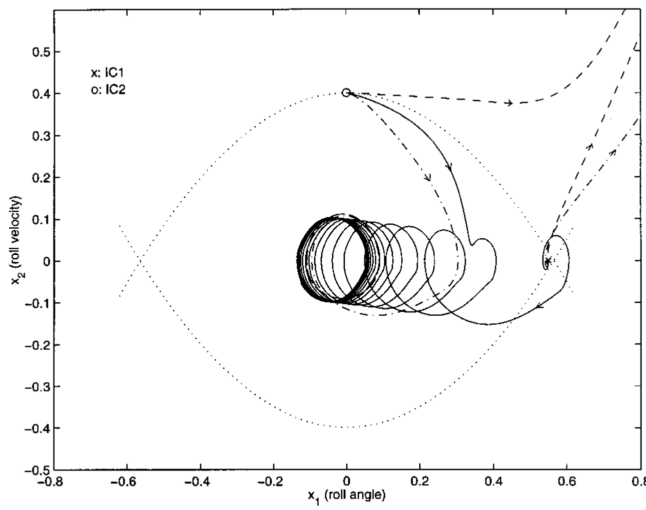

Fig. 4 Roll dynamics without parameter variations under different controllers: nonlinear feedback (solid), linear feedback (dash–dot), and open loop (dashed)

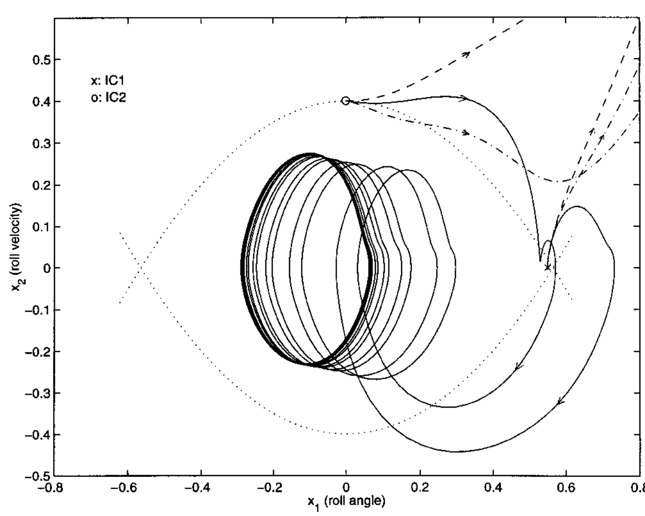

Fig. 5 Roll dynamics with parameter variations under different controllers: nonlinear feedback (solid), linear feedback (dash–dot), and open loop (dashed)

The position of the CG and the corresponding control effort have been monitored for several simulation runs. It is observed that during transients y_G can reach as large as 0.2 m (the worst case), although it quickly settles down to smaller levels, just like the control effort. For antiroll tanks using 5 percent of the ship weight, this accounts for a 4 m movement of the CG of the water. The maximum peak flow rate encountered for the pump was about 200 L/s (corresponding to $u \approx 8$, the worst case). If this peak control effort goes beyond practical limitations, it will be necessary to tune down the design parameters.

Fortunately, the nonlinear controller developed in this study has a large flexibility in tuning its parameters. For the linear feedback controller, the tuning is restricted in that high feedback gains must in general be used in order to stabilize initial conditions far away from the origin. In simulation runs, it was generally observed that the peak control effort required for the linear controller near capsize conditions was roughly twice that of the nonlinear controller, and this may lead to significant practical difficulties in implementation of a linear controller.

5 Conclusions

In this study, a nonlinear state feedback controller was designed using a Lyapunov-based approach in order to stabilize a nonlinear 3-DOF ship model in beam seas. The nonlinear controller is robust in the sense that it takes into account model uncertainties, resulting primarily from unknown hydrodynamic contributions. The design procedure follows the idea of composite control for singularly perturbed systems. The slow control for the dynamics on the slow manifold consists of two parts, linked by the backstepping technique, both of which use a smooth version of sliding mode control which can handle the uncertainties. It is shown by a Lyapunov analysis that the slow control alone can restrict the roll motions to a small region in the state space, and, at the same time, bounds the motions in the other degrees of freedom.

Numerical simulations for a fishing vessel, the clam dredge *Patti-B*, were carried out for the open-loop system, the closed-loop system with linear feedback, and the closed-loop system with the designed nonlinear feedback controller. It was shown that only the nonlinear controller can effectively stabilize the system against capsizing using a reasonable amount of control effort over a wide range of initial states and ship model uncertainties.

Many of the details regarding the model development and details of the proof of the main theorem presented here can be found in [7,10].

References

[1] Bennett, S., 1991, ''Ship Stabilization: History,'' *Concise Encyclopedia of Traffic and Transportation Systems*, M. Papageorgiou, ed., Pergamon, New York, NY, pp. 454–459.

[2] Lewis, E. V., 1989, *Principles of Naval Architecture*, 2nd ed., SNAME, NJ.

[3] Chadwick, J. H., 1955, ''On the Stabilization of Roll,'' SNAME Trans., **63**, pp. 234–280.

[4] Fortuna, L., and Muscato, G., 1996, ''A Roll Stabilization System for a Monohull Ship: Modeling, Identification, and Adaptive Control,'' IEEE Trans. Control Systems Technol., **4**, pp. 18–28.

[5] Fossen, T. I., 1995, *Guidance and Control of Ocean Vehicles*, Wiley, New York, NY.

[6] van Amerongen, J., 1991, ''Ship rudder roll stabilization,'' *Concise Encyclopedia of Traffic and Transportation Systems*, M. Papageorgiou, ed., Pergamon, New York, NY, pp. 448–454.

[7] Chen, Shyh-Leh, 1996, Modeling, *Dynamics and Control of Large Amplitude Motions of Vessels in Beam Seas*, Department of Mechanical Engineering, Michigan State University, East Lansing, MI.

[8] Hsieh, S. R., Shaw, S. W., and Troesch, A. W., 1994, ''A Nonlinear Probabilistic Method for Predicting Vessel Capsizing in Random Beam Seas,'' Proc. R. Soc. London, Ser. A, **446**, pp. 195–211.

[9] Khalil, Hassan K., 1996, *Nonlinear Systems*, 2nd ed., Prentice-Hall, Upper Saddle River, NJ.

[10] Chen, Shyh-Leh, Shaw, Steven, W., and Troesch, Armin W., 1999, ''A Systematic Approach to Modeling Nonlinear Multi-DOF Ship Motions in Regular Seas,'' J. Ship Res., **43**, pp. 25–37.

[11] Kokotovic, Petar V., Khalil, Hassan K., and O'Reilly, J., 1986, *Singular Perturbations Methods in Control: Analysis and Design*, Academic, New York, NY.

[12] Kristic, M., Kanellakopoulos, I., and Kokotovic, P., 1995, *Nonlinear and Adaptive Control Design*, Wiley-Interscience, New York, NY.