Steady-State Responses in Systems of Nearly-Identical Torsional Vibration Absorbers

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In this paper we consider the steady-state response of a rotor fitted with a system of nearly identical torsional vibration absorbers. The absorbers are of the centrifugal pendulum type, which provide an effective mean of attenuating torsional vibrations of the rotor at a given order. The model considered employs absorbers that are tuned close to the order of the excitation, with an intentional mistuning that is selected by design, and imperfections among the absorbers which arise from manufacturing, wear, and other effects. It is shown that these systems can experience localized responses in which the response amplitude of one or more absorbers can become relatively large as compared to the response of the corresponding system with identical absorbers. The results are based on an exact steady-state analysis of the mathematical model, and they show that the strength of the localization depends on the average level of absorber mistuning (a design parameter), the magnitude of the relative imperfections among the absorbers, and the absorber damping. It is found that the most desirable situation is one in which the relative imperfections are kept as small as possible, and that this becomes more crucial when the levels of mistuning and damping are very small. The results of the analysis are confirmed by simulations of the fully nonlinear equations of motion of the rotor/absorber system. It is concluded that the presence of localization should be accounted for in absorber designs, since its presence makes the absorbers less effective in terms of vibration reduction and, perhaps more significantly, it can drastically reduce their operating range, since such absorbers typically have limited rattle space. [DOI: 10.1115/1.1522420]

1 Introduction

Research on the phenomenon of localization has shown that when the degrees of freedom of a periodic structure are weakly coupled and there are some irregularities or disorder among them, mode localization may occur. This results in confined regions of the system (that is, specific degrees of freedom) where vibration energy is concentrated. The underlying feature of these systems is the presence of multiple system modes with close natural frequencies. Some of the relevant work on localization in mechanical vibrations includes the following papers: Hodges [1] and Hodges and Woodhouse [2] showed that structural irregularities can result in localized motions in elastic systems. Pierre and Dowell [3] investigated localization phenomenon in a chain of coupled oscillators, and Pierre et al. [4] theoretically and experimentally investigated localization of the free modes of vibration of disordered multi-span beams constrained at slightly irregular intervals. Wei and Pierre [5,6] studied both free and forced vibration localization in nearly periodic mistuned assemblies with cyclic symmetry. A singular perturbation approach has been shown to be very useful in describing localization behavior by Happawana et al. [7]. Also, it has more recently been found that localization can occur in nonlinear systems, even when the subsystems are perfectly tuned. In this case, the mistuning is caused by the amplitude dependence of the subsystems' frequencies and other nonlinear effects. See, for example, Vakakis and Centikaya [8], King and Layne [9], and Chao and Shaw [10].

In a previous study of localization in vibration absorber systems, the authors showed that when the ratio of the coupling between the absorbers to the imperfections among them is small, the free vibration modes of the undamped rotor/absorber system localize [11]. An interesting feature of this system is that the coupling is through the primary (rotor) inertia, and the coupling parameter is an inertia ratio. The results of that study motivated the present investigation, which focuses on the steady-state response of systems of nearly-identical centrifugal pendulum vibration absorbers (CPVAs). An important feature of these absorbers is that they are tuned to a given order of rotation, rather than to a given frequency, and are therefore effective at all rotation speeds. Historical background and applications of CPVAs can be found in [12,13], and the papers by Shaw and co-workers listed in the references.

Since this work spans the fields of localization and vibration absorber design, a comment on terminology is in order. Specifically, the term *mistuning* is used in different contexts in these fields. In this paper the term *mistuning* refers to the relative difference between the order of the absorber, that is, the order for which it is tuned and the order of the applied torque.[2] This is a parameter that is set by the designer of the absorber system, and absorber systems are designed so that they all have the same tuning, and thus the same mistuning. The term *imperfection* here refers to a deviation from the desired tuning, which results from manufacturing tolerances, wear, and other factors, and is not dictated by design. These imperfections will vary among absorbers in a given system.

The paper is organized as follows. Section 2 describes the absorber system and the mathematical models used in the analysis and simulations. Section 3 describes the analytical expressions for the steady state response of the linearized system and considers some special cases. Section 4 presents sample results from three examples and a discussion of some general parameter trends. The paper closes with a discussion in Section 5.

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[2]These absorbers use the centrifugal field to establish the restoring force, and can thus be tuned to given order, rather than a given frequency, and are effective at that order at all rotor speeds.

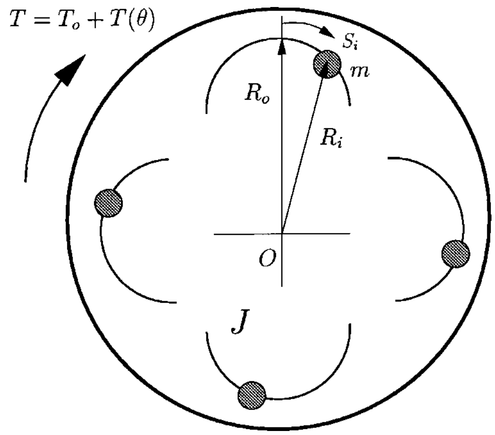

Fig. 1 Schematic diagram of a rotor fitted with multiple CPVAs

2 Mathematical Formulation

The idealized system consists of N CPVAs, modeled as point masses m, mounted on a rotor with moment of inertia J, as schematically shown in Fig. 1. The nonlinear, dimensionless equations of motion for this system are obtained by Lagrange's method and are given by [14]

$$ws_i'' + [s_i' + \tilde{g}_i(s_i)]w' - \frac{1}{2}\frac{dx_i}{ds_i}(s_i)w = -\mu_a s_i', \quad i = 1, \ldots, N \tag{1}$$

$$\frac{\nu}{N}\sum_{i=1}^{N}\left[\frac{dx_i}{ds_i}s_i' w^2 + x_i(s_i)ww' + \tilde{g}_i(s_i)s_i' ww' + \tilde{g}_i(s_i)s_i'' w^2 + \frac{d\tilde{g}_i}{ds_i}(s_i)s_i^2 w^2\right] + ww' = \frac{\nu}{N}\sum_{i=1}^{N}\mu_a\tilde{g}_i(s_i)s_i' w - \mu_o w + \Gamma_o + \Gamma(\theta), \tag{2}$$

where $(\cdot)'$ represents differentiation with respect to the rotor angular orientation, θ, and the various terms are defined below. Note that the equations of motion have been formulated in such a manner that the rotor angle, θ, is the independent variable, in place of time. This converts the nonlinear forcing term, $\Gamma(\theta)$, into a periodic forcing term, which facilitates the analysis.[3] The ith absorber is riding on a path specified by the function $R_i(S_i)$, chosen by design. R_i denotes the distance from a point on the ith absorber path to the (fixed) center of rotation, and S_i is an arc length variable along the path. s_i is the nondimensional arc length variable given by $s_i = S_i/R_o$, where R_o is the value of R_i at the vertex of the path, i.e., $R_o = R_i(S_i = 0)$. w is the ratio of the rotor angular velocity, $\dot{\theta}$, to the nominal rotor angular velocity, Ω, i.e., $w = \dot{\theta}/\Omega$. The variables s_i and w represent the generalized coordinates for this $N+1$ degree of freedom system. The parameters μ_a and μ_o represent the nondimensional damping coefficients for the absorbers and the rotor, respectively, i.e., $\mu_a = c_a/m\Omega$ and $\mu_o = c_o/J\Omega$, where c_a, and c_o are the equivalent viscous damping constants for the absorber/rotor and rotor/ground interfaces. Γ_o and $\Gamma(\theta)$ are the nondimensional mean and fluctuating components of the applied torque, that is, $\Gamma_o = T_o/J\Omega^2$ and $\Gamma(\theta) = T_\theta/J\Omega^2$. It is assumed that all absorbers have the same mass, m, the same damping, μ_a, and the same value of R_o. The parameter ν represents the ratio of the total moment of inertia of all absorbers about the center of rotation to the rotor inertia, i.e., $\nu = I_o/J$, where $I_o = m_o R_o^2$ and $m_o = Nm$. The dimensionless path functions $x_i(s_i)$ and $\tilde{g}_i(x_i)$ are defined as follows:

$$x_i(s_i) = \left(\frac{R_i(s_i R_o)}{R_o}\right)^2$$
$$\tilde{g}_i(s_i) = \sqrt{x_i(s_i) - \frac{1}{4}\left(\frac{dx_i}{ds_i}(s_i)\right)^2}, \tag{3}$$

where x_i is simply $(R_i/R_0)^2$ and $\tilde{g}_i$ is related to the tangent along the absorber path. The fluctuating component of the applied torque is periodic and generally contains several harmonics. In most situations only one or two harmonics have significant amplitude, and therefore we approximate the fluctuating torque by its dominant harmonic, taken to be of order n. For example, in four-stroke IC engines, n is equal to half the number of cylinders.

Equation (1) represents the dynamics of the absorber masses, and it is clear that the ith absorber is only indirectly coupled to the other absorbers through the dynamics of the rotor, represented by w. Equation (2) represents the torque balance on the rotor, where it is seen that the dynamic effect that each absorber has on the rotor arises in an identical manner, and their total effect is captured in the summation terms. This type of coupling leads to an interesting symmetry in the system, which can result in a variety of interesting instabilities and bifurcations [10,14,15].

The phenomena of interest here can be captured using a linearized model of the equations of motion. To that end, the absorber path function $x_i(s_i)$ is expanded as follows [16]

$$x_i(s_i) = 1 - \tilde{n}_i^2 s_i^2 + O(s_i^4), \tag{4}$$

where $\tilde{n}_i$ is the (order) tuning of the ith absorber. This tuning is determined by the curvature of the absorber path at its vertex [13]. It is convenient to express the path order, $\tilde{n}_i$, as follows:

$$\tilde{n}_i = n(1 + \sigma_i),$$

where σ_i is a (typically small) quantity that describes the mistuning and imperfection of the ith path. The mistuning represents a nominal value of σ_i that is the same for all absorbers, while small differences between absorbers are caused by imperfections.

For realistic systems, many of the dimensionless parameters are small, including the inertia ratio ν, the absorber damping level μ_a, the mistunings σ_i, and the fluctuating torque $\Gamma(\theta)$. These observations follow, respectively, since one typically uses a relatively small amount of inertia for the absorbers, the absorbers should be lightly damped for good performance (since they remain tuned at all rotor speeds), they are tuned near the order of the applied torque, and the torque is scaled by the kinetic energy of the rotor, which is typically large. These facts will allow for some useful approximations in the analytical results.

The model of interest is obtained by first cancelling the mean rotor torque with the mean bearing torque, $\mu_o = \Gamma_o$, which dictates the nominal dimensionless rotor speed, $w = 1$. The dynamic variables are then linearized about the operating condition $(s_i, w) = (0,1)$. This results in linear equations of motion for both the rotor and the absorbers. The linearized rotor equation can be solved for w', resulting in

$$w' = \left(\frac{1}{1+\nu}\right)\left[\Gamma(\theta) - \frac{\nu}{N}\sum_{k=1}^{N}(s_k'' - \mu_a s_k')\right] \tag{5}$$

which is the linearized rotor angular acceleration expressed in terms of the absorber dynamics. This quantity is a useful measure of the torsional vibration level, since it is zero in the desired operating condition, that is, when the rotor spins at a constant rate. However, it must be noted that the rotor acceleration is dominated by nonlinear effects, even when the absorbers behave in an essentially linear manner [10]. This is due to the fact that the absorbers effectively cancel the linear component of the rotor acceleration. This will be evident in the numerical simulations.

The linearized absorber equations are given by

[3]In rotating systems the applied torques generally depend explicitly on the rotor angle, rather than time.

$$s_i'' + \mu_a s_i' + \tilde{n}_i^2 s_i = -w', \quad i = 1, \ldots, N, \tag{6}$$

from which it is clear that the rotor acceleration acts as a rotational base excitation, applied identically to each absorber. It is now straightforward to eliminate the rotor equation of motion, leaving a set of coupled linear equations that describe the dynamics of the absorbers, as follows,

$$s_i'' + \mu_a s_i' + \tilde{n}_i^2 s_i - \frac{\nu}{N(1+\nu)} \sum_{k=1}^{N} (s_k'' - \mu_a s_k') = -\frac{\Gamma(\theta)}{1+\nu},$$
$$i = 1, \ldots, N. \tag{7}$$

These equations have a very special structure that arises from the physical nature of the coupling: each absorber is identically coupled to all other absorbers, including itself, through the summation term, which arises from the rotor acceleration.

The steady-state response of this systems of equations, and the conclusions drawn from it, form the main results of this paper.

3 The Steady-State Response

Exact Solution. We use a complex formulation of the problem to conveniently determine the steady state response of the system. The periodic torque of order n is modeled by $\Gamma(\theta) = fe^{jn\theta}$ where $f \in C$ is the complex torque amplitude and $j = \sqrt{-1}$. The resulting steady-state absorber responses are expressed as $s_i = A_i e^{jn\theta}$ where each $A_i \in C$, and the steady-state rotor response, expressed in terms of its angular acceleration, is taken to be $w' = Be^{jn\theta}$ where $B \in C$.

When these are substituted into Eq. (7), the following equations for the steady-state absorber amplitudes are obtained:

$$A_i \gamma_i + \rho \sum_{k=1}^{N} A_k = \beta, \tag{8}$$

where

$$\left.\begin{aligned} \gamma_i &= \tilde{n}_i^2 - n^2 + jn\mu_a, \\ \rho &= \frac{\nu}{N(1+\nu)}(n^2 + jn\mu_a), \\ \beta &= -\frac{f}{1+\nu}. \end{aligned}\right\} \tag{9}$$

It is not difficult to uncouple this system of equations. This is accomplished by noting from Eq. (8) that the quantity

$$A_i \gamma_i = \beta - \rho \sum_{k=1}^{N} A_k = \Delta \tag{10}$$

is *independent* of i, and Δ is defined for convenience. The fact that $A_i \gamma_i$ is the same for each absorber is not surprising, since it is simply a frequency domain statement of Eq. (6), which results from the fact that each absorber is driven identically by the rotor acceleration. Thus, the absorber amplitudes can be expressed as $A_i = \Delta/\gamma_i$. From this result, it can also be seen that $\Delta = B$, which represents the complex magnitude of the rotor acceleration.

In order to uncouple the amplitude equations, we use

$$A_k \gamma_k = A_i \gamma_i, \quad \forall k, i, \tag{11}$$

solve for each A_k in terms of A_i, substitute this into Eq. (8), and solve for A_i, resulting in

$$A_i = \frac{\beta}{\gamma_i \left(1 + \rho \Sigma_{k=1}^{N} \frac{1}{\gamma_k}\right)}, \quad i = 1, \ldots, N. \tag{12}$$

This result expresses the complex response amplitude of the ith absorber in terms of known system and excitation parameters.

Note that the excitation order n plays the role of the excitation frequency, but that this quantity is fixed in these applications. Therefore, these results are not used to quantify the absorbers' frequency responses, but rather as means of investigating how features of the response depend on parameters such as the absorber damping, the inertia ratio, the number of absorbers, and the mistuning and imperfections of the absorbers.

These results also allow for the solution of the complex rotor acceleration amplitude, which can be expressed in several forms, including

$$B = \Delta = \beta - \rho \sum_{k=1}^{N} A_k = \frac{\beta}{1 + \rho \Sigma_{k=1}^{N} \frac{1}{\gamma_k}}, \tag{13}$$

where the latter is very useful since it expresses the result directly in terms of the system and excitation parameters. This amplitude contains information about the ultimate system performance, that is, the rotor vibration, as measured by its angular acceleration. However, as noted above, an accurate measure of the rotor acceleration requires inclusion of at least the leading order (quadratic) nonlinear terms.

It is interesting to note that the ratios between the absorber amplitudes are given directly by Eq. (11). Thus, one can immediately compute the ratio of largest to smallest absorber amplitudes by using $\max(A_i/A_k) = \max(\gamma_k/\gamma_i)$ and noting that, when the absorber dampings are equal, this is determined by using the absorber with the most mistuning for k and that with the smallest mistuning for i. Thus, the largest amplitude ratio is given by

$$\max|A_i/A_k| \cong \sqrt{\frac{\sigma_{\max}^2 + (\mu_a/n)^2}{\sigma_{\min}^2 + (\mu_a/n)^2}}, \tag{14}$$

where small terms involving σ^2 have been ignored. Thus, it is seen that for small damping the maximum ratio of mistunings plays the key role in the degree of localization in the system. Also, for moderate levels of damping, any differences in mistunings have a less pronounced effect.

Some Special Cases. Insight about the system response can be obtained by considering some special cases of interest. The results from these cases yield convenient and insightful forms when approximations are made based on the small parameter assumptions described above. Specifically, when the "small parameter assumption" is made in the following developments, it implies that the result has been expanded in terms of the dimensionless parameters ν, σ_i, μ_a and f, and quantities involving products of these parameters have been ignored.

The first special case is that of identical absorbers, such that $\tilde{n}_i = \tilde{n}_0 \forall i$, which implies that $\sigma_i = \sigma_0 \forall i$ and $\gamma_i = \gamma_0 \forall i$. This analysis is useful for setting the average level of mistuning to be designed into the absorbers. The small parameter assumption yields

$$A_i \cong \frac{-f}{n(2\sigma_0 n + j\mu_a + \nu n)}, \tag{15}$$

and

$$B \cong \frac{-f(2\sigma_0 n + j\mu_a)}{2\sigma_0 n + j\mu_a + \nu n}. \tag{16}$$

Note that there is a resonance for small damping when the mistuning satisfies $\sigma_0 = -\nu/2$. This corresponds to the case when the

Table 1 Data for example 1

Absorber	a. σ	b. σ	c. σ	d. σ
1	0.0016	0.0016	0.0016	0.0016
2	0.0120	0.0013	0.0013	0.0013
3	0.0080	0.0080	0.0015	0.0015
4	0.0100	0.0100	0.0100	0.0018

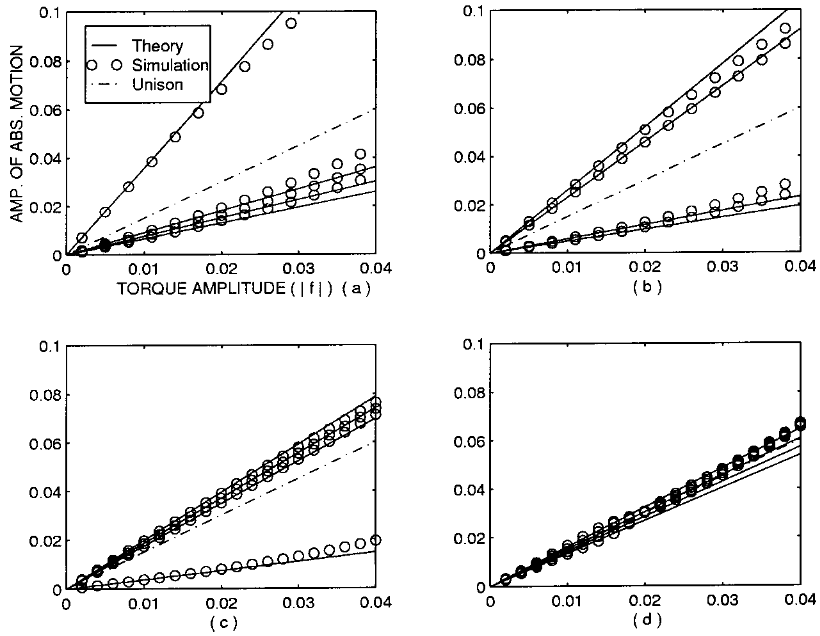

Fig. 2 Absorber amplitudes versus fluctuating torque level for example 1. See Table 1.

excitation order matches that of a natural mode of the system in which all absorbers move synchronously and out of phase with respect to the rotor. For small values of the inertia ratio ν, this resonance is very close to the ideal tuning point, $\sigma_0=0$. Note also that the rotor acceleration is equal to zero when the absorber damping and mistuning are both zero. Due to the close proximity of the resonance to the ideal tuning point, a small amount of positive mistuning is typically employed to provide robustness against potential resonance problems. In fact, this resonance effect has an important nonlinear character and is quite well understood [12,14].

The next special case is when two sets of absorbers are taken to have mutually identical levels of mistuning, one of which is zero and the other σ_0. The goal here is to observe what happens if a subset of absorbers is perfectly tuned, while another group is mistuned. Without loss of generality (for this case), it is assumed that the first M absorbers are identically mistuned with $\sigma_i=\sigma_0$, $i=1,2,\ldots,M(<N)$ and the remaining $N-M$ absorbers are perfectly tuned, that is, $\sigma_i=0$, $i=M+1, M+2,\ldots,N$. Again using the small parameter assumptions, the results for the complex amplitudes of the absorbers and the rotor are given, respectively, as follows:

$$\left.\begin{aligned} A_1=\ldots=A_M &\cong \frac{-fNj\mu_a}{n(N(2\sigma_0 n+j\mu_a)(j\mu_a+\nu n)-M2\sigma_0 n^2\nu)},\\ A_{M+1}=\ldots=A_N &\cong \frac{-fN(2\sigma_0 n+j\mu_a)}{n(N(2\sigma_0 n+j\mu_a)(j\mu_a+\nu n)-M2\sigma_0 n^2\nu)}, \end{aligned}\right\} \tag{17}$$

and

$$B\cong \frac{-fj\mu_a(2\sigma_0 n+j\mu_a)}{N(2\sigma_0 n+j\mu_a)(j\mu_a+\nu n)-M2\sigma_0 n^2\nu}. \tag{18}$$

Of course, the $M=N$ case matches the previous special case. These results offer useful insights into the response that are made clear by considering the zero damping case, $\mu_a=0$. Here the rotor acceleration is zero ($B=0$) as are the amplitudes of the first M absorbers, that is, those that are mistuned. The response amplitude of the perfectly tuned absorbers in this case is given by $A_i=-fN/((N-M)\nu n^2)$, $i=M+1,\ldots,N$. In this case, the mistuned absorbers do absolutely nothing—they do not move and do not dynamically contribute to the rotor vibration (other than acting like a flywheel). The rotor vibration is completely attenuated by the set of tuned absorbers, whose amplitudes are magnified by the ratio $N/(N-M)$ when compared to the case when all absorbers are ideally tuned. This amplification allows them to make up for the lack of contribution from the mistuned absorbers. Here the localization is clearly evident. The most severe case is when only one absorber is perfectly tuned, in which case its amplitude is given by $A_N=-fN/(\nu n^2)$ while the remaining absorbers have zero amplitude. The localization becomes less severe if more absorbers are perfectly tuned, until the case $M=N$, considered above, where no localization occurs. Of course, the presence of damping makes these results less sharp, in terms of the tuning and the degree of localization. Generalizations of these observations can be made for cases in which there are two or more groups of absorbers, each mutually identically mistuned.

In order to examine these effects for more general conditions, specifically with imperfections, we turn to some numerical examples in which the absorbers are given individual values of tuning.

4 Numerical Examples and Parameter Trends

For all cases in this section we consider a system of four absorbers, $N=4$, with the following numerical data: inertia ratio $\nu=0.1662$ and torque order $n=2$.[4]

The Effects of Imperfections Among Absorbers—Example 1. For this example we take the damping to be $\mu_a/N=0.0013$.[5] Four cases are considered, for which the mistuning levels are shown in Table 1. The first case corresponds to a situation where the absorbers are mistuned in a positive manner, but one absorber has an imperfection such that it has a smaller mistuning level than the remaining three. The second case corresponds to a situation where two absorbers have smaller mistuning

[4]These values are borrowed from the study of a particular in-line, four-cylinder engine by Denman [13].

[5]The absorber damping is modeled as an equivalent linear viscous damping that does not depend on the total mass of the absorber system, i.e., the quantity $\mu_a/N=c_a/m_o\Omega$ is a fixed physical quantity.

Table 2 Data for example 2

Absorber	a. σ (%)	b. σ (%)
1	0.0003 (0.03)	0.0002 (0.02)
2	0.0024 (0.24)	0.0012 (0.12)
3	0.0016 (0.16)	0.0008 (0.08)
4	0.0020 (0.20)	0.0010 (0.10)

levels than the other two. The third case corresponds to a situation where the absorbers are only slightly mistuned, and one absorber has an imperfection that gives it a larger mistuning level than the other three. The fourth case corresponds to a situation where the the mistuning level is small and there are imperfections among the absorbers.

The amplitudes of the steady-state responses of the four absorbers are plotted versus the applied torque level for these four cases in Fig. 2. The simulation results shown are obtained by numerically solving the full nonlinear equations of motion for the case of epicycloidal absorber paths (these paths are the closest to being linear over a wide range of amplitudes, cf. [13,15]). Also note that the dashed line represents the absorbers' response for the system when all absorbers are identical and perfectly tuned to the order of the applied torque. It is clear from this figure that localization indeed occurs for this system. The severity of the localization depends on the tuning differences, that is, the imperfections, between the absorbers. The absorbers with smaller mistuning levels localize whenever any of the other absorbers have larger levels of mistuning. This follows since the absorbers with tuning closest to the ideal tuning will do most of the work in counteracting the applied torque. It is also clear from the figure that the strength of the localized response depends on the number of absorbers that localize. As expected, the most severe case occurs when one absorber has a small level of mistuning compared to the remaining absorbers, that is, case 1(a), as shown in Fig. 2(*a*). These observations are consistent with the special cases considered in the previous section.

The Effects of the Average Level of Imperfections—Example 2. This example is limited to the most severe case in which only one absorber localizes. Here, the damping level is kept the same as that in example 1, but the overall levels of imperfections are reduced. The two cases shown in Table 2 are considered. The first case has imperfection levels one fifth of those in example 1(a), while the second case has imperfection levels one tenth those of example 1(a). The amplitudes of the absorbers' steady-state responses versus the applied torque level for these two cases are shown in Fig. 3. It is clear that localization becomes weaker when the imperfection levels are reduced.

The Effects of the Average Level of Mistuning—Example 3. In this example three cases are considered, in which nominal intentional mistuning levels of 0.016, 0.033, and 0.05 are added to all four absorber paths of example 1(a), respectively. The relative imperfections among the absorbers' paths are similar to those of example 1(a), and are shown in Table 3. The amplitudes of the absorbers' steady-state responses versus the applied torque level for these three cases are shown in Fig. 4. It is clear that the system becomes less localized when a positive intentional mistuning is introduced, especially when it is large compared to the imperfections in the absorbers' paths.

The Effects of Damping. To see the effects of the absorber damping level, the cases of examples 1(a), 2(a) and 2(b) are reconsidered with variable damping levels. The ratios of the maximum to the minimum absorber amplitudes, $|A_{\max}/A_{\min}|$, are plotted versus the damping level μ_a for these examples in Fig. 5.[6] These results clearly demonstrate that increasing the absorber damping decreases the strength of the localized response. Also note that the strength of the localization is greater as the magnitude of the imperfections is increased. This is due to the fact that as the absorber tunings become more spread out, the absorber closest to perfect tuning will do the bulk of the absorbing.

The Effects of Varying the Tuning of a Single Absorber. In each of the two cases considered here, a certain mistuning level is assigned to three of the absorbers and the mistuning of the fourth absorber is varied from zero to the corresponding value of the other absorbers. The levels of mistuning for the three absorbers are taken to be 0.008 and 0.002 for the two cases. Figure 6 shows the maximum absorber amplitude ratio versus a measure of the difference in mistuning between the absorbers. Note that as the difference in mistuning between the fourth absorber and the other

[6]Note that all curves start at the ratio between the largest and smallest levels of mistuning, which is 7.5 for all cases considered here.

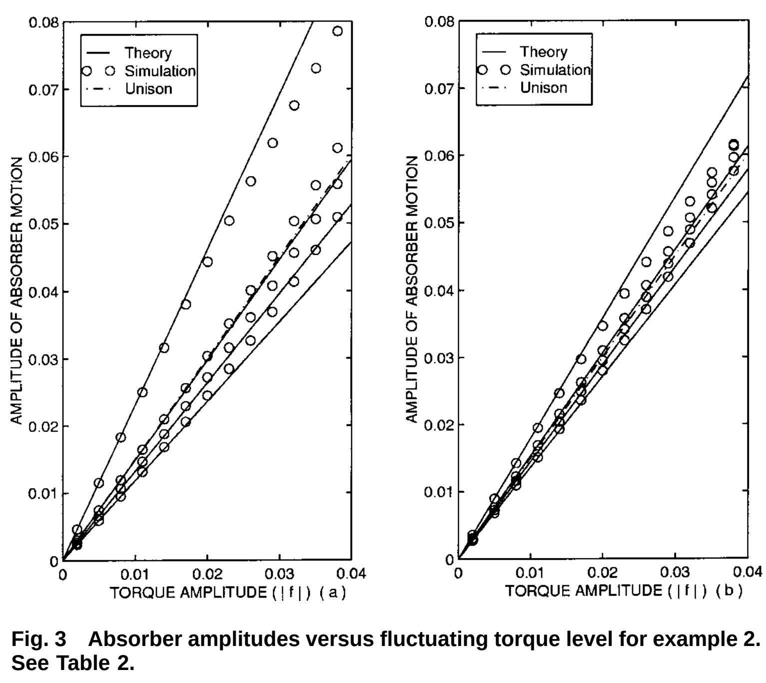

Fig. 3 Absorber amplitudes versus fluctuating torque level for example 2. See Table 2.

Table 3 Data for example 3

Absorber	a. σ	b. σ	c. σ
1	0.018	0.035	0.052
2	0.028	0.045	0.062
3	0.025	0.042	0.058
4	0.026	0.043	0.060

three becomes larger, the degree of localization becomes stronger. It is also clear that the localization is stronger for higher absolute magnitudes of mistuning. This is due to the fact that the larger mistuning prevents the three absorbers from working effectively, while the forth absorber does virtually all the absorbing when its mistuning is relatively small (that is, on the right side of the graph).

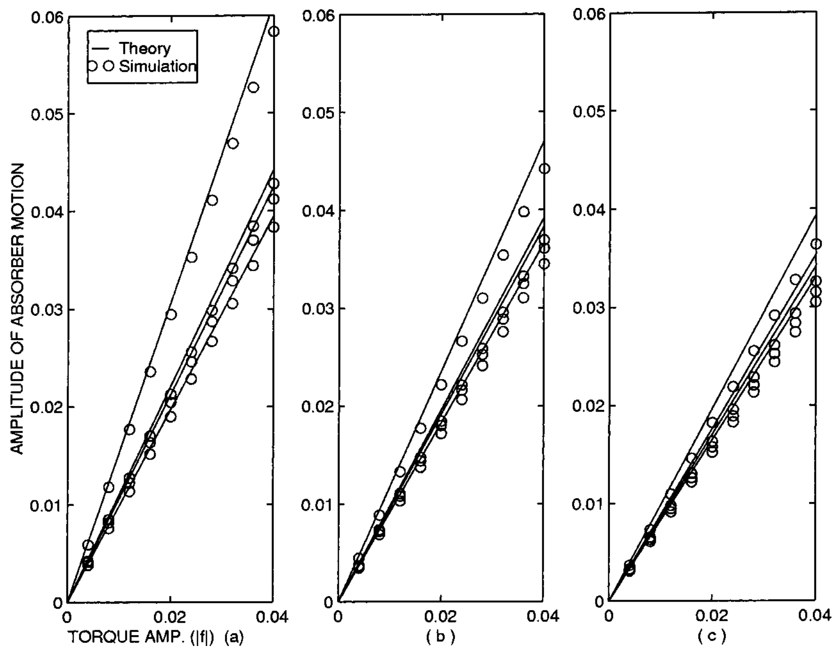

Fig. 4 Absorber amplitudes versus fluctuating torque level for example 3. (a) σ_n=0.016, (b) σ_n=0.033, (c) σ_n=0.050. See Table 3.

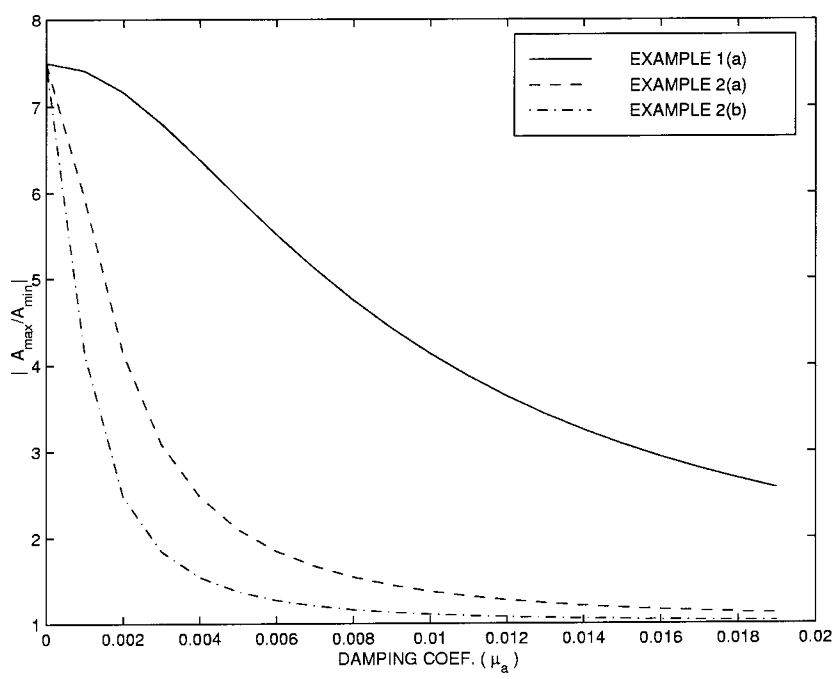

Fig. 5 Effect of damping level on localization

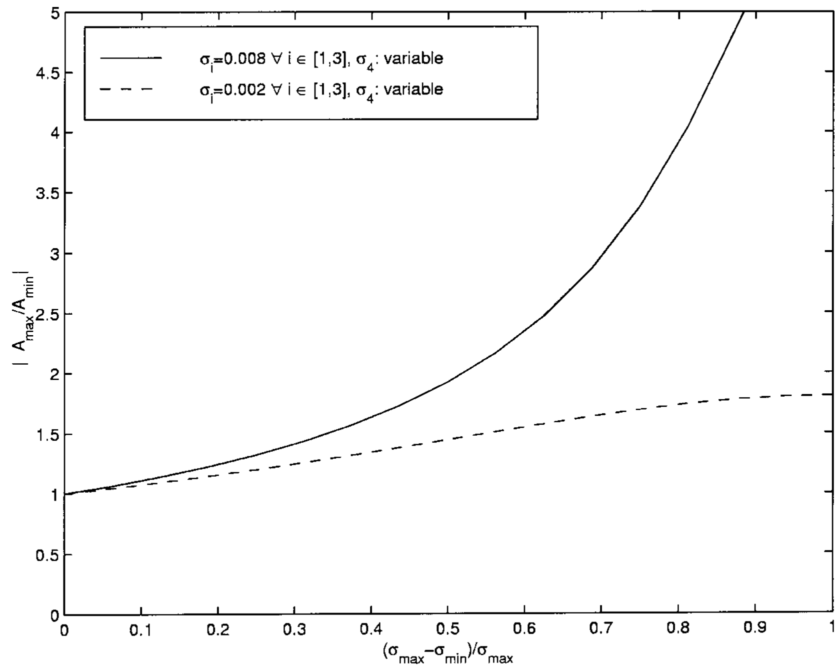

Fig. 6 Effect of mistuning differences on localization

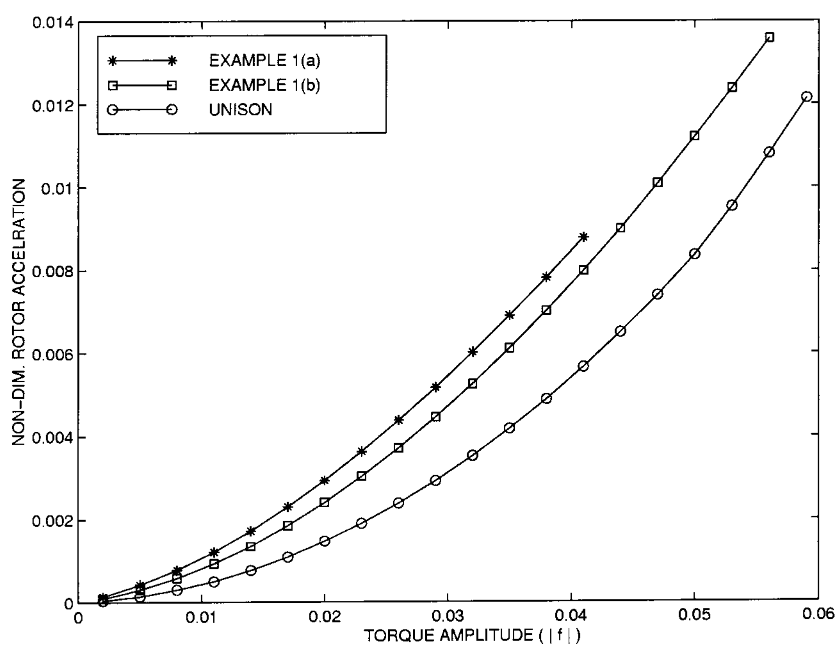

Fig. 7 Nondimensional rotor acceleration versus fluctuating torque level for example 1. From numerical simulations.

5 Discussion

Summary. In the presence of small imperfections between the absorber paths, localization can occur in the forced steady-state response of CPVA systems. The severity of the localized responses depends on (i) the level of damping, (ii) the imperfections among the absorber tunings, (iii) the average level of mistuning, and (iv) the number of absorbers experiencing localization. These results can be quantified using the analytical results, which can be used to guide absorber design specifications.

A Note on System Performance. There are two primary reasons why localized responses should be avoided in these absorber systems. The first is that they degrade system performance, in terms of the ability of the absorbers to attenuate torsional vibrations. The second is the fact that the localizing absorber(s) will hit its (their) amplitude limits at a smaller level of applied torque than if there were no localization. This means that for a limited rattle space, localization decreases the system's operating range.

These consequences are demonstrated in Fig. 7, which shows the amplitude of the nondimensional angular acceleration of the rotor for the systems given in examples 1(a) and 1(b). The figure also shows the case of the perfectly tuned system in which all absorbers move at the same amplitude. This figure was obtained by numerically simulating the full nonlinear equations for the case of epicycloidal paths. Note the essentially nonlinear character of this response, even though the absorber motions are quite accurately described by their linearized equations.

The simulations are run up to a torque level at which at least one absorber reaches a limit in amplitude that is imposed by the nature of this path [13]. It is seen that the perfectly tuned paths offer the lowest rotor torsional vibration levels over the widest range of torques. A modest amount of localization, given by example 1(b), does not have much effect on the torque range, but results in larger vibration levels. The more severe localization of example 1(a) yields slightly larger vibration levels, but, more importantly, it causes a significant reduction in the operating range.

Recommendations. Increasing the damping level reduces the severity of localization (when it occurs), but this is not a desirable solution to the localization issue, since it severely affects absorber performance (see Eq. (18) and note the strong dependence on μ_a). In addition, damping is difficult to implement into absorber designs in a controlled manner. A more effective solution to avoid localization in CPVA systems is to introduce a small amount of intentional mistuning in the absorber paths, as demonstrated in Fig. 4. The mistuning level can be implemented by kinematic configuration of the absorber paths, and should be selected to be relatively large compared to the imperfections among the absorbers. This strategy, which requires information about the tolerances caused by manufacturing, wear, and thermal effects, will render a CPVA system robust against localization, and will maintain good absorber performance, since mistuning does not affect rotor acceleration as severely as does damping (cf. Eq. (18)).

It has long been known that positive mistuning is useful for avoiding the nonlinear jump behavior that occurs for absorbers with the popular circular path [12]. In fact, virtually all absorbers used in practice employ some type of positive mistuning, either linear (achieved by order selection) or nonlinear (achieved by employing a cycloidal absorber path [14]. More recently the authors have shown that this type of mistuning is also beneficial for avoiding a symmetry-induced nonlinear instability that occurs for a wide range of absorber paths [14].

Therefore, a suggested design strategy would include consideration of both localization and dynamic instabilities, based on estimates of system parameters such as damping, the inertia ratio, etc., as well as bounds on the uncertainties in the absorber parameters. The selection of the mistuning level could then be made based on avoiding all types of dynamic behaviors that reduce the effectiveness of the absorbers.

Finally, many of these features of absorber systems are currently being investigated using an experimental facility that permits one to systematically control the excitation, and measure the responses of the rotor and the individual absorbers [17].

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