Nonlinear Dynamics of Longitudinal Ground Vehicle Traction

Brian J. Olson

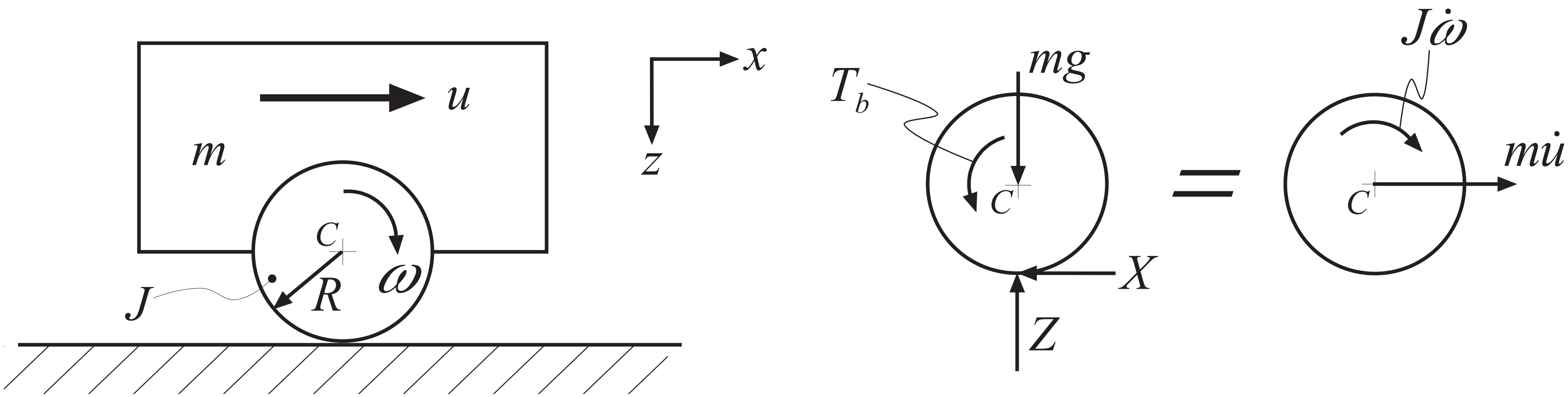
Graduate Research Assistant

Michigan State University

Outline

- Introduction
- Single Wheel Models
  - Braking (SWBM)
  - Acceleration (SWAM)
- Two Wheel Model
  - Braking (2WBM)
- Conclusions and Directions for Future Work

The Single Wheel Braking Model



Dynamics are governed by

$$
\begin{aligned}
Z &= mg \\
m\dot{u} &= -X \\
J\dot{\omega} &= RX - T_b
\end{aligned}
$$

The Tire/Road Interface

- **Need an expression relating X and Z** •

Friction Law (Creep Force Equation)

$$X = \mu(s)Z$$

μ longitudinal force coefficient
s longitudinal wheel slip

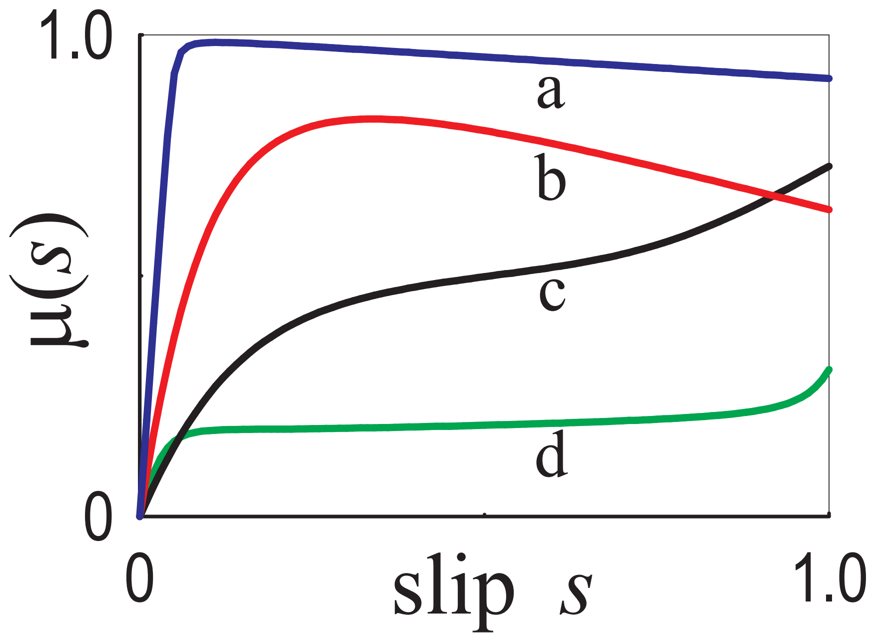
Longitudinal Wheel Slip

- Measure of the difference between vehicle speed u and rolling speed ωR of wheel

$$s \equiv \frac{u - \omega R}{\text{max}(u, \omega R)}, \quad \begin{cases} \omega R \leq u & \text{(braking)} \\ u \leq \omega R & \text{(acceleration)} \end{cases}$$

$$\begin{aligned} s = -1 \quad & (u = 0) & \Rightarrow \quad & \text{pure slip} \\ s = 0 \quad & (u = \omega R) & \Rightarrow \quad & \text{no slip, or free rolling} \\ s = 1 \quad & (\omega R = 0) & \Rightarrow \quad & \text{wheel lockup} \end{aligned}$$

Friction Characteristics



$a)$ Dry asphalt

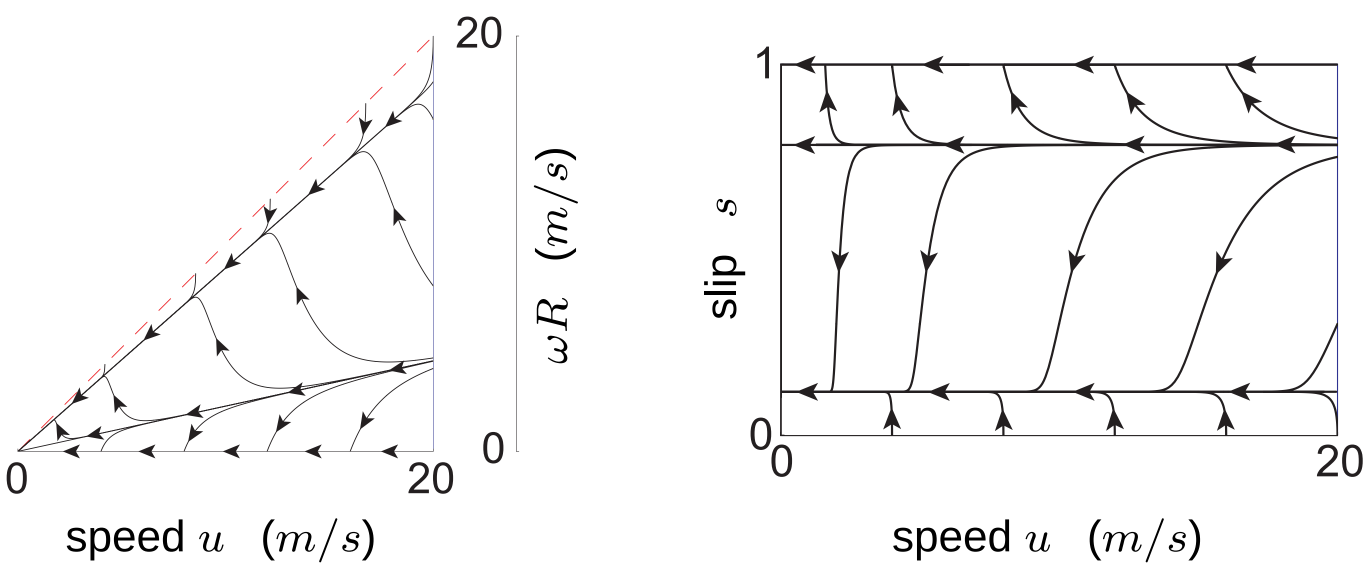
$b)$ Wet asphalt

$c)$ Gravel

$d)$ Packed Snow

Choice of Dynamic States

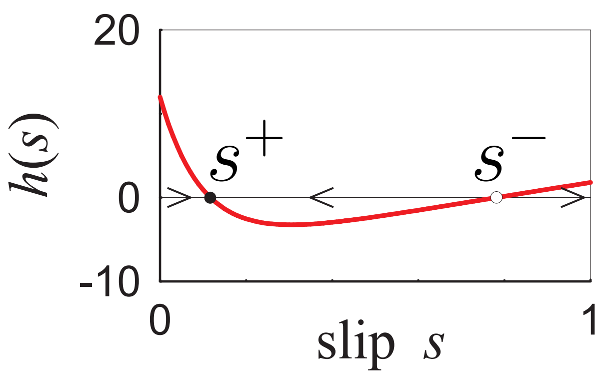
$(u, \omega R)$ vs. (u, s) state space



Equations of Motion

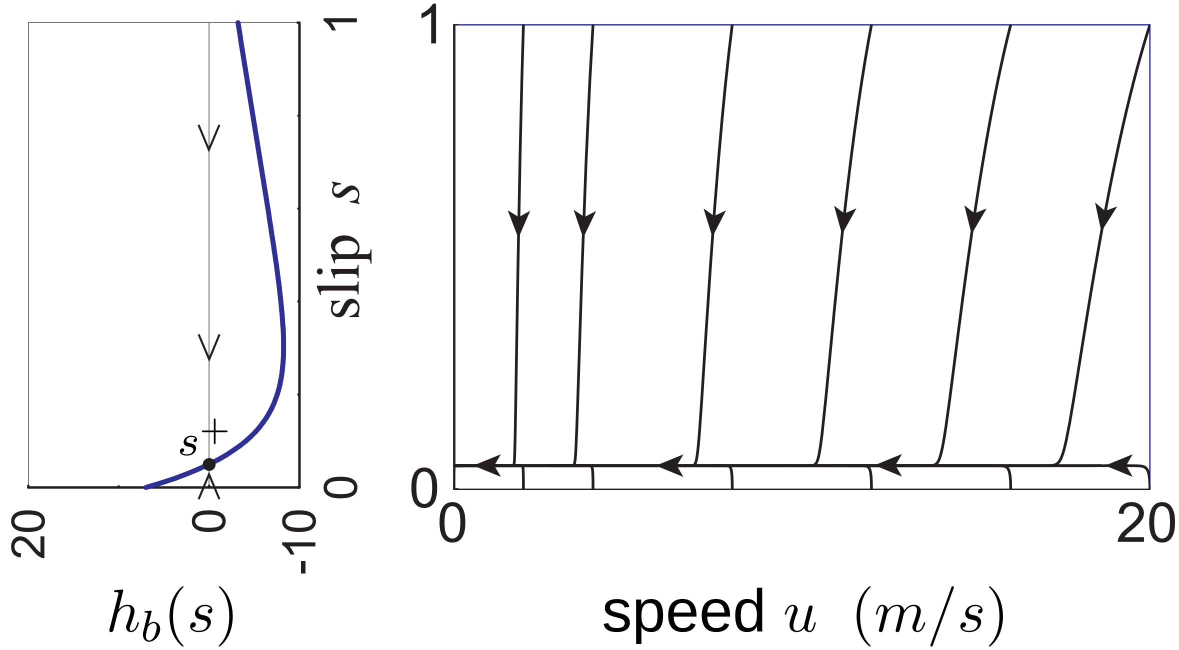
$$\left.\begin{array}{rcl}\dot{u} & = & -\mu(s)g \\ \dot{s} & = & \frac{g}{u}h_b(s)\end{array}\right\} \quad u > 0,\ \ s \in I$$

$$h_b(s) = \mu(s)(s - 1 - \Psi) + \Upsilon_b$$



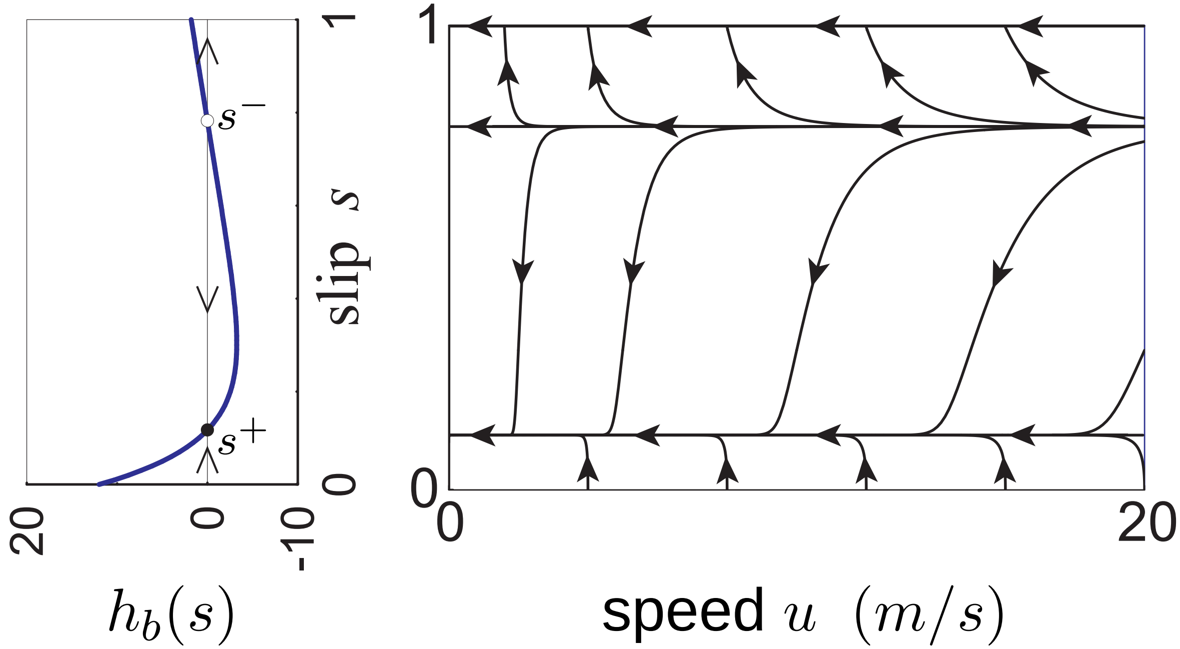
$$\Upsilon_b = 12$$
$$\Psi = 15$$

State Space Description



$\Upsilon_b = 7$ **Stable Braking**

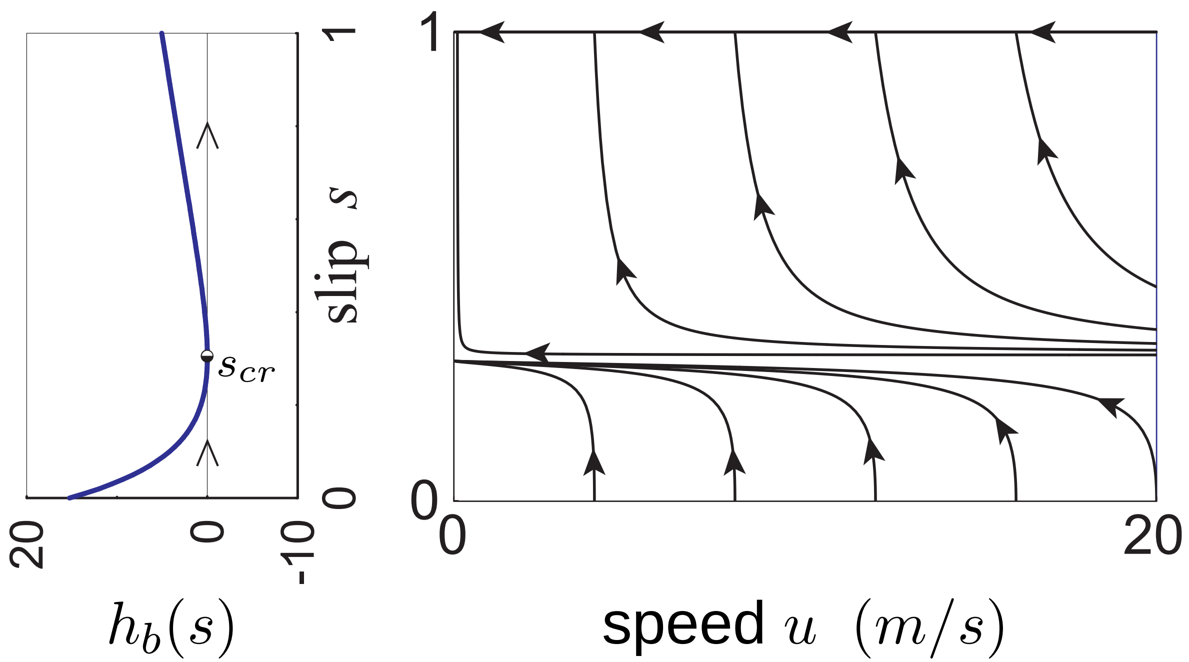
State Space Description



$$\Upsilon_b = 12$$

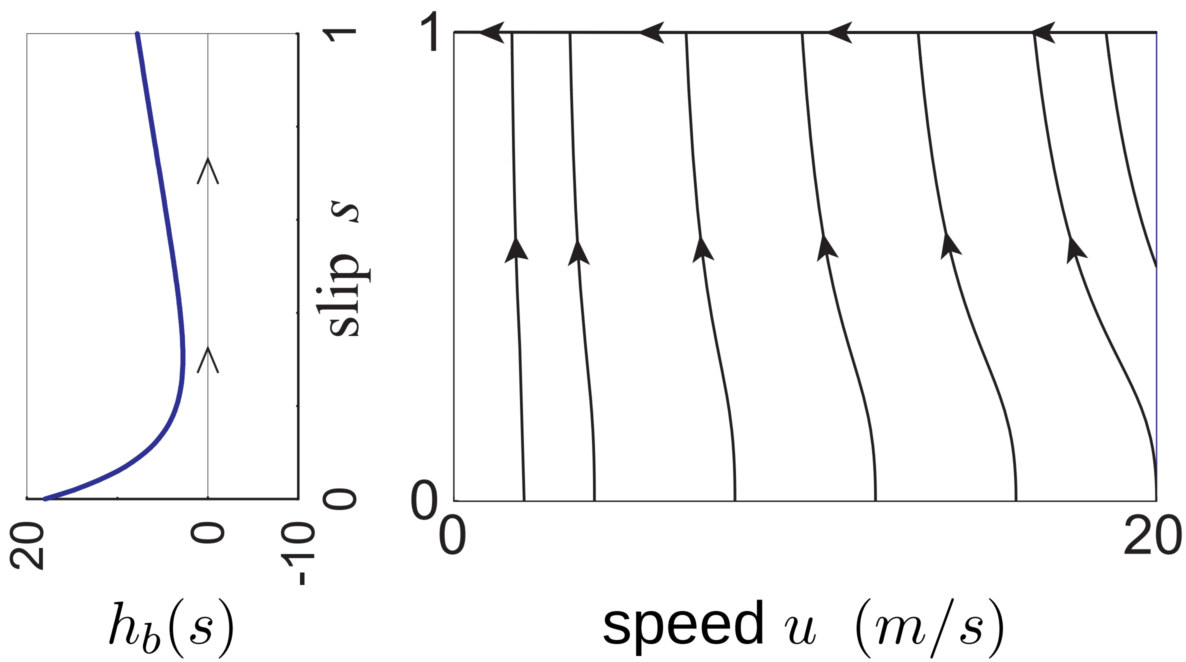
Mixed Braking

State Space Description



$$\Upsilon_b = \Upsilon_{b_{cr}} = 15.250$$ **Unstable Braking**

State Space Description



$\Upsilon_b = 18$ **Unstable Braking**

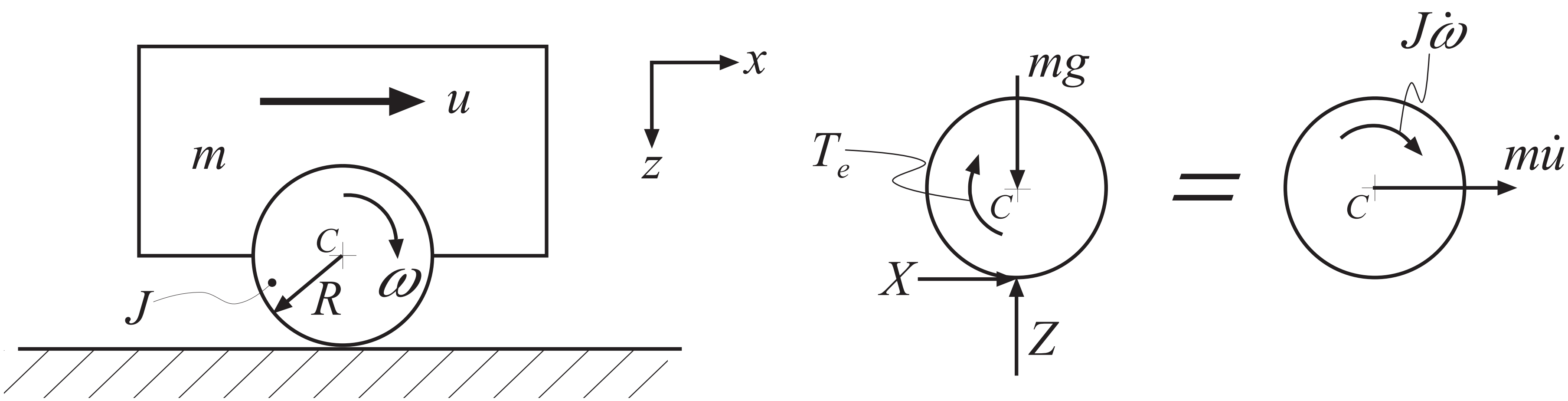
The Transition to Unstable Braking

- Maximum brake torque:

$$T_{b_p} = \begin{cases} mgR\mu(s_p) & \text{(assumed)} \\ mgR\mu(s_{cr})\left[1 + \frac{1}{\Psi}(1 - s_{cr})\right] & \text{(actual)} \end{cases}$$

- Fundamental assumptions:
  1. $\Psi \gg 1$.
  2. s_p can be reached.

The Single Wheel Acceleration Model

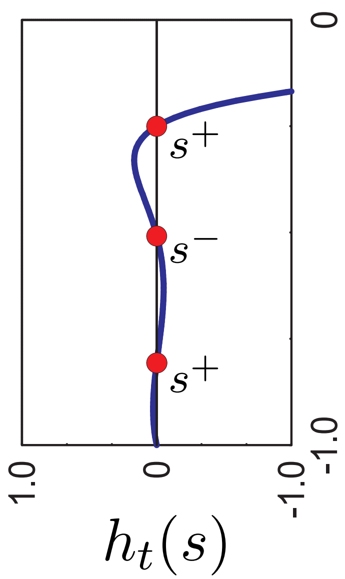


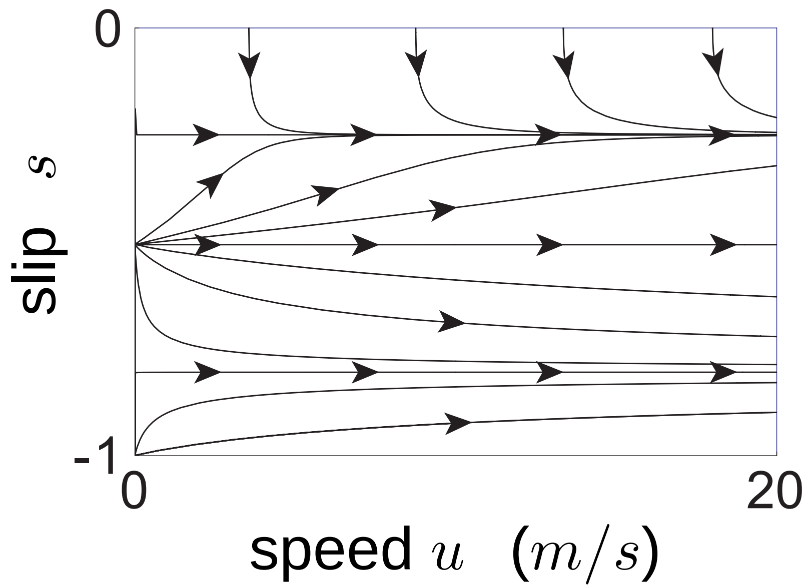
Equations of Motion:

$$\left.\begin{aligned}\dot{u} &= \mu(s)g \\ \dot{s} &= \frac{g}{u}h_t(s)\end{aligned}\right\}, \quad u > 0,\ s \in [-1, 0].$$

Example State Space

$$h_t(s) = (s+1)^2 \left[(s+1)^{-1}\mu(s) + \Psi\mu(s) - \Upsilon_e\right]$$

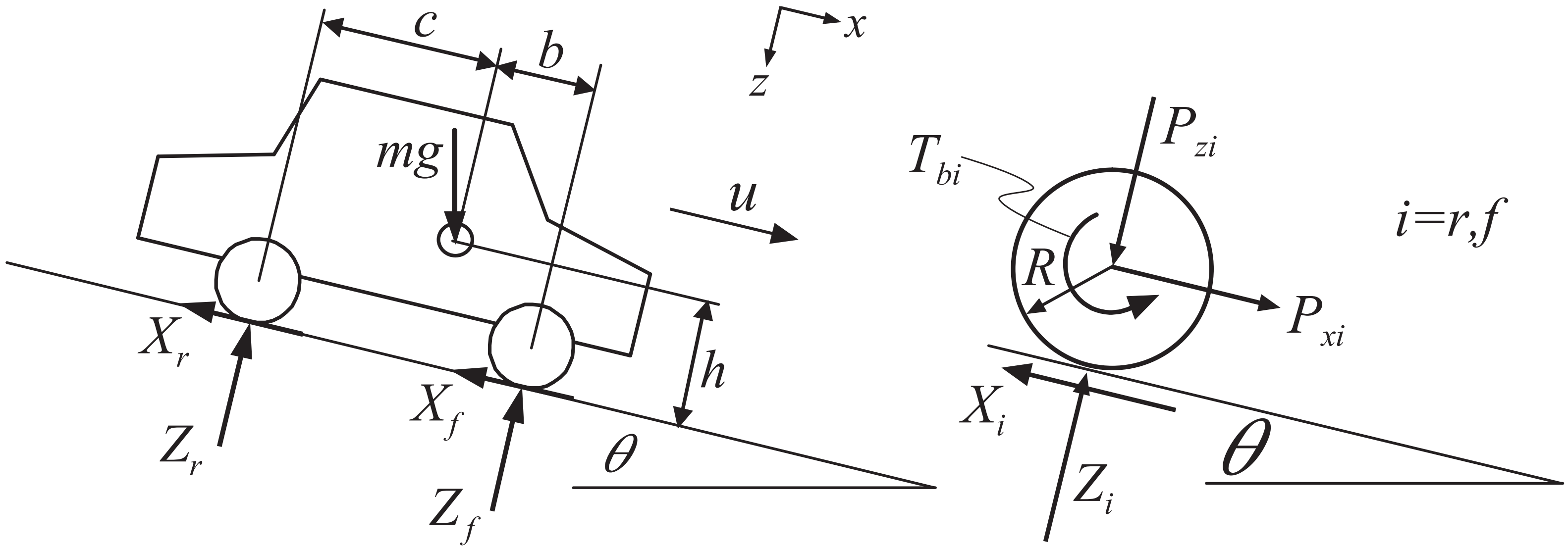




$\Upsilon_e = 15.65$

Mixed Acceleration

The Two Wheel Braking Model



$$X_i = \mu(s_i) Z_i, \qquad s_i \equiv \frac{u - \omega_i R}{u}, \qquad i = r, f$$

$$Z_r = mg\left(\frac{b}{l}\cos\theta - \frac{h}{l}\sin\theta\right) + m\dot{u}\frac{h}{l}$$

$$Z_f = mg\left(\frac{c}{l}\cos\theta + \frac{h}{l}\sin\theta\right) - m\dot{u}\frac{h}{l}$$

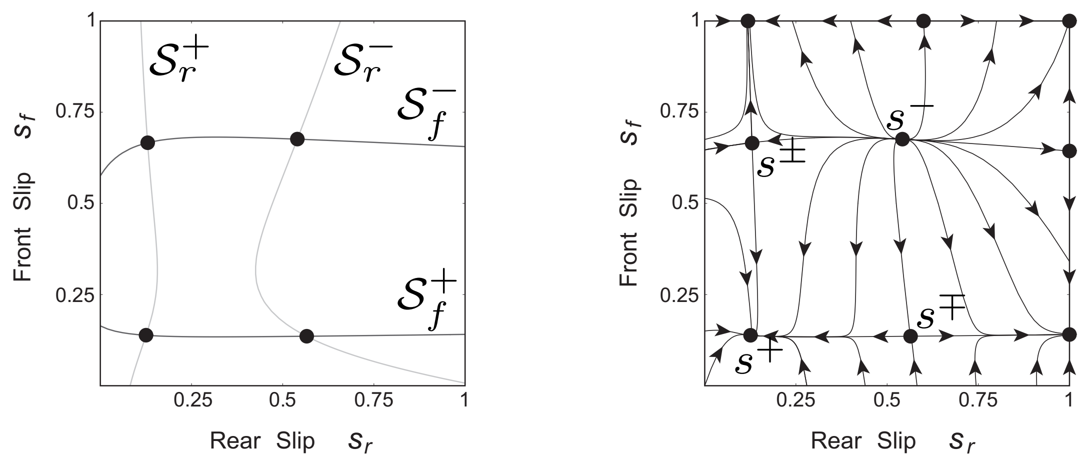
Equations of Motion

$$\left.\begin{array}{rcl} \dot{u} & = & -g\left(\Lambda_b(\mathbf{s})\cos\theta - \sin\theta\right) \\ \dot{s}_r & = & \frac{g}{u} h_{br}(\mathbf{s}) \\ \dot{s}_f & = & \frac{g}{u} h_{bf}(\mathbf{s}) \end{array}\right\}, \qquad u > 0,\ \ \mathbf{s} \in I \times I.$$

$$\mathbf{s} = \left(s_r, s_f\right)$$

$$h_{bi}(\mathbf{s}) = (s_i - 1)\left(\Lambda_b(\mathbf{s})\cos\theta - \sin\theta\right) - \mu(s_i)\Psi\lambda_i(\mathbf{s}) + \Upsilon_{bi},$$
$$i = r, f$$

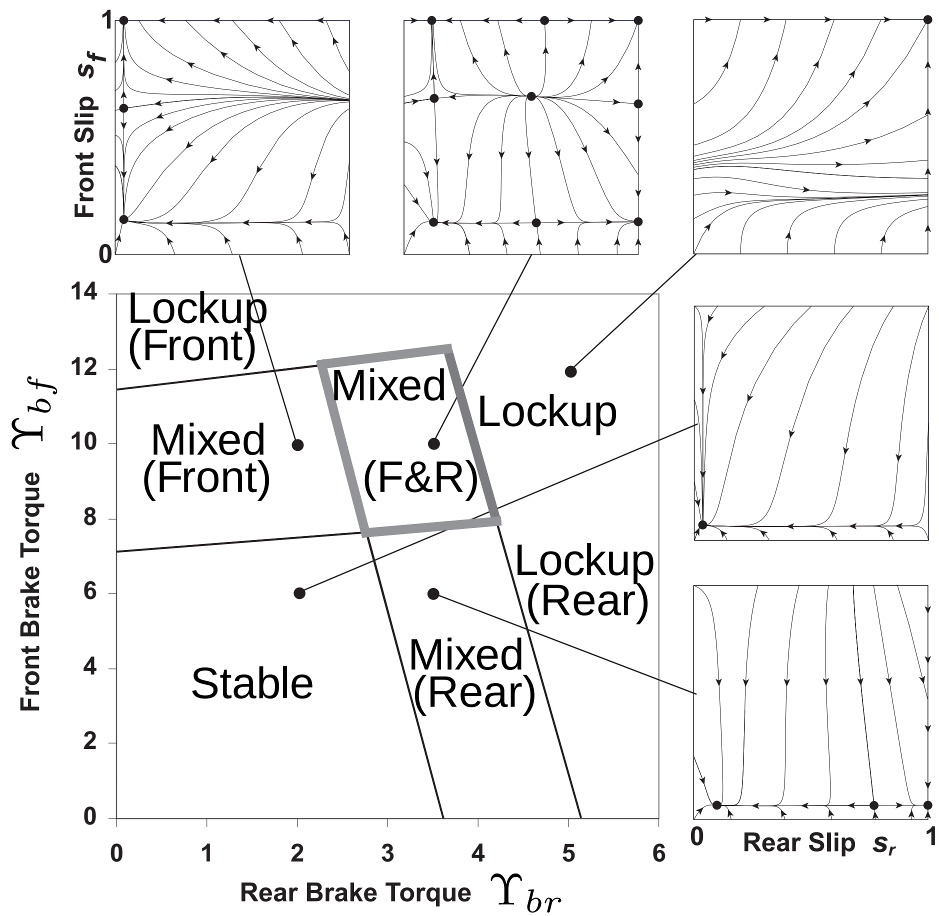
Steady Slip Conditions



$$\mathcal{S}_i^{\pm} = \{\mathbf{s} \mid h_{bi}(\mathbf{s}) = 0, h'_{bi}(\mathbf{s}) \lesseqgtr 0\}, \quad i = r, f.$$

$\mathbf{s}^+$	$=$	$\mathcal{S}_r^+ \cap \mathcal{S}_f^+$	Stable Node
$\mathbf{s}^-$	$=$	$\mathcal{S}_r^- \cap \mathcal{S}_f^-$	Unstable Node
$\mathbf{s}^\pm$	$=$	$\mathcal{S}_r^+ \cap \mathcal{S}_f^-$	Saddle
$\mathbf{s}^\mp$	$=$	$\mathcal{S}_r^- \cap \mathcal{S}_f^+$	Saddle

State Space Description



Directions for Future Work

- More detailed parameter studies
- Use of the two-wheel braking model to assess brake proportioning strategies
- A more thorough study of the two wheel acceleration model
- The effects of cornering
- the incorporation of these models into ABS/TCS development

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