A Nonlinear Model for Vehicle Braking

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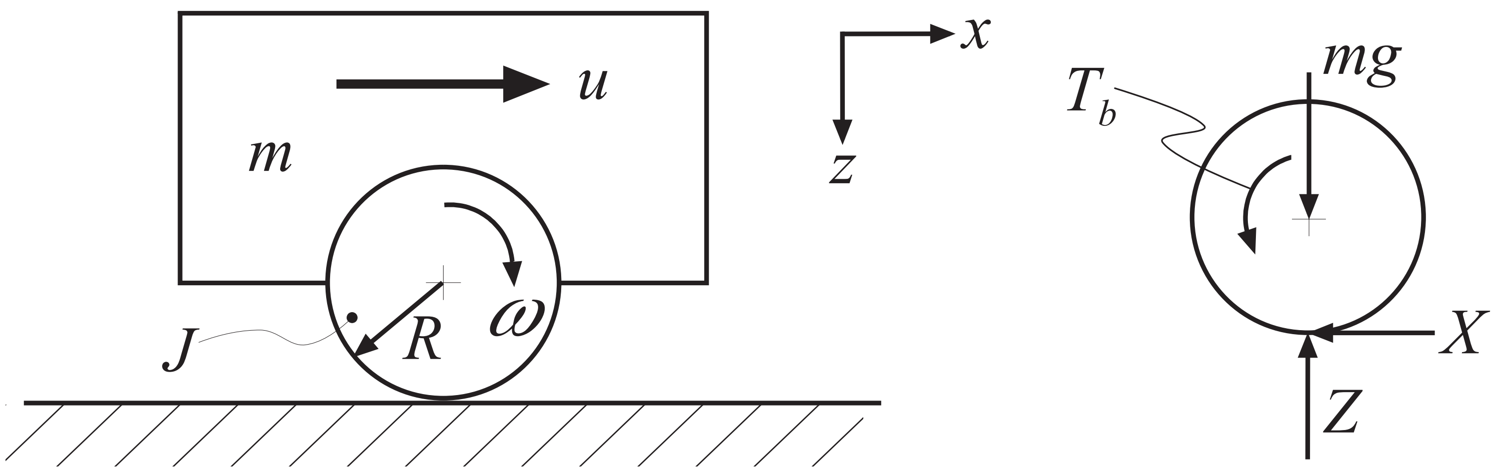
Outline

- Introduction
- Vehicle Braking
  - Single Wheel Model (SWM)
  - Two Wheel Model (2WM)
- Vehicle Acceleration Model
- Conclusions

The Single Wheel Model



SWM dynamics are governed by

$$\begin{aligned} Z &= mg \\ m\dot{u} &= -X \\ J\dot{\omega} &= RX - T_b \end{aligned}$$

The Tire-to-Road Interface

- **Need an expression relating X and Z** •

Friction Law (Creep Force Equation)

$$X = \mu(s)Z$$

μ longitudinal force coefficient (experimentally derived)
s wheel slip

Wheel Slip

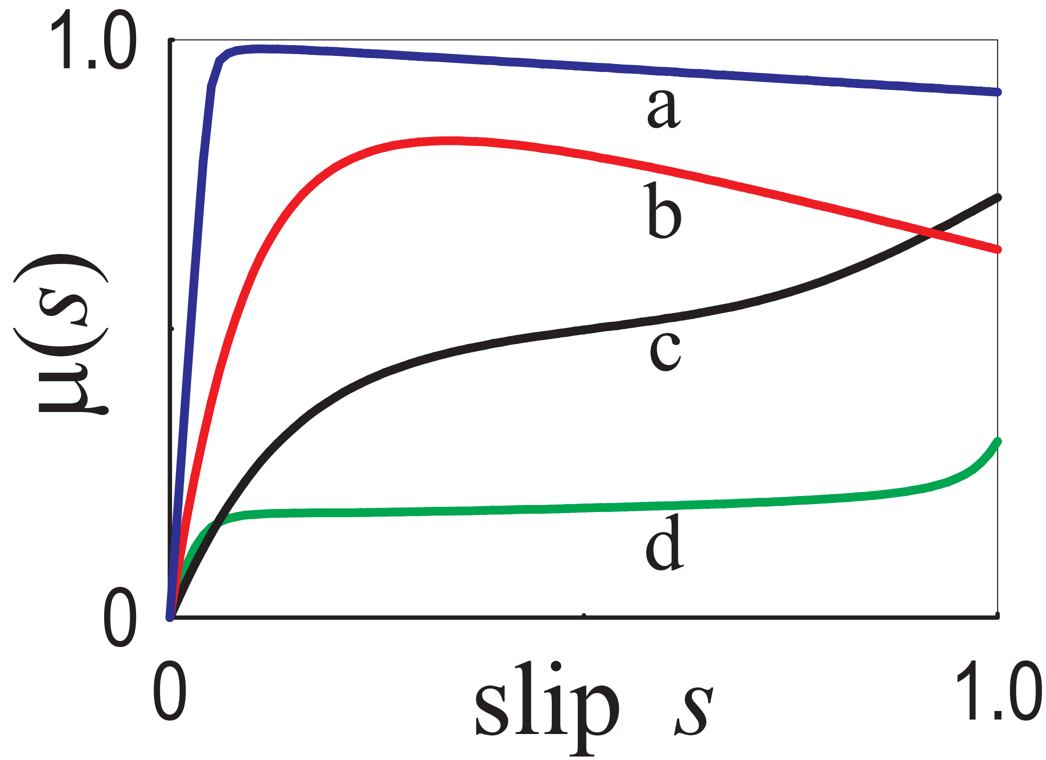
- Relative deviation of wheel speed compared to vehicle speed.

$$s \equiv \frac{u - \omega R}{u}, \qquad 0 \leq \omega R \leq u$$

$s = 0$ (u=ωR) $\Rightarrow$ no slip, or free rolling

$s = 1$ ($\omega R = 0$) $\Rightarrow$ wheel lockup!

Force Coefficient Characteristics



a) Dry asphalt

b) Wet asphalt

c) Gravel

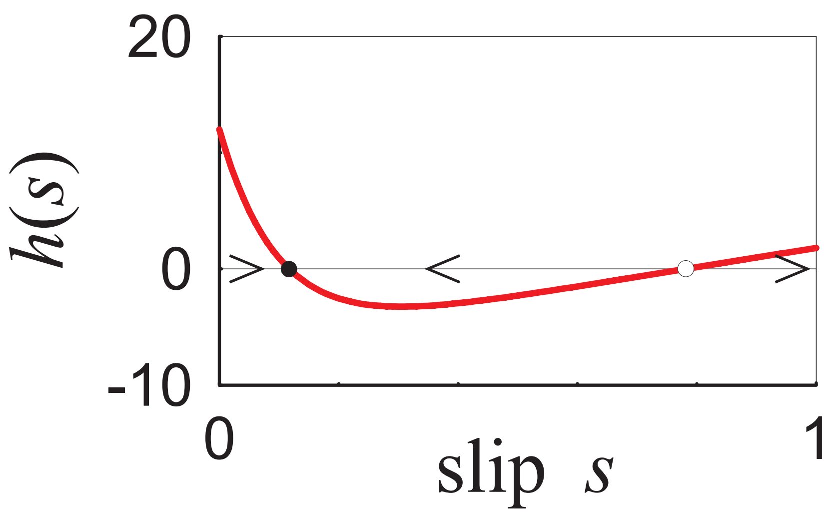
d) Packed Snow

Equations of Motion for the SWM

$$\dot{u} = -\mu(s)g$$

$$\dot{s} = \frac{g}{u}h(s)$$

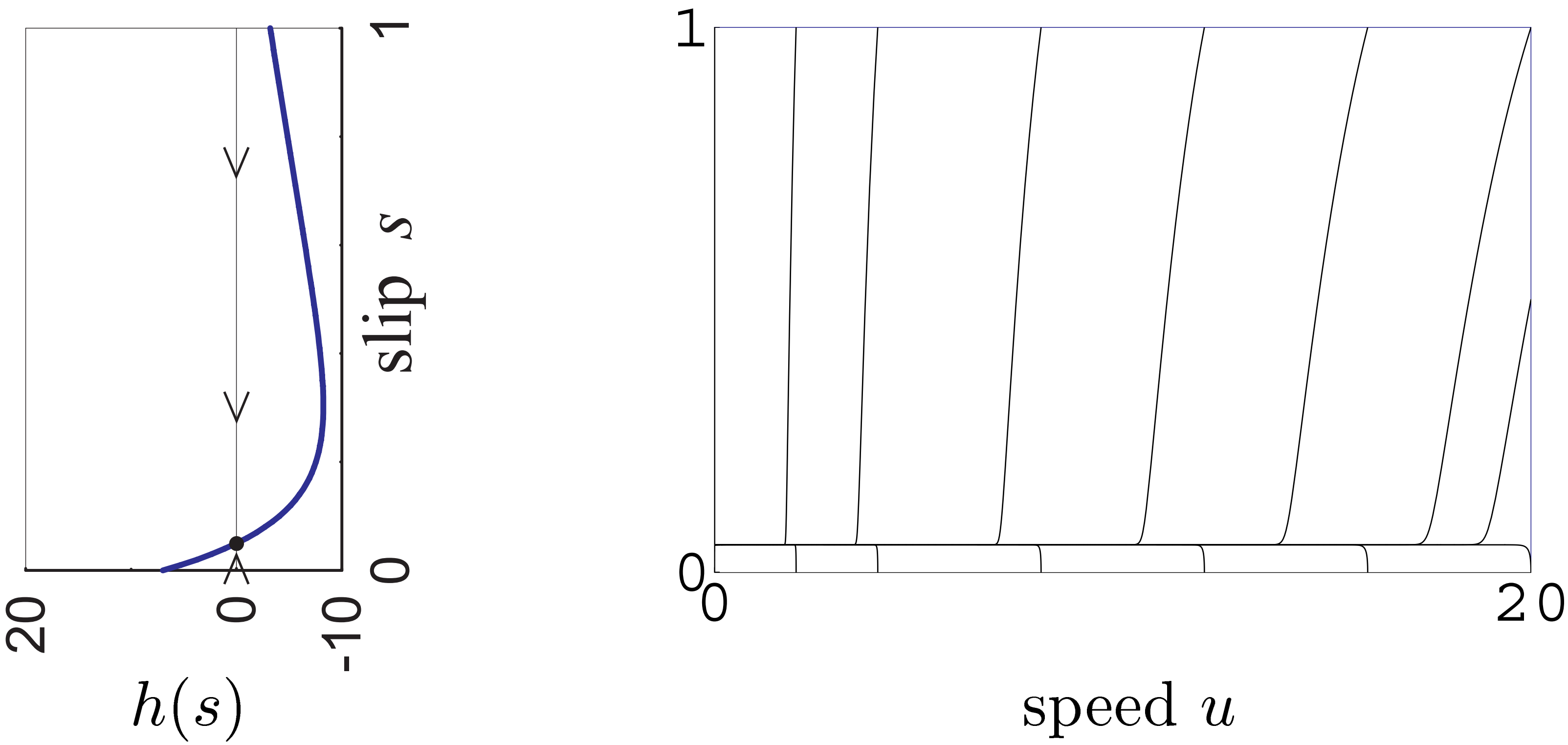
$$\blacktriangleright \quad h(s) = \mu(s)\,(s - 1 - \nu) + \Upsilon_b$$



$\Upsilon_b = 12$

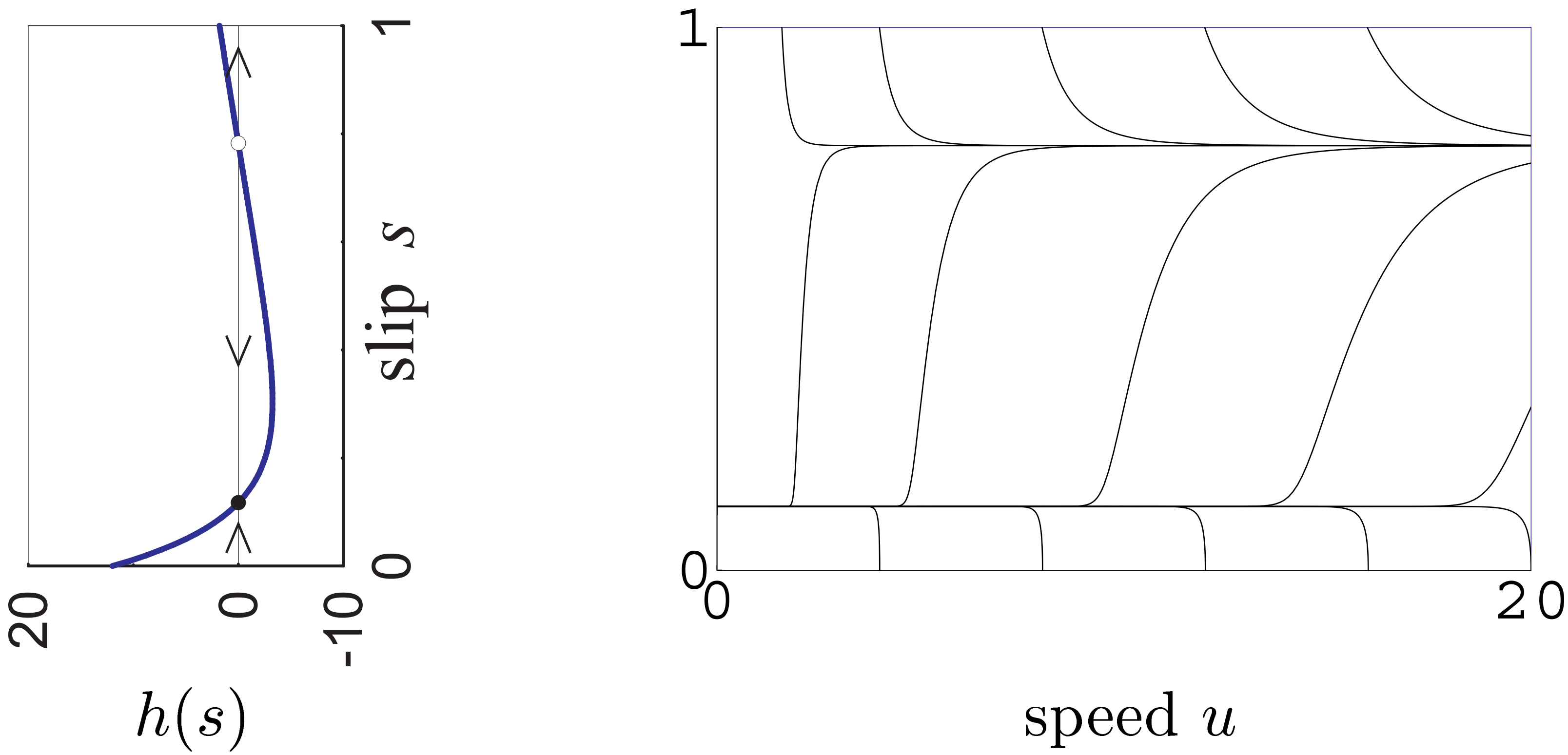
$\nu = 15$

State Space Description of the SWM



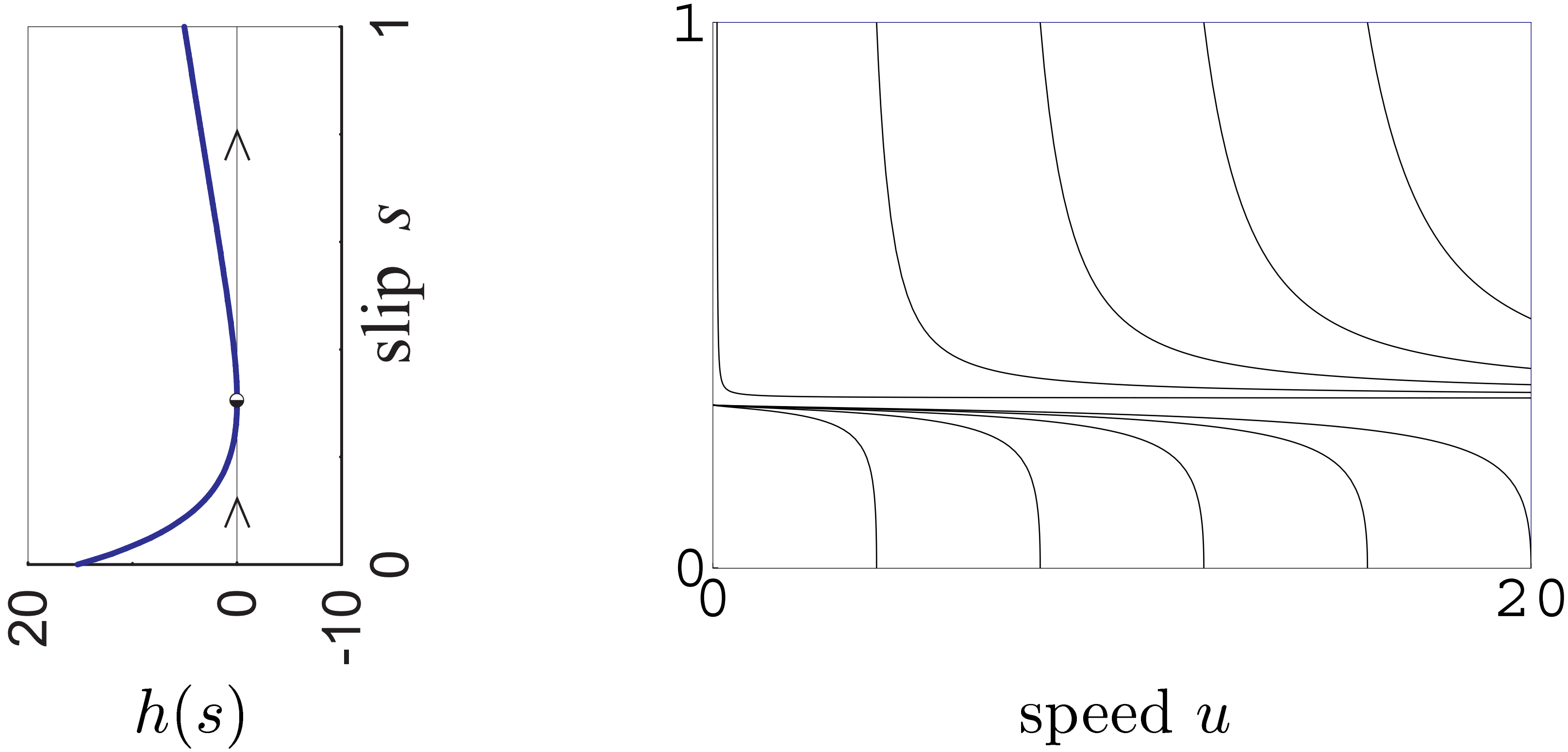
$\Upsilon_b = 7$ **Stable Braking**

State Space Description of the SWM



$\Upsilon_b = 12$ **Mixed Braking**

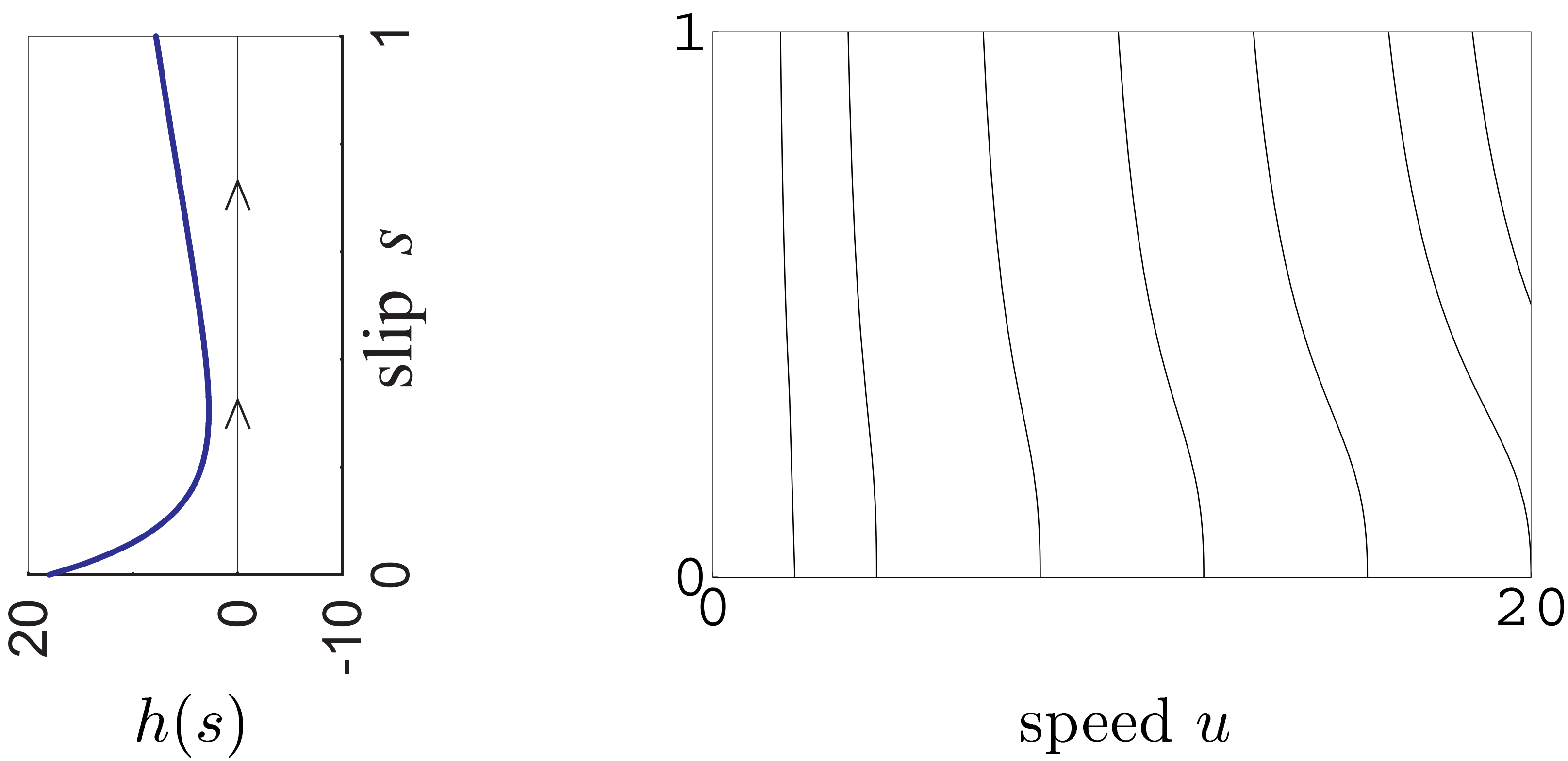
State Space Description of the SWM



$\Upsilon_b \cong 15.248$

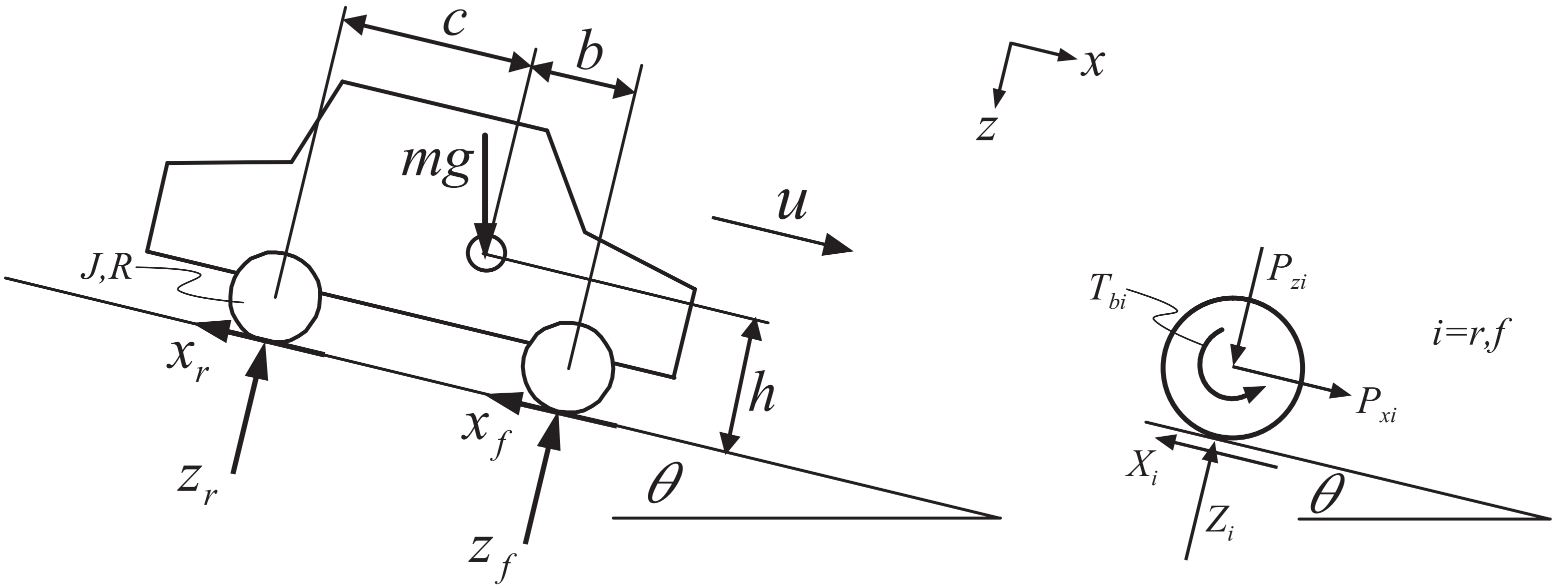
Unstable Braking

State Space Description of the SWM



$\Upsilon_b = 18$ **Unstable Braking**

The Two Wheel Model



$$X_i = \mu(s_i)Z_i, \qquad s_i \equiv \frac{u - \omega_i R}{u}, \qquad i = r, f$$

$$Z_r = mg\left(\frac{b}{l}\cos\theta - \frac{h}{l}\sin\theta\right) + m\dot{u}\frac{h}{l}$$

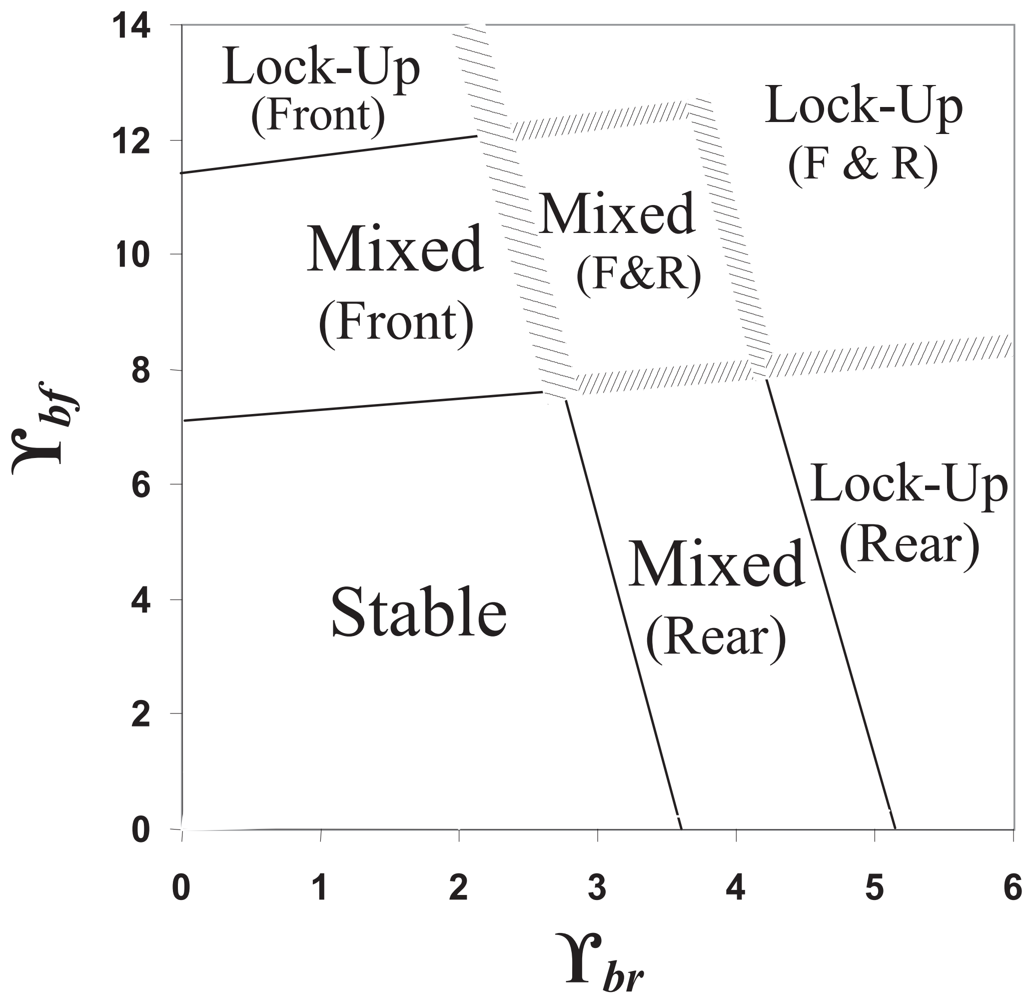
$$Z_f = mg\left(\frac{c}{l}\cos\theta + \frac{h}{l}\sin\theta\right) - m\dot{u}\frac{h}{l}$$

Equations of Motion for the 2WM

$$
\begin{aligned}
\dot{u} &= -g\left[\Lambda(s_r, s_f)\cos\theta - \sin\theta\right] \\
\dot{s_r} &= \frac{g}{u} h_r(s_r, s_f) \\
\dot{s_f} &= \frac{g}{u} h_f(s_r, s_f)
\end{aligned}
$$

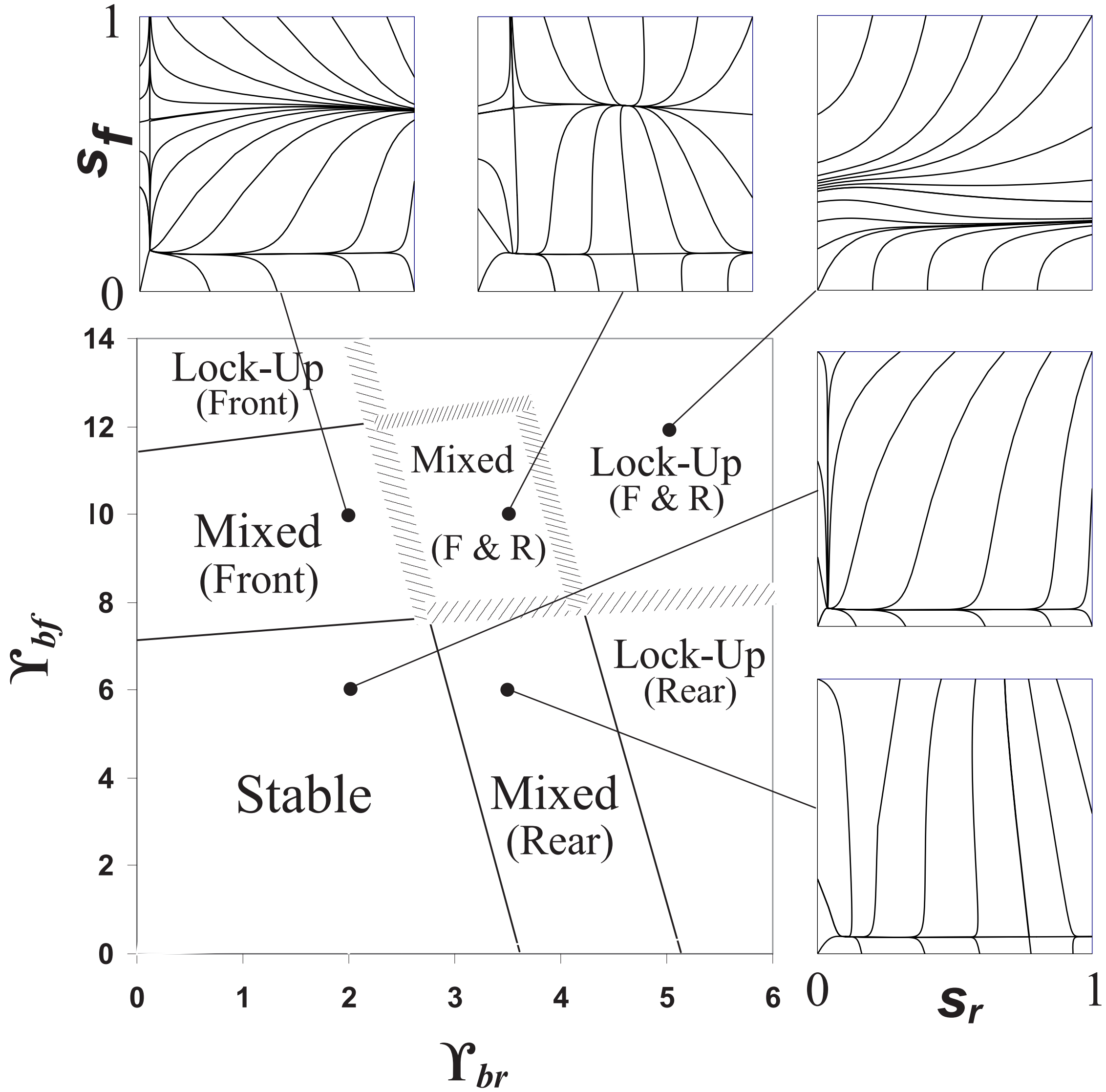
$$
h_i(s_r, s_f) = (1 - s_i)\Gamma(s_r, s_f) - \mu(s_i)\nu\lambda_i(s_r, s_f) + \Upsilon_{bi}
$$

Stability

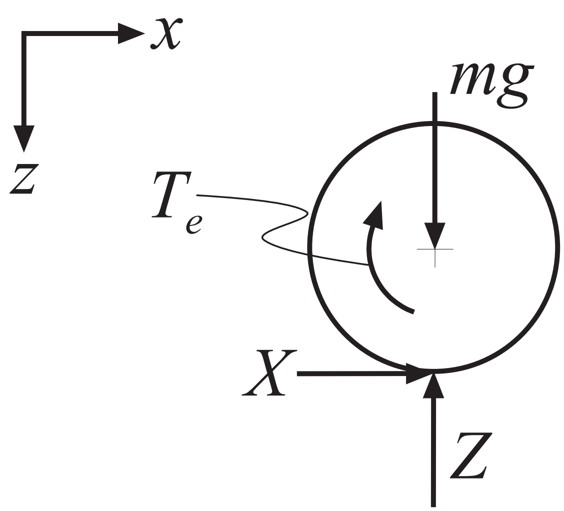


Lock-Up
(Front)
Lock-Up
(F & R)
Mixed
(F&R)
Mixed
(Front)
Lock-Up
(Rear)
Mixed
(Rear)
Stable
14
12
10
8
6
4
2
0
0
1
2
3
4
5
6
Υ_{bf}
Υ_{br}

State Space Description



Vehicle Acceleration Model



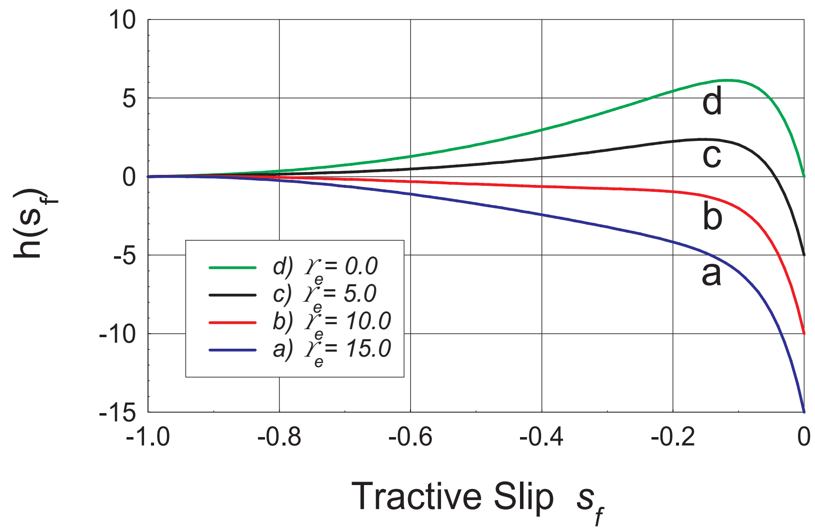
Equations of Motion:

$$\dot{u} = \mu(s_t) g$$

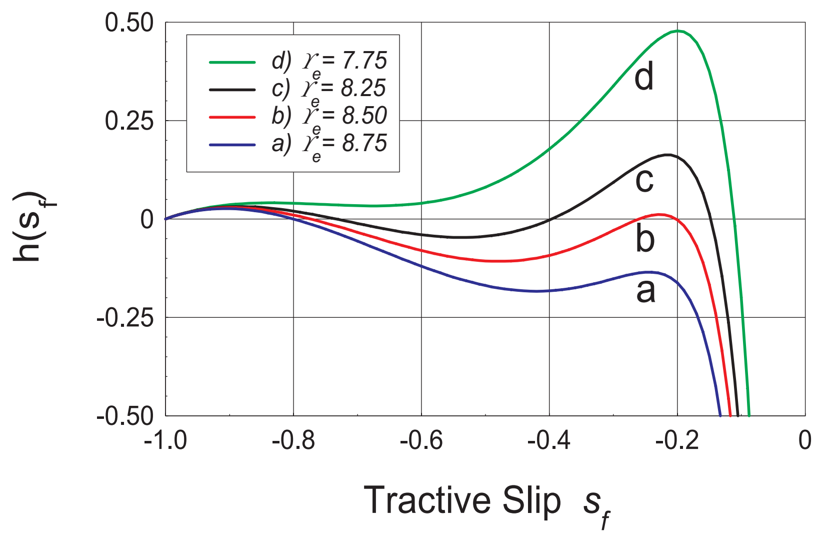
$$\dot{s}_t = \frac{g}{u} h_t(s_t)$$

$$s_t \equiv \frac{u - \omega R}{\omega R}$$

$$h_t(s_t) = (1 + s_t)^2 \left[(1 + s_t)^{-1} \Gamma(s_t) + \mu(s_t)\nu - \Upsilon_e \right]$$



10
5
0
-5
-10
-15
-1.0
-0.8
-0.6
-0.4
-0.2
0
h(s_f)
Tractive Slip s_f
d) γ_e = 0.0
c) γ_e = 5.0
b) γ_e = 10.0
a) γ_e = 15.0
d
c
b
a



0.50
0.25
0
-0.25
-0.50
-1.0
-0.8
-0.6
-0.4
-0.2
0
h(s_f)
Tractive Slip s_f
d) γ_e = 7.75
c) γ_e = 8.25
b) γ_e = 8.50
a) γ_e = 8.75
d
c
b
a

Conclusions

- A vehicle braking model described in terms of the independent variables u and s offers new insight into the dynamics and the stability of the system.
- Transition to lockup does not occur at the peak of $\mu(s)$, but at the minimum of $h(s)$.
- This new approach can be easily extended to include a simple model for vehicle acceleration.