A Nonlinear Model for Vehicle Braking

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ABSTRACT

The purpose of this study is to investigate and understand the dynamics of motor vehicles under straight ahead braking and accelerating conditions. Two vehicle braking models are considered—a quarter-car, or single wheel model, and a half-car, or two wheel model—and a nonlinear analysis is undertaken for each. A brief survey of the main characteristics and features of a single wheel acceleration model is also conducted.

The equations of motion describing the single wheel model shown in Figure 1 are presented and the need to quantify the available friction force for braking is specified. This motivates an investigation of the tire/road interface and leads to the introduction of force coefficient characteristics μ as a function of longitudinal wheel slip s. The equations of motion are hence cast into a framework which is convenient for a nonlinear dynamic analysis. The forward vehicle speed u is customarily chosen as an independent variable for such an investigation. Whereas the angular velocity of the wheel ω is typically chosen as the second independent variable for braking analysis [1], this paper utilizes wheel slip in its stead. This results in a single dimensionless function $h(s)$ of longitudinal wheel slip that completely characterizes the nonlinear dynamic behavior of the system. It will be shown that considerable insight is gained from such a description. Criteria for the local stability of finite slip in braking and wheel lockup are outlined and a qualitative vector field description of the braking model is presented and discussed. The main results of the investigation include domains of stable and unstable braking, local stability criteria, and a description of the transition to wheel lock-up as the brake torque is varied.

The single wheel model equations of motion can be cast in the form

$$\begin{aligned} \dot{u} &= -\mu(s)g \\ \dot{s} &= \frac{g}{u}h(s) \end{aligned}$$

where g is the acceleration due to gravity and $\mu(s)$ is an experimentally derived friction coefficient characteristic. It is defined by the ratio of the longitudinal braking force X to the reaction force Z of the single wheel model (See Figure 1.) Overdots denote differentiation with respect to time. The variable $s \in [0, 1]$ is *wheel slip* and is defined by $s \equiv \frac{u-\omega R}{u}$, where R is the radius of the wheel. $s = 0$ corresponds to free rolling ($u = \omega R$) while $s = 1$ implies wheel lockup ($\omega R = 0$). The dimensionless function $h(s)$ completely characterizes the dynamics of the system. It is given by

$$h(s) = \mu(s)\left(s - 1 - \frac{mR^2}{J}\right) + \Upsilon_b$$

where m is the mass of the vehicle, J is the mass moment of inertia of the wheel, and $\Upsilon_b = \frac{R}{Jg}T_b$ is the dimensionless brake torque.

Figure 2 summarizes the dynamics of the single wheel model for increasing Υ_b and distinguishes between qualitatively different braking dynamics as a function of this parameter.

Next, the single wheel model is extended to a half-car, or two wheel model, and it becomes permissible to investigate the effects of braking on an incline. With the addition of another dynamic state, *dynamic load transfer* [2] becomes important and is briefly discussed. The development of the two wheel model follows similarly to its single wheel counterpart, resulting also in a qualitative state space description of its dynamics. A total of three states—front and rear slip, and the forward vehicle speed—are used to describe its dynamics, although it is shown that only two are necessary to completely characterize its nonlinear dynamic behavior.

Finally, a simple vehicle acceleration model is briefly presented and its dynamic characteristics are compared to the single wheel braking model.

This work offers new insight into vehicle dynamics in braking and acceleration conditions and may prove to be useful for simulation of these systems and/or in the development of control systems.

References

[1] Society of Automotive Engineers (SAE): Antilock Brake System Review. SAE J2246 **25**, 1992, pp.90-102.

[2] Gillespie, T.D.: *Fundamentals of Vehicle Dynamics*. Society of Automotive Engineers (SAE), Inc., 1992.

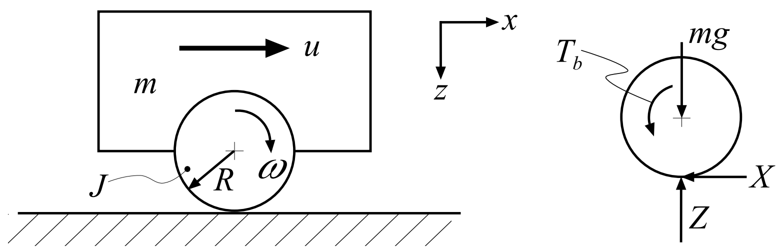

Figure 1: Schematic of the single wheel model and corresponding free body diagram. It consists of a disk constrained to move longitudinally in the x-direction at a speed u with wheel angular velocity ω , where R and J are its radius and moment of inertia respectively. The entire weight mg of the vehicle is assumed to be distributed uniformly throughout the wheel or is otherwise concentrated at its center of rotation. The effect of a braking mechanism on the vehicle wheel is captured by the single brake torque T_b, which opposes the forward motion of the system. The reaction force Z balances the static weight, while the longitudinal force X serves to slow the vehicle in braking.

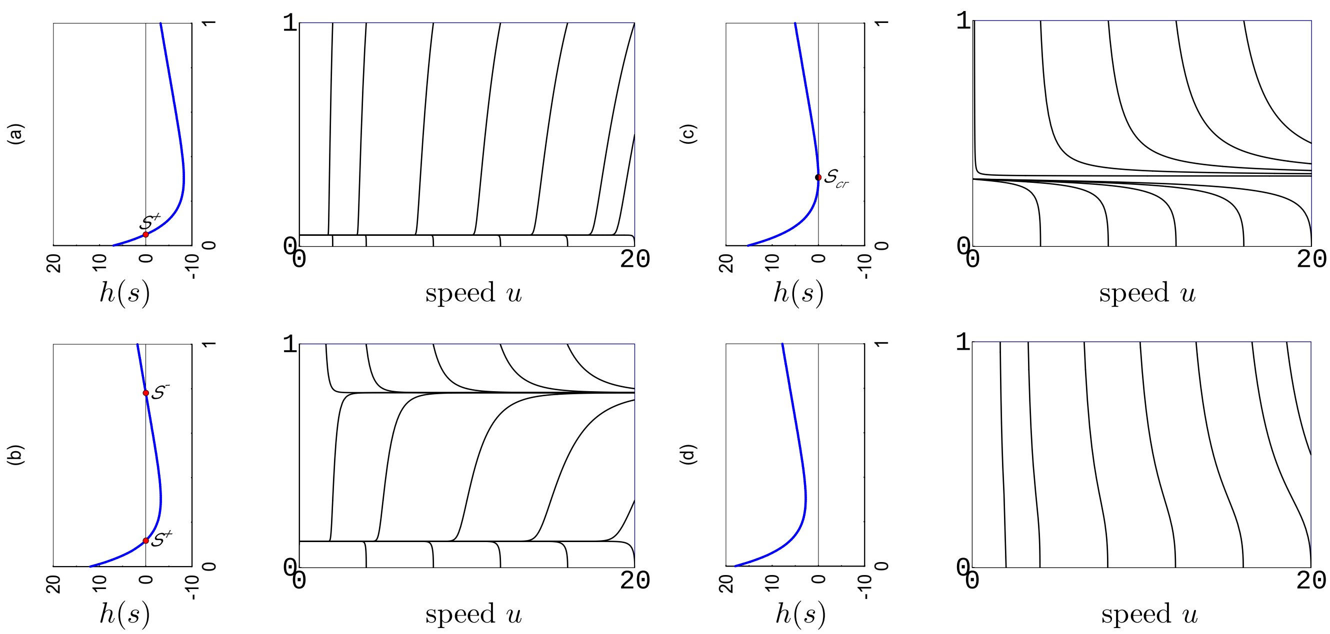

Figure 2: The function $h(s)$ versus slip and corresponding state space description in u and s for nondimensional brake torque values a) $\Upsilon_b = 7$, b) $\Upsilon_b = 12$, c) $\Upsilon_b = \Upsilon_{cr} \approx 15.248$, and d) $\Upsilon_b = 18$. The intersection of $h(s)$ with the line $h = 0$ defines the fixed points of the system in the variable s (wheel slip). Stable and unstable steady slip values are denoted by s^+ and s^- respectively. Note that wheel slip is defined along the ordinate.