# Finite-Element-Based Modal Reduction Of a Rotating Blade with Large-Amplitude Motion Using Nonlinear Normal Modes

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The vibrations of a rotating blade are strongly influenced by the axial forces that are induced by centrifugal loads [1]. To correctly account for these effects, one must develop a model that describes the nonlinear coupling between the axial and transverse blade dynamics. This coupling causes modal convergence problems, whether one models the blade as a continuous system or using finite elements [1], and the resulting system has many more degrees of freedom than should be needed for accurate simulation results. In turn, the computational costs may be unacceptably high [1]. To overcome this difficulty, the concept of nonlinear normal modes based on invariant manifolds [2] is used in the present work for systematic model size reduction.

A finite element representation of a cantilevered, Euler-Bernoulli beam attached to a rigid rotating hub is used as a simplified model of a rotorcraft blade. Although simple to analyze, this model features the same convergence problems as rotating blades with complex geometries. The equations of motion in (finite element) physical coordinates are generated from the "weak" formulation: $M$ one-dimensional beam elements are used, with standard linear shape functions to represent the axial displacements and cubic shape functions for the transverse displacements, yielding linear mass and stiffness matrices and quadratic and cubic nonlinear force vectors. Since there are three degrees of freedom at each node, and accounting for the boundary conditions, one obtains $3M$ nonlinear equations of motion in the nodal coordinates. Next, the "static" axial blade deflection is calculated and taken as a new equilibrium position about which a relative axial displacement is defined. Finally, the equations of motion about the zero equilibrium position are transformed into linear modal coordinates, using the axial and transverse normal modes of the linearized rotating beam, and the resulting set is truncated to $Q= N_t+N_a$ equations, where $N_t$ is number of kept transverse modes and $N_a$ the number of kept axial modes. These equations are uncoupled at linear order, with quadratic and cubic coupling terms.

The elements of the nonlinear coefficient matrices are generated automatically from the above finite element formulation and coordinate transformations. Nonlinear modal analysis is then applied to the system in linear modal coordinates. This consists of calculating the nonlinear normal mode of interest—typically one of the transverse (flap) blade modes—and reducing the equations of motion to a single-degree of freedom system describing the dynamics in the selected nonlinear mode. The invariant manifold that describes the selected nonlinear mode is solved for using a new nonlinear Galerkin technique which was recently developed by the authors. It allows for very accurate results to be obtained over a large amplitude range [2]. The nonlinear modal analysis process has also been automated, and it produces a single, conservative, nonlinear oscillator which systematically accounts for the coupling between the linear axial and transverse modes over a wide range of amplitudes. The essential features of the response of this reduced system can be described by the frequency/amplitude relation of the nonlinear modal oscillator.

The figures below depict the response frequency for the first nonlinear flapping mode as a function of beam tip amplitude, for various numbers of finite elements *M*. In the figures, *M* is number of elements, *Q* the number of modes retained, $N_t$ the number of transverse modes, and $N_a$ the number of axial modes ($Q= N_t+N_a$). In Fig. 1.A, the modes associated with lowest *Q* natural frequencies are retained. In Fig. 1.B, *Q* modes are also retained, but selected so that all axial modes in the first *3M/2* modes are retained, and the lowest transverse modes complete the set of *Q* modes. Figure 1.B displays much faster convergence than Fig. 1.A in the large amplitude regime. This is because when the rotating beam experiences large transverse displacements, foreshortening effects become more pronounced, thus requiring the axial deformation to be captured accurately. Figures 1.A and 1.B also depict the case labeled "Modal," obtained by direct linear modal expansions of the original coupled partial differential equations. By using a large number of modes and searching for the periodic solution of the resulting equations, this approach gives results that are very close to the exact result, which are also close to the results obtained by the converged finite element approach, *M=20*. The accuracy of the present method demonstrates the utility of modal reduction based on nonlinear normal modes.

Figure 1. Response frequency of the first flapping mode as a function of modal amplitude, for various numbers of finite elements. The number of axial modes is selected in two different ways, as described in the text. The maximum amplitude shown corresponds to blade tip peak-to-peak vibration amplitudes that are 11% of the blade length.

**References**

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2. E. Pesheck, C. Pierre and S.W. Shaw, "A new Galerkin-based approach for accurate nonlinear normal modes through invariant manifolds," preprint, 2000.