

ME/ECE859-Spring 2008 Homework 3, Due date: 2/06/08 Wed

1. **(Gradient systems)** A dynamical system $\dot{x} = F(x)$, where $F(x) = -\text{grad}V(x) = -\left(\frac{\partial V(x)}{\partial x}\right)^T$ (minus gradient of a smooth function V), is called a gradient system.

a If $x(t)$ is a trajectory of a gradient system $\dot{x} = -\text{grad}V(x)$, show that

$$\frac{d}{dt}V(x(t)) = -\|\text{grad}V(x(t))\|^2.$$

b If the Hessian

$$D^2V = \begin{pmatrix} \frac{\partial^2 V}{\partial x_1^2} & \cdots & \frac{\partial^2 V}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 V}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 V}{\partial x_n^2} \end{pmatrix}$$

of V (i.e., the matrix of second partial derivatives of V) is nonsingular at the equilibria of $\dot{x} = -\text{grad}V(x)$, show that the equilibria cannot be spirals or centers.

2. Exercise 4.3-part (2) of the textbook.
3. Exercise 4.4 of the textbook.