ME 812
Conductive Heat Transfer

Wilke Mixture Rule

The Chapman-Enskog model can be used for predicting of thermal conductivity of gas mixtures. Using Wilke's approach we have

$$k_{mix} = \sum_{i=1}^{n} \frac{x_i k_i}{\sum_{j=1}^{n} x_j \Phi_{ij}}$$

where

x_i is the mole fraction of the ith component

$$\Phi_{ij} = \frac{\left[1 + \left(\frac{\mu_i}{\mu_j}\right)^{0.5} \left(\frac{MW_j}{MW_i}\right)^{0.25}\right]^2}{\sqrt{8}[1 + (MW_i/MW_j)]^{0.5}}$$

Note that this equation reduces to

$\Phi_{ii} = 1$

The μ in the above equation is the dynamic viscosity for the gas, which can be calculated from the Chapman-Enskog model to be

$$\mu = (2.6709 \text{ x } 10^{-6}) \frac{\sqrt{MW \cdot T}}{\sigma^2 \Omega_\mu}$$

where Ω_μ is equal to Ω_k.

Example: Thermal Conductivity of Air as a Gas Mixture

We wish to determine the thermal conductivity of air at 500 K treating it as a gas mixture of N_2 (72%) and O_2 (28%).

Solution:

We will use the Chapman-Enskog model with the Wilke mixture rule. We set up the following table:

Gas	**MW**	**σ**	**ε/κ**	**Tκ/ε**	**Ω_k**	**μ**	**k**
N_2	28.013						
O_2	31.999						

Going to Table 7.1 to obtain σ and ε/κ.

Gas	**MW**	**σ**	**ε/κ**	**Tκ/ε**	**Ω_k**	**μ**	**k**
N_2	28.013	3.798	71				
O_2	31.999	3.467	107				

Calculating Tκ/ε and reading Ω_k from Table 7.2.

Gas	MW	σ	ε/κ	Tκ/ε	Ω_k	μ	k
N_2	28.013	3.798	71	7.042	0.8719		
O_2	31.999	3.467	107	4.673	0.9393		

For the dynamic viscosity and thermal conductivity we will use

$$\mu = (2.6709 \text{ x } 10^{-6})\frac{\sqrt{MW \cdot T}}{\sigma^2 \Omega_\mu}$$

$$k = k_{monatomic} + 1.32(c_P - 5/2\frac{R_u}{MW})(2.6709 \text{ x } 10^{-6})\frac{\sqrt{MW \cdot T}}{\sigma^2 \Omega_k}$$

$$k_{monatomic} = (8.3127 \times 10^{-2})\frac{\sqrt{T/MW}}{\sigma^2 \Omega_k}$$

The specific heat, c_P, will be calculated form the equations on Table A.11SI. We then obtain

Gas	MW	σ	ε/κ	Tκ/ε	Ω_k	μ	k
N_2	28.013	3.798	71	7.042	0.8719	$2.5133 \text{ x } 10^{-05}$	0.0383
O_2	31.999	3.467	107	4.673	0.9393	$2.9923 \text{ x } 10^{-05}$	0.0416

We can now apply our mixture rule,

$$k_{mix} = \sum_{i=1}^{n} \frac{x_i k_i}{\sum_{j=1}^{n} x_j \Phi_{ij}}$$

where

$$\Phi_{ij} = \frac{\left[1 + \left(\frac{\mu_i}{\mu_j}\right)^{0.5}\left(\frac{MW_j}{MW_i}\right)^{0.25}\right]^2}{\sqrt{8}[1 + (MW_i/MW_j)]^{0.5}}$$

Then

$\Phi_{N2N2} = 1$

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$\Phi_{N2O2} = 0.9791$

$\Phi_{O2N2} = 1.0205$

Finally, our mixture thermal conductivity is calculated:

$k_{mix} = 0.0392$ W/(m·K)

From Icropera & DeWitt, we find that at 500 K they give an air thermal conductivity of 0.0407 W/(m·K) or a difference of 3.6%. Hence, this is a pretty good approximation.