1. The finite element stiffness and mass matrices for a linear two-noded bar element are:

\[
[k] = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}
\]

\[
[m] = \frac{\rho AL}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}
\]

The general element equations for a free vibration (modal analysis) problem take the form:

\[
[m] \ddot{\{u\}} + [k] \{u\} = \{0\}
\]

Recall that for synchronous motion, we can assume that: \(u(x,t) = \{u'(x)\}e^{i\omega t}\).

Using the finite element mesh of bar elements shown below, estimate the first two natural frequencies of the system. Consider only axial motion.

Bars: Young’s modulus: 200 E9 Pa
Cross-sectional area: 0.0004 square meters
Density: 7.86 E3 kg/m³
2. The interpolation functions \( N_1 \), \( N_2 \) and \( N_3 \) for a quadratic 3-noded bar element as shown below were developed in Homework 3.

![Diagram of a quadratic 3-noded bar element](image)

If the three nodal displacements are known to be \( u_1 = 4 \), \( u_2 = 10 \), and \( u_3 = 12 \),
(a) what is the elemental displacement at the point \( x = 6 \)?
(b) what is the elemental normal strain at the point \( x = 6 \)?

3. Consider the mesh of linear two-noded truss elements shown in the figure below. If the finite element equations for each element are given by:

\[
\begin{bmatrix}
  k_{11}^{(e)} & k_{12}^{(e)} & k_{13}^{(e)} & k_{14}^{(e)} \\
  k_{21}^{(e)} & k_{22}^{(e)} & k_{23}^{(e)} & k_{24}^{(e)} \\
  k_{31}^{(e)} & k_{32}^{(e)} & k_{33}^{(e)} & k_{34}^{(e)} \\
  k_{41}^{(e)} & k_{42}^{(e)} & k_{43}^{(e)} & k_{44}^{(e)}
\end{bmatrix}
\begin{bmatrix}
  u_{x1}^{(e)} \\
  u_{y1}^{(e)} \\
  u_{x2}^{(e)} \\
  u_{y2}^{(e)}
\end{bmatrix}
= 
\begin{bmatrix}
  f_{x1}^{(e)} \\
  f_{y1}^{(e)} \\
  f_{x2}^{(e)} \\
  f_{y2}^{(e)}
\end{bmatrix}
\]

(a) write the Boolean array,
(b) assemble the global system of finite element equations associated with this mesh, and
(c) list the boundary conditions or show them as applied to the assembled equations.

![Diagram of a truss element mesh](image)