

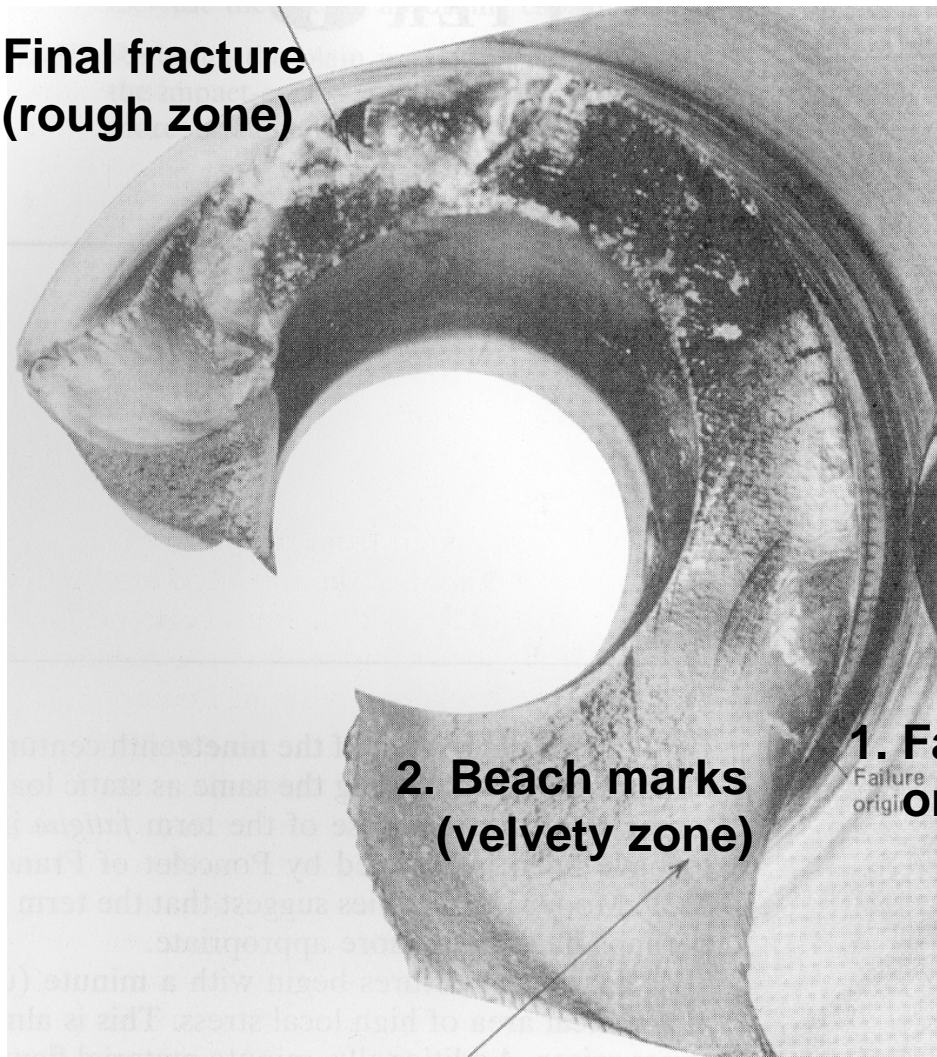
# Fatigue

Term **fatigue** introduced by Poncelet (France) 1839  
**progressive fracture** is more descriptive

1. Minute crack at critical area of high local stress  
(geometric stress raiser, flaws, preexisting cracks)
2. Crack gradually enlarges  
(creating “beach marks”)
3. Final fracture  
(suddenly, when section sufficiently weakened)

**Fatigue: no or only microscopic distortion**  
static failure: gross distortion

3. Final fracture  
(rough zone)

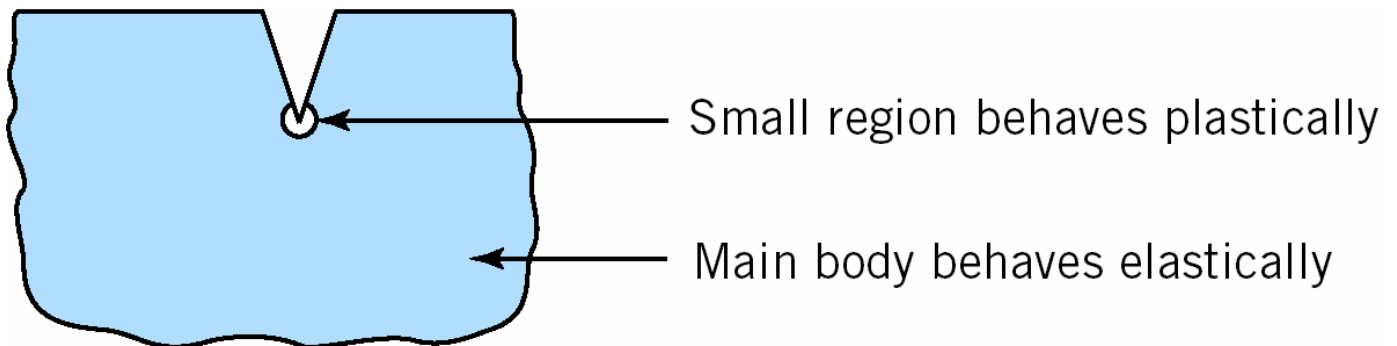


2. Beach marks  
(velvety zone)

1. Fatigue  
Failure  
origin

# Fatigue

- Repeated plastic deformation
- Thousands/millions of microscopic yielding (far below conventional yield or elastic point)
- Highly localized plastic yielding (holes, sharp corners, threads, keyways, scratches, corrosion)

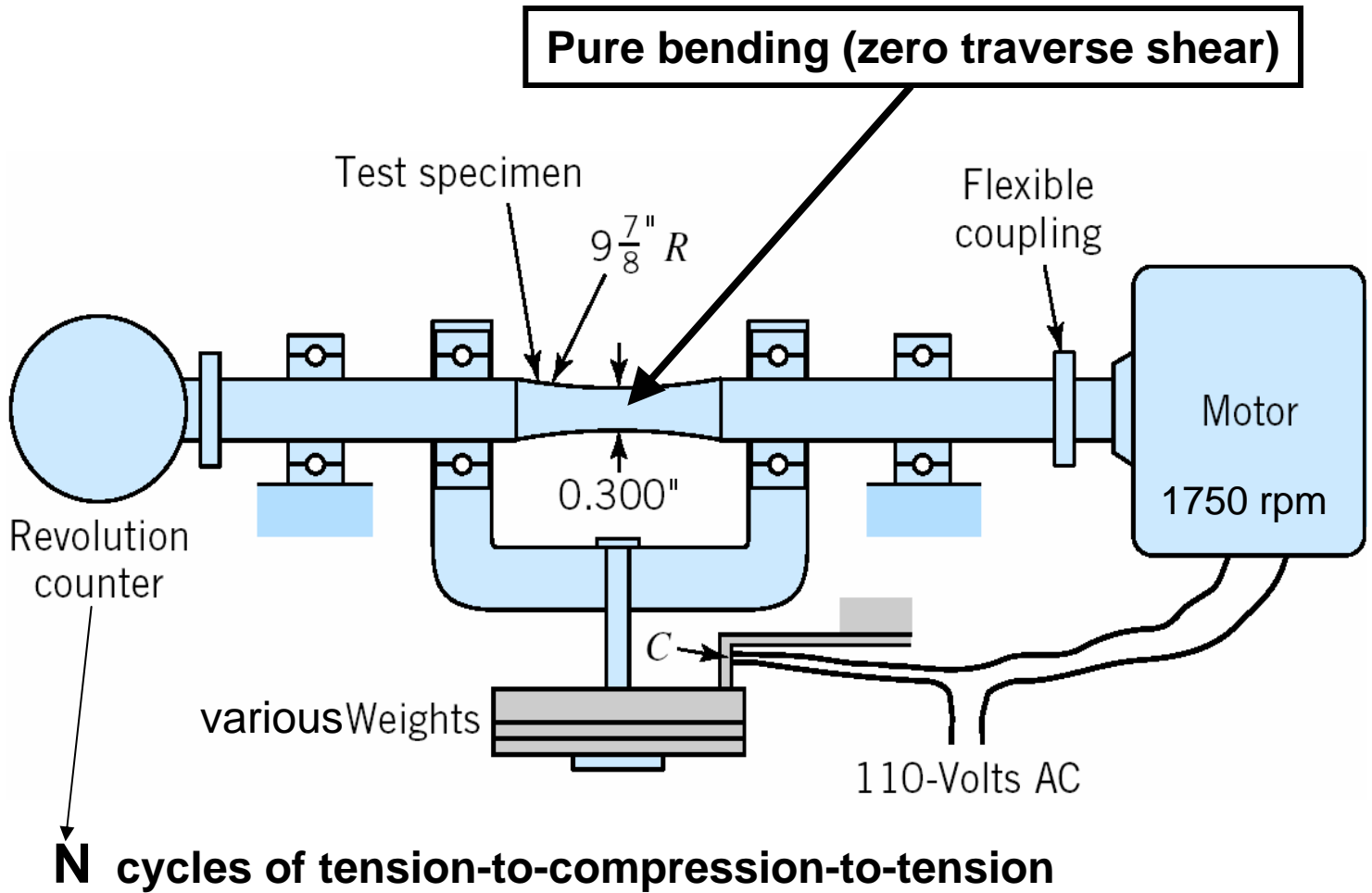


- **Strengthen vulnerable location** often as effective as choosing a stronger material
- (If local yielding is sufficiently minute strain-strengthen may stop the yielding)

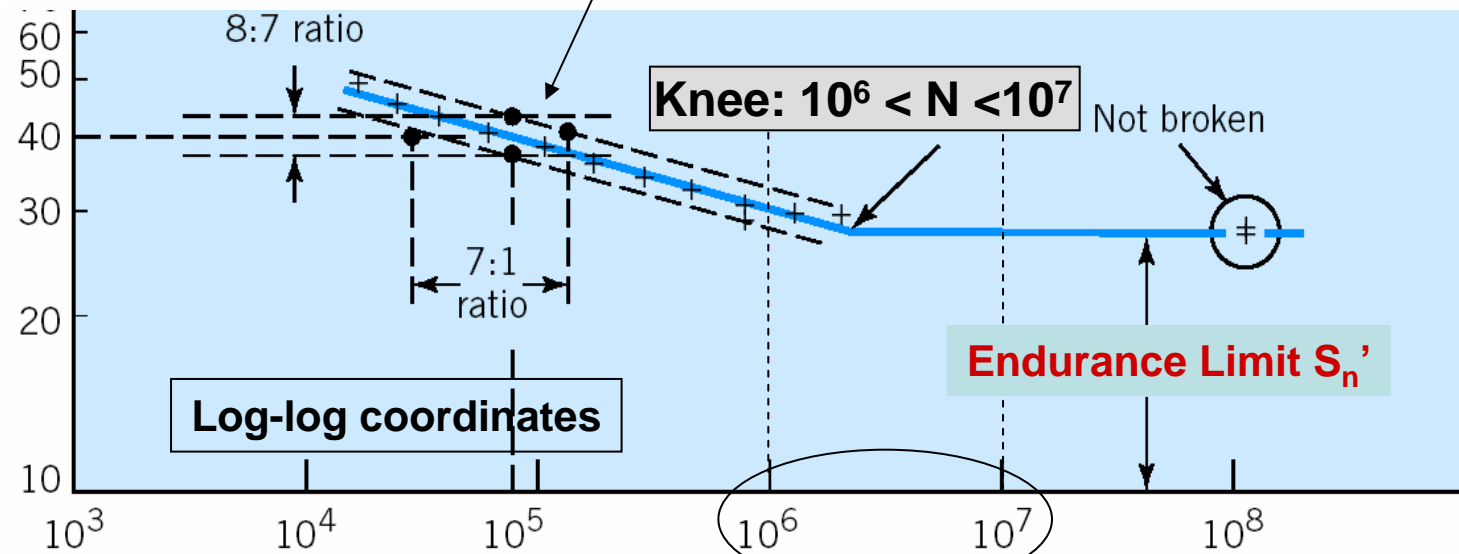
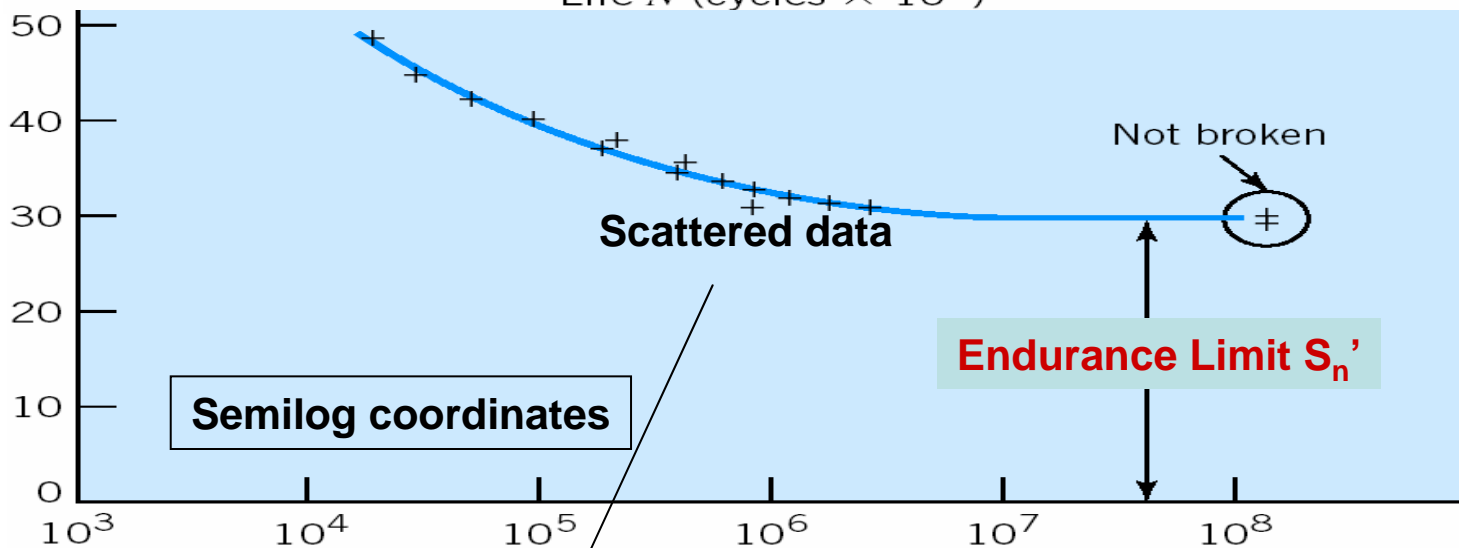
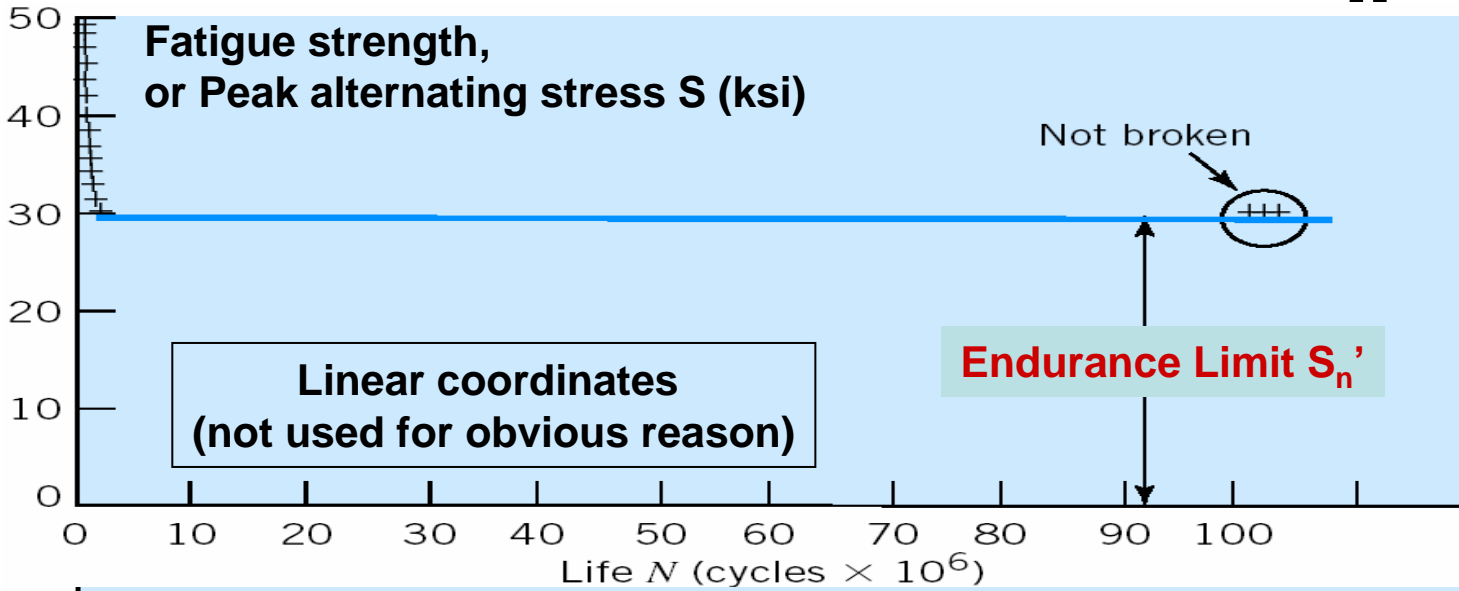
# Standard Fatigue Strength $S_n'$

Empirical data from *R.R. Moore fatigue test*  
(Highly standardized and restricted conditions)

## Rotating-beam fatigue-testing machine



# Standard Fatigue Strength $S_n'$



Ferrous materials: for life cycle  $N > 10^6 \rightarrow \sigma < S_n'$

# Fatigue Strength

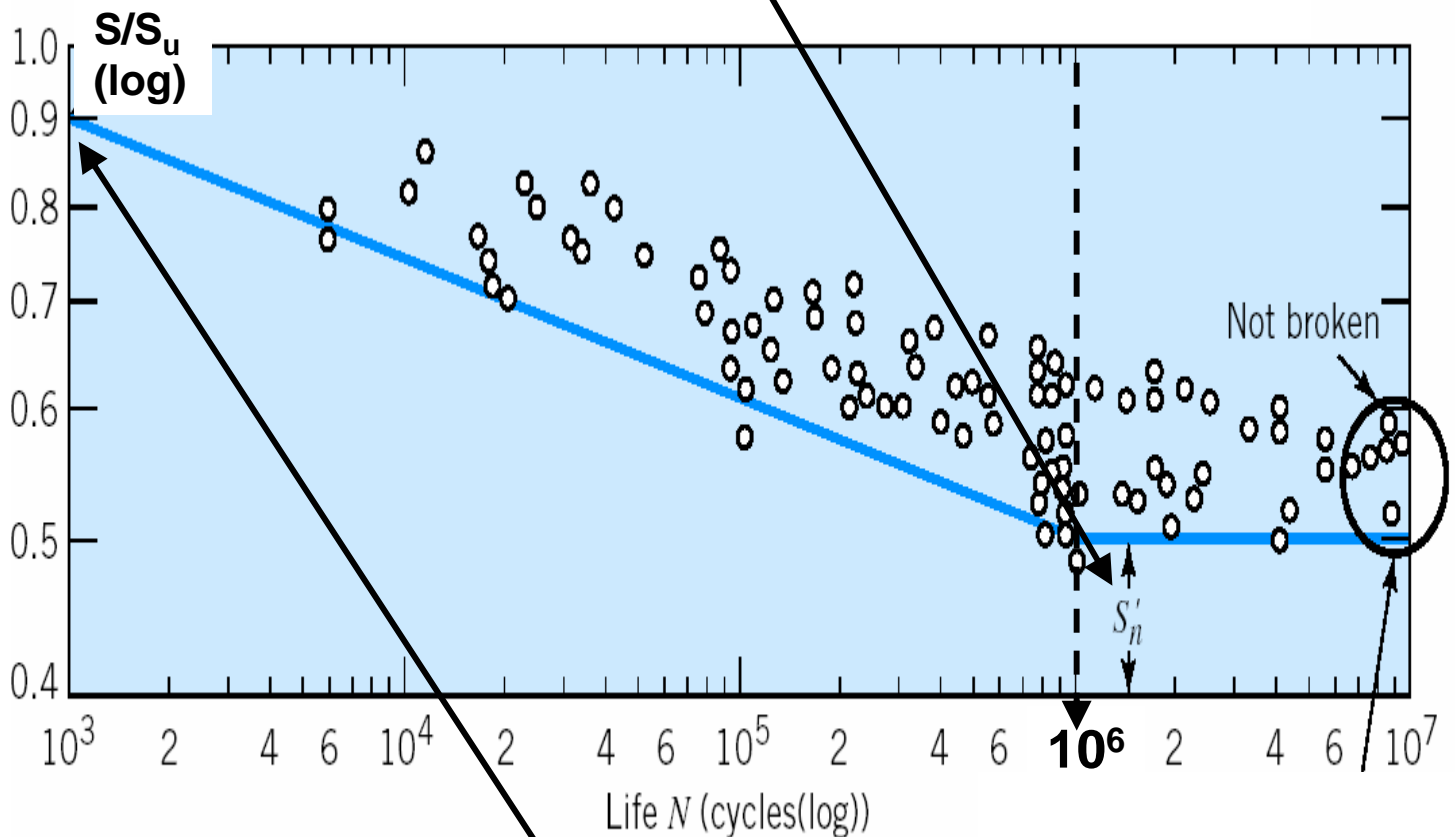
S-N curve approximation for steel

$$S_n' = 0.5 \times S_u$$

(0.4 x  $S_u$  for cast iron)

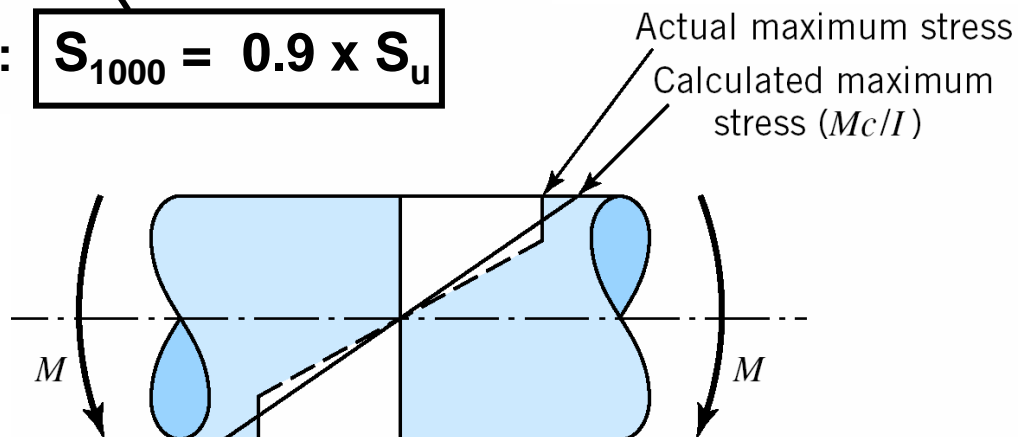
$S_u$ /ksi = 0.5 x  $H_B$ ... Brinell Hardness (also Bhn)

Hence  $S_n'$  /ksi = 0.25 x  $H_B$  for  $H_B < 400$



$10^3$ -cycle fatigue:

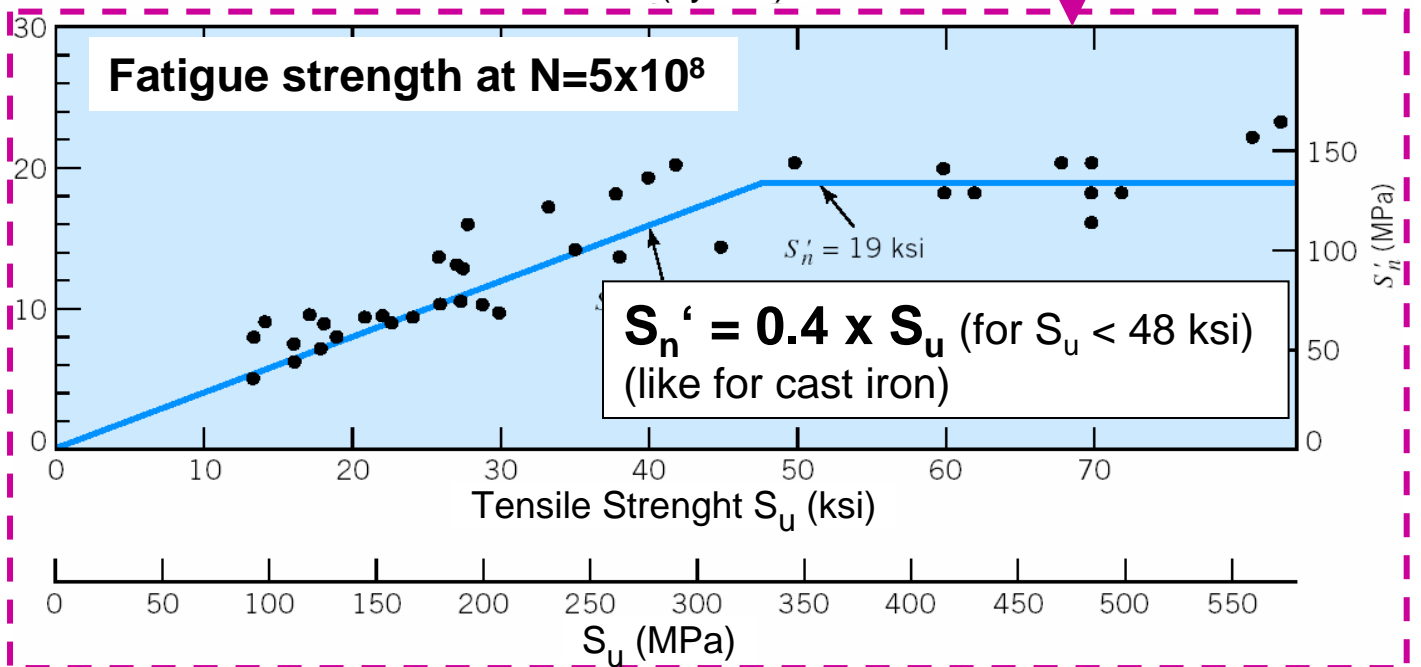
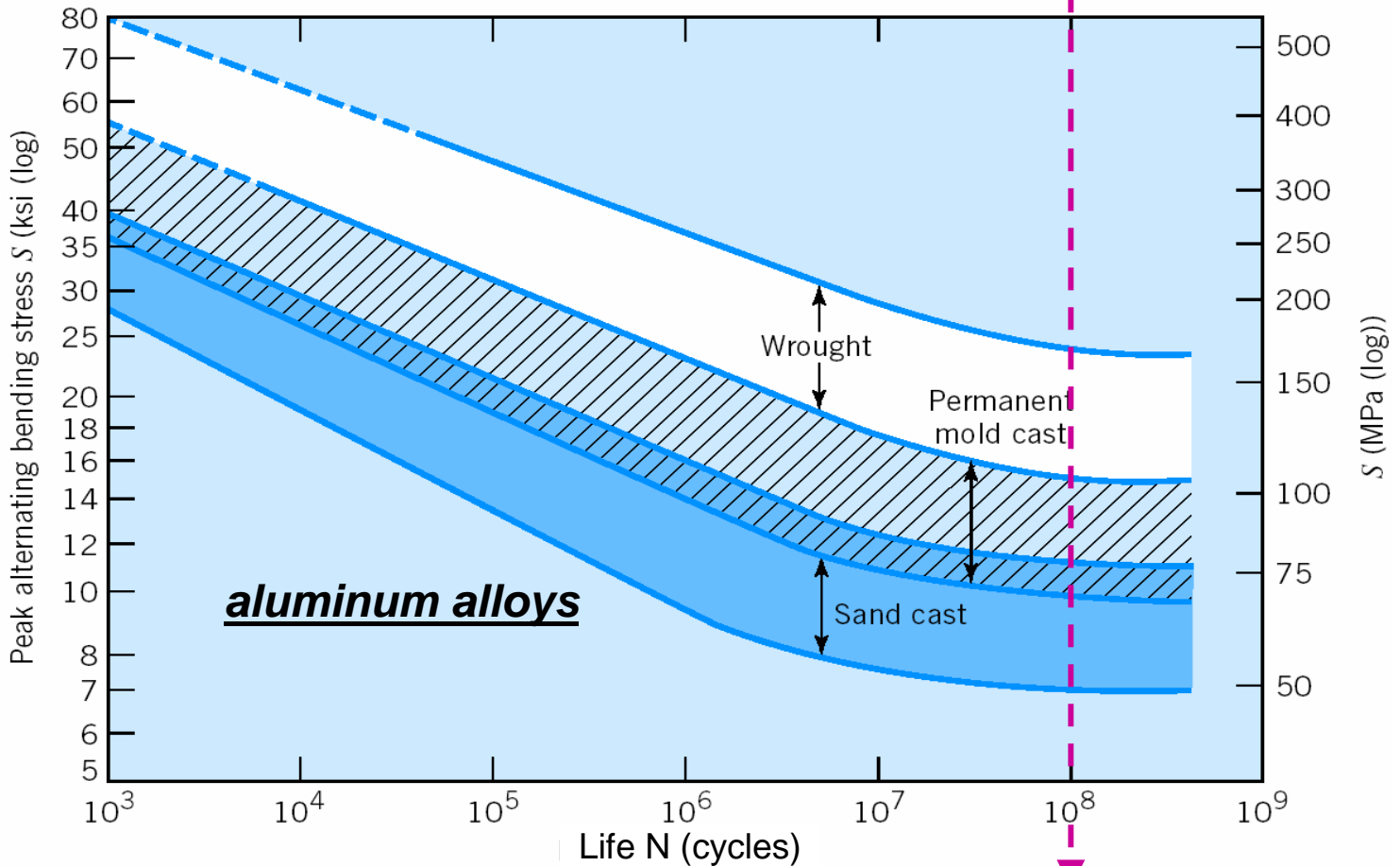
$$S_{1000} = 0.9 \times S_u$$



# Endurance Limit

## S-N curve for nonferrous metals

- No Sharply defined knee and
- No True endurance limit (Fatigue strength at  $N=5 \times 10^8$  often used)



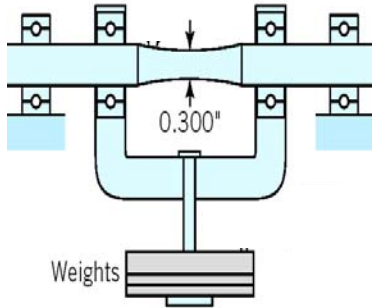
# Endurance Limit

## S-N curve approximation

<u>Endurance limit</u>		
Steel	$S_n' = 0.5$	$x S_u$ @ $N=10^6$
Titanium	$S_n' = 0.45 \dots 0.6$	$x S_u$
Cast Iron	$S_n' = 0.4$	$x S_u$
Aluminum		@ $N=10^8$
Magnesium	$S_n' = 0.35$	$x S_u$
Nickel alloys	$S_n' = 0.35 \dots 0.5$	$x S_u$
Cooper alloys	$S_n' = 0.25 \dots 0.5$	$x S_u$

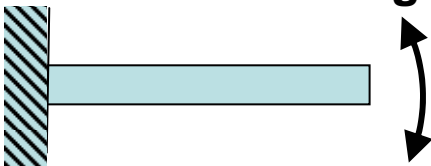
# Endurance Limit

## Rotating Bending (Moore testing)



maximum stresses **on surface**  
weakest point → fatigue start

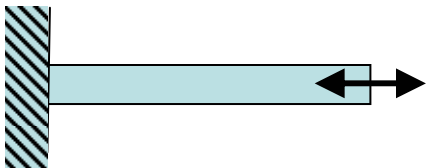
## Reversed Bending (not rotating bending like in Moore testing)



maximum stresses **only @ top and bottom**  
high probability not weakest point

**Fatigue strength usually slightly greater**  
deliberately neglected → safe side

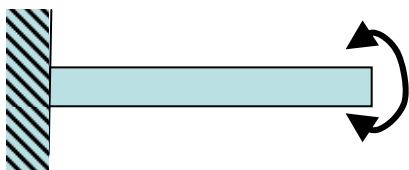
## Reversed Axial Loading



maximum stresses **entire cross section**  
no reserve !

**Fatigue strength about 10% less**  
eccentric loads about 20...30% less }  $C_G = 0.7 \dots 0.9$   
*gradient factor*

## Reversed Torsional Loading

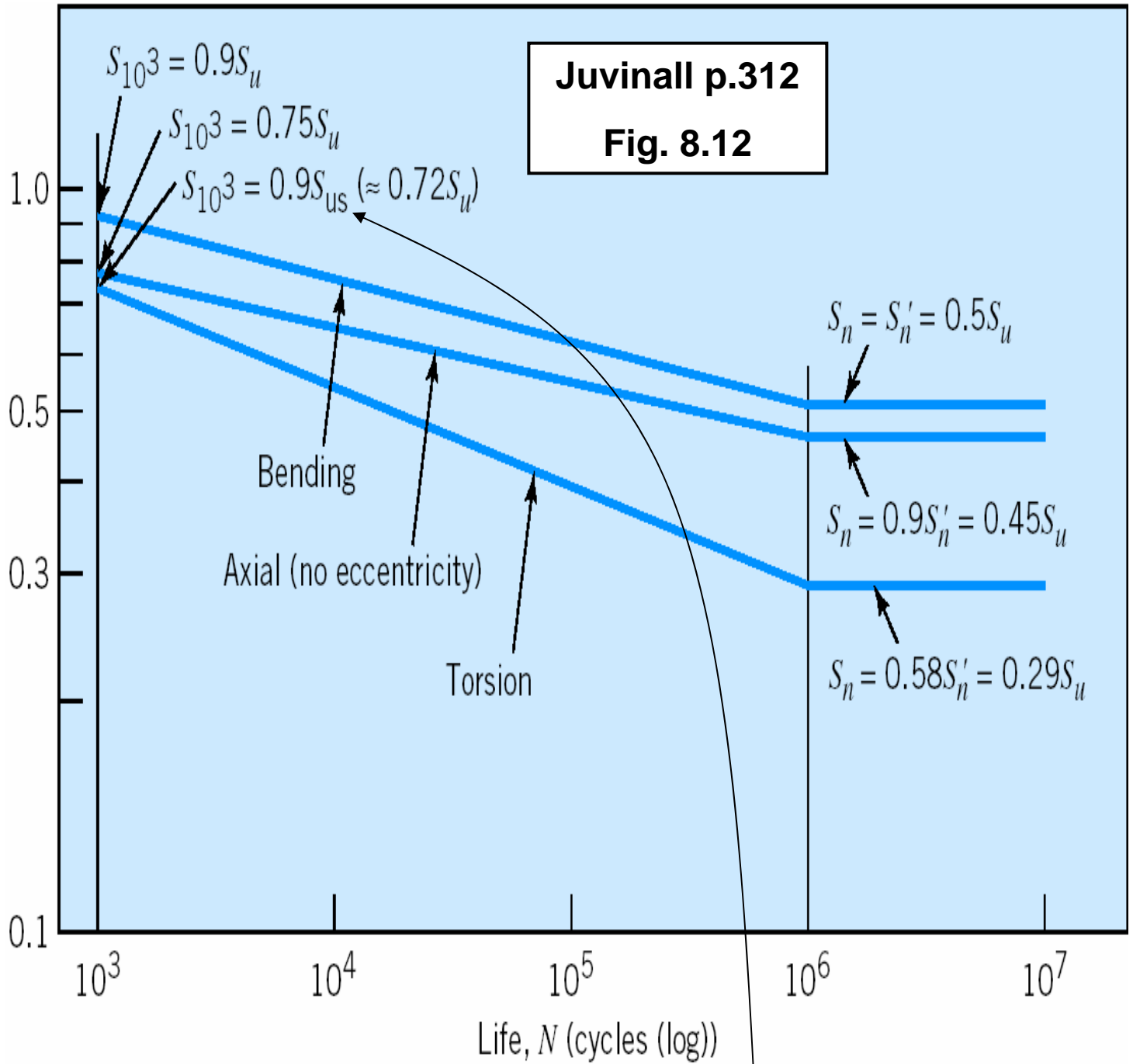


maximum stresses **on surface**  
**shear stresses** → fatigue start

reversed biaxial stress  
**distortion energy theory** → **58%** }  $C_L = 0.58$   
*load factor*



# Fatigue Strength



**Steel**  
 $S_{us} = 0.8 S_u$

**Other ductile material**  
 $S_{us} = 0.7 S_u$

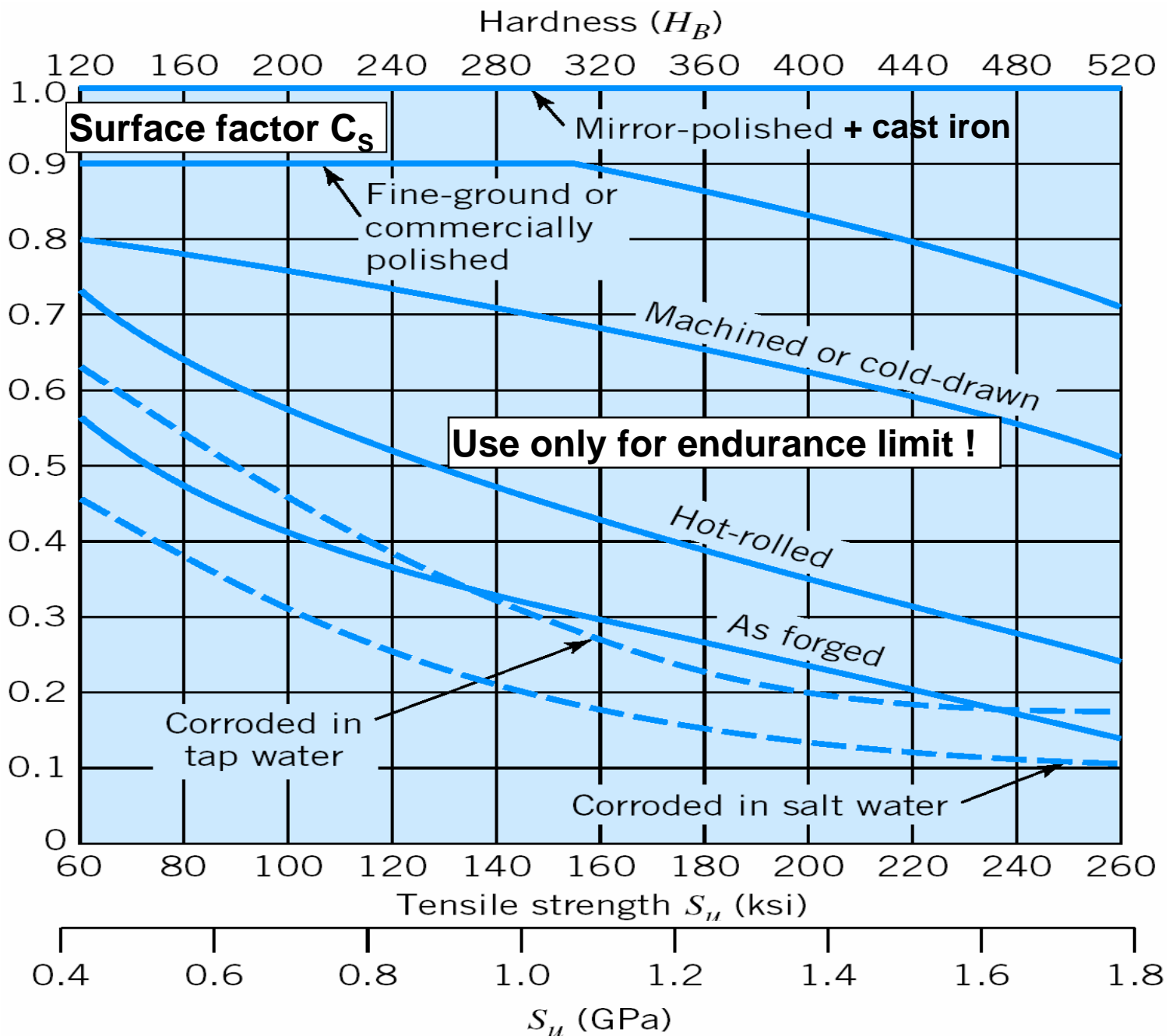
# Fatigue Strength

## Influence of Surface

So far special “mirror polish” surface (only in laboratory !)

- Minimizes
- 1.) surface scratches (stress concentration)
  - 2.) differences of surface & interior material
  - 3.) residual stresses from finishing

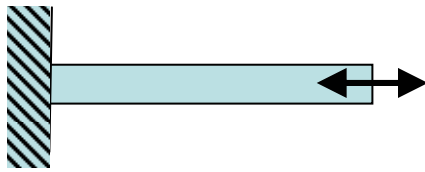
**Commercial surfaces have localized points of greater fatigue vulnerability.**



# Fatigue Strength

## Influence of Size

### Reversed Axial Loading



maximum stresses **entire cross section**  
no reserve !

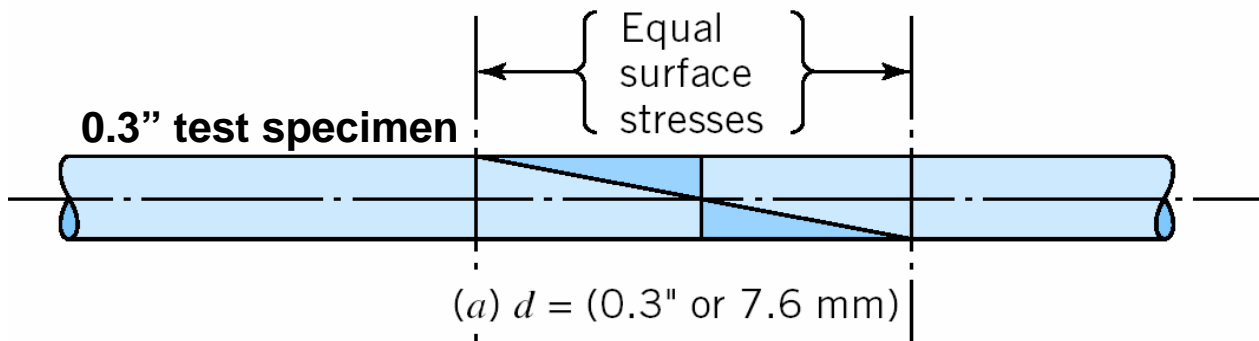
Fatigue strength about 10% less

eccentric loads about 20...30% less

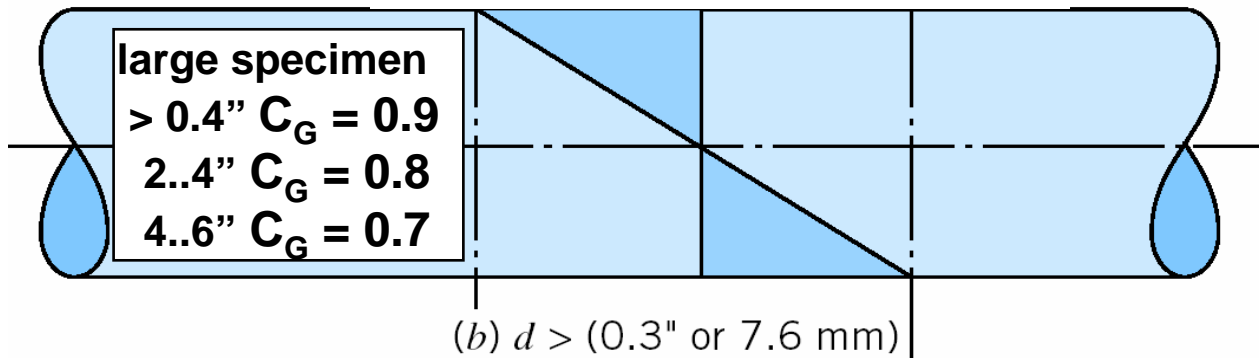
$C_G = 0.7 \dots 0.9$

gradient factor

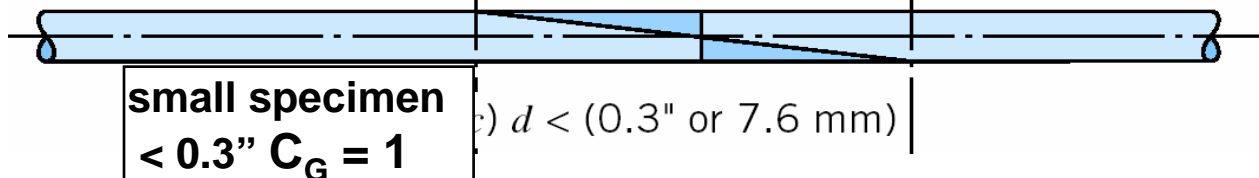
### Bending & Torsional Reversed Loading



Use equivalent round section !



large specimen  
> 0.4"  $C_G = 0.9$   
2..4"  $C_G = 0.8$   
4..6"  $C_G = 0.7$



small specimen  
< 0.3"  $C_G = 1$

# Fatigue Strength

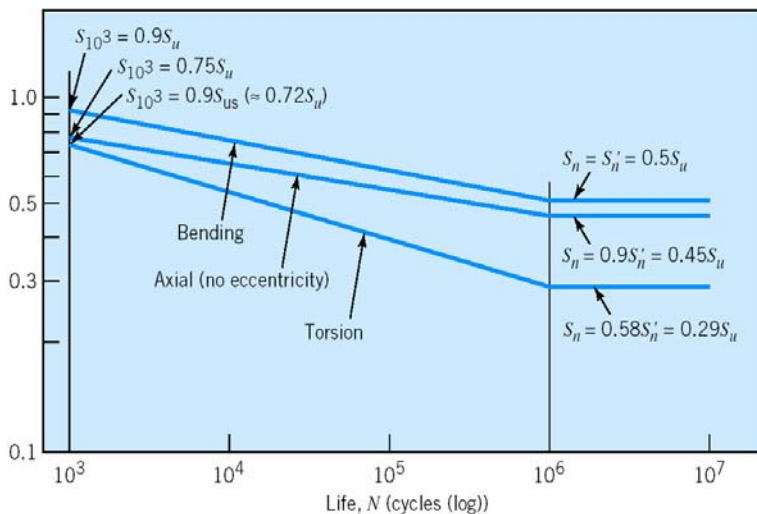
Summery use Table 8.1 Juvinall p.316

## 10<sup>6</sup>-cycle strength (endurance limit)

$S_n = \underline{S_n'} \underline{C_L} \underline{C_G} \underline{C_S}$	$S_n'$ ... Moore endurance limit		
	<b>Bending</b>	<b>Axial</b>	<b>Torsion</b>
$C_L$ (load factor)	<b>1</b>	<b>1</b>	<b>0.58</b>
$C_G$ (gradient factor) <b>&lt; 0.4''</b> 0.4''...2'' 2''...4'' 4''...6''	<b>1</b> 0.9	<b>0.7...0.9</b> 0.7...0.9 reduce - 0.1 reduce - 0.2	<b>1</b> 0.9
$C_S$ (surface factor)	<b>see Fig. 8.13</b>		

## 1000-cycle strength

	<b>Bending</b>	<b>Axial</b>	<b>Torsion</b>
$C_L$ (load factor)	<b>0.9S<sub>u</sub></b>	<b>0.75S<sub>u</sub></b>	<b>0.9S<sub>us</sub></b>



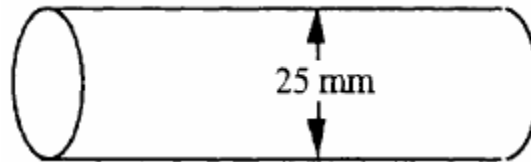
steel:  $S_{us} = 0.8S_u$   
 other ductile metals:  $S_{us} = 0.7S_u$

# Example

P 8.18

Known:  $D=25\text{mm}$ ,  $S_u = 950\text{MPa}$ ,  $S_y = 600\text{MPa}$ ,  
reversed axially loaded, steel, hot-rolled surface

Find:  $S_n(2 \times 10^5 \text{ life cycles})$



$S_u = 950 \text{ MPa}$
$S_y = 600 \text{ MPa}$

Endurance limit ( $10^6$  cycle strength)

$$S_n = S_n' C_L C_G C_s$$

For axial,

$$S_n' = 0.5 S_u = [0.5(950) = 475] \text{ MPa}$$

$$C_L = 1$$

$$C_G = 0.8 \quad (\text{between } 0.7 \text{ and } 0.9)$$

$$C_s = 0.475$$

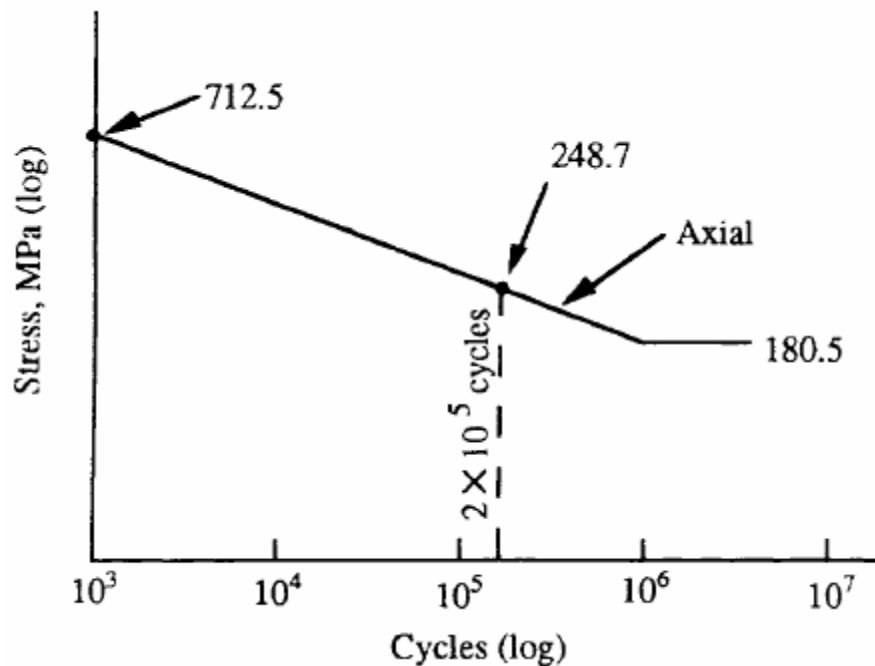
$$S_n = [(475)(1)(0.8)(0.475) = 180.5] \text{ MPa}$$

$10^3$  cycle strength

For axial,

$$0.75 S_u = [0.75(950) = 712.5] \text{ MPa}$$

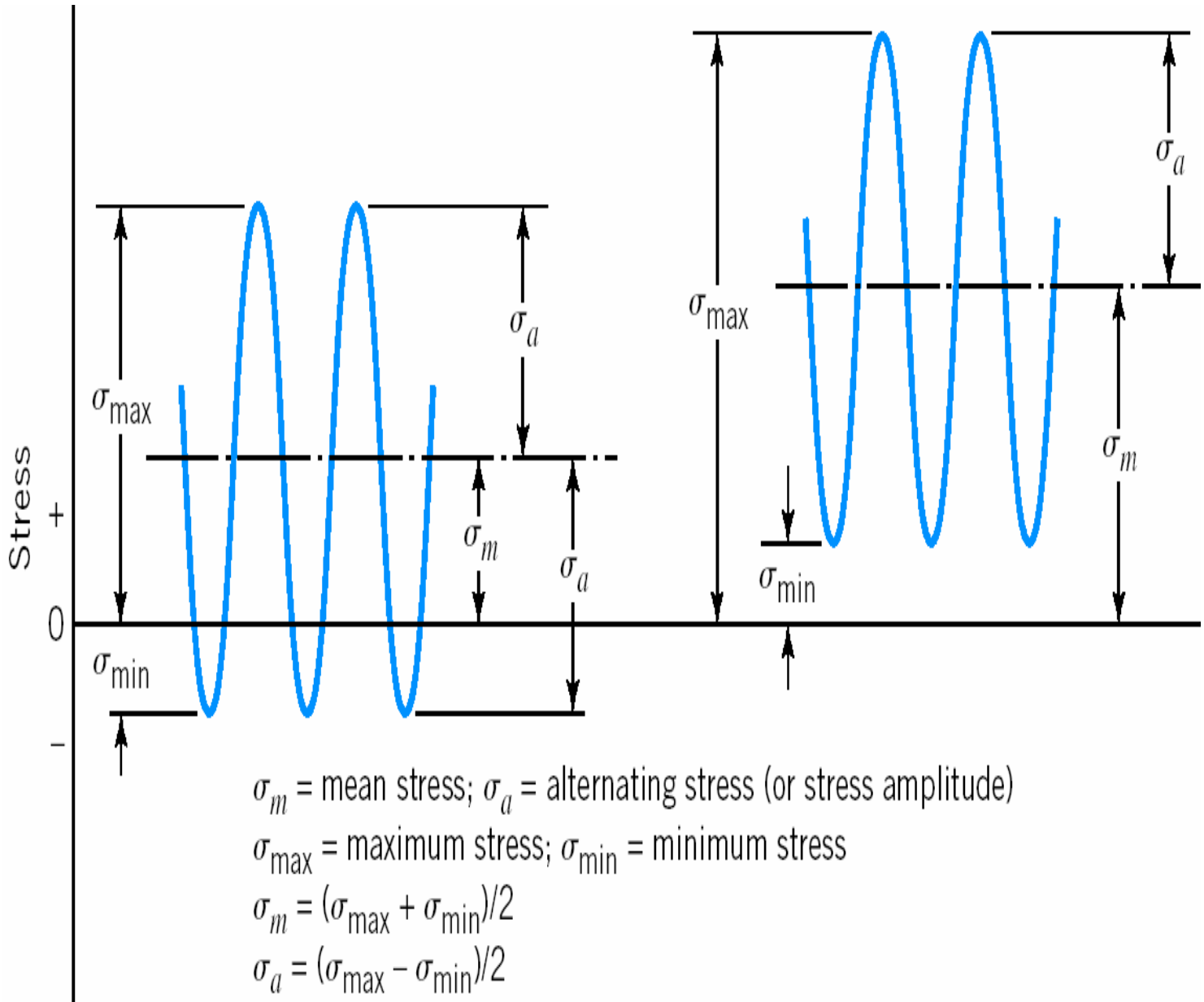
S-N curves



# Fatigue Strength

## Effect of mean stress

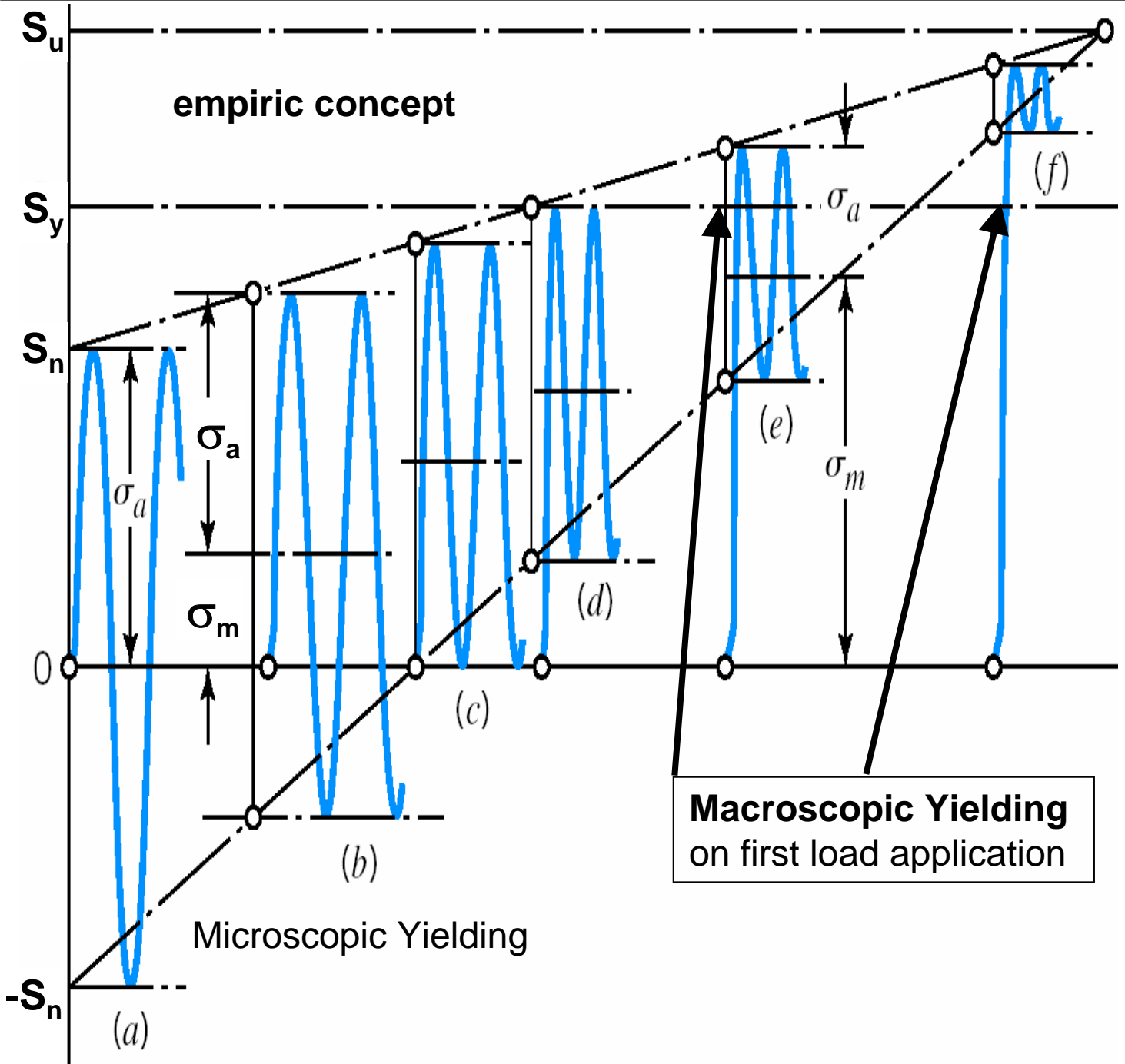
Fluctuating stress = static stress + completely reversed stress  
mean + alternating



# Fatigue Strength

## Effect of mean stress

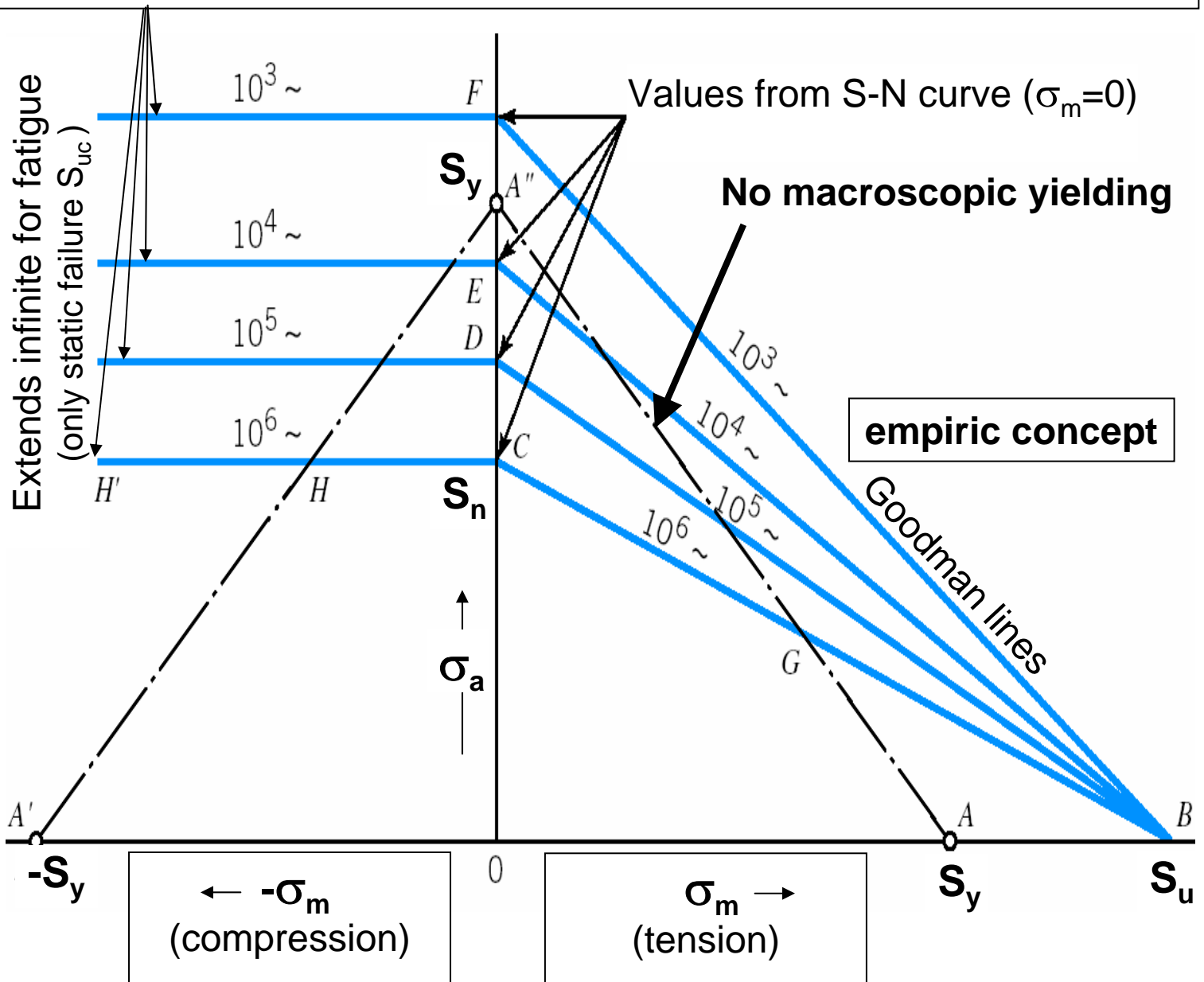
Static tensile stress *reduces amplitude* of reversed stress that can be superimposed



# Fatigue Strength

## Effect of mean stress

Compressive mean stress *does not reduce amplitude* that can be superimposed



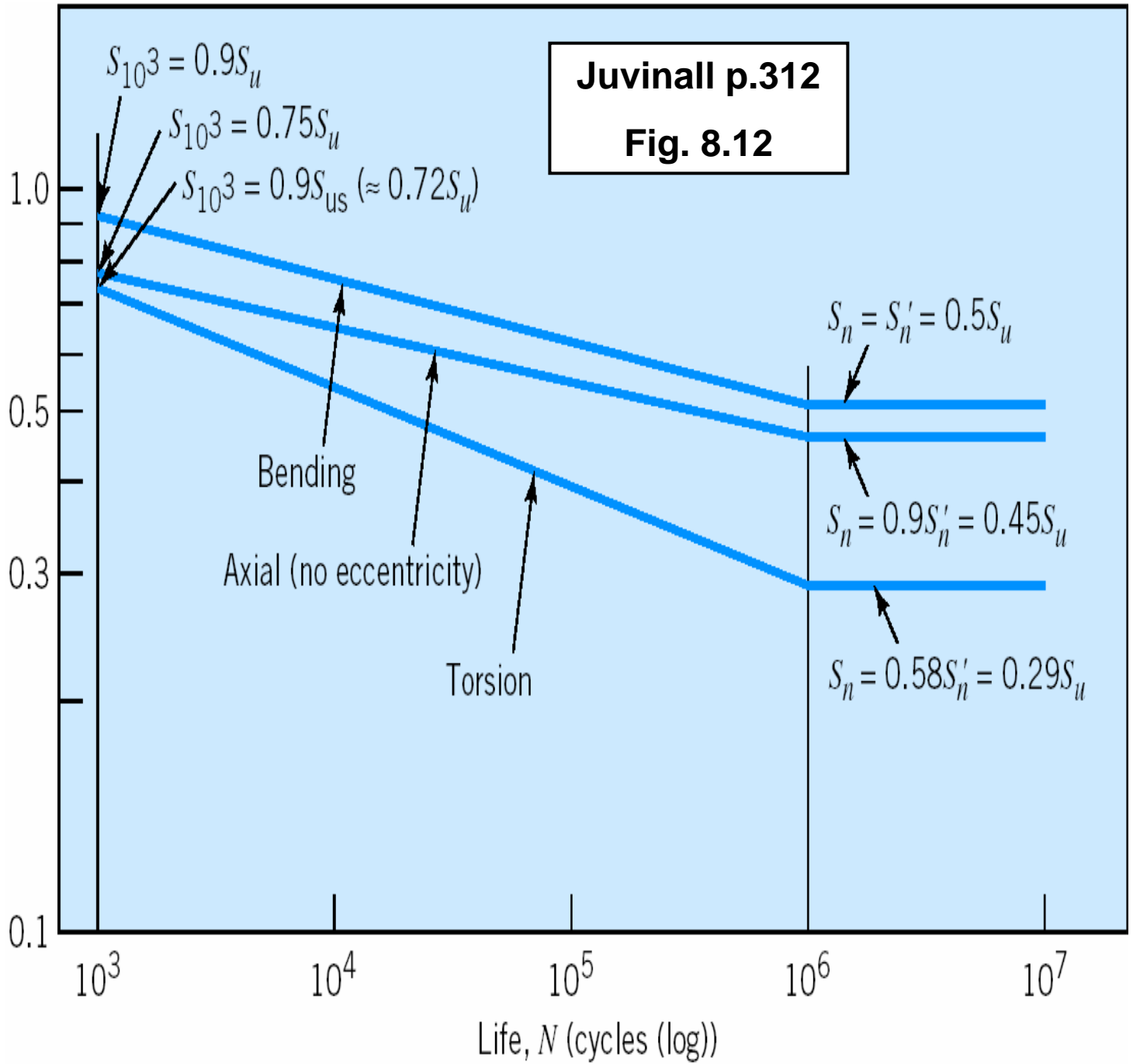
constant-life fatigue diagram

Juvinall p.318 Fig. 8.16

# Fatigue Strength

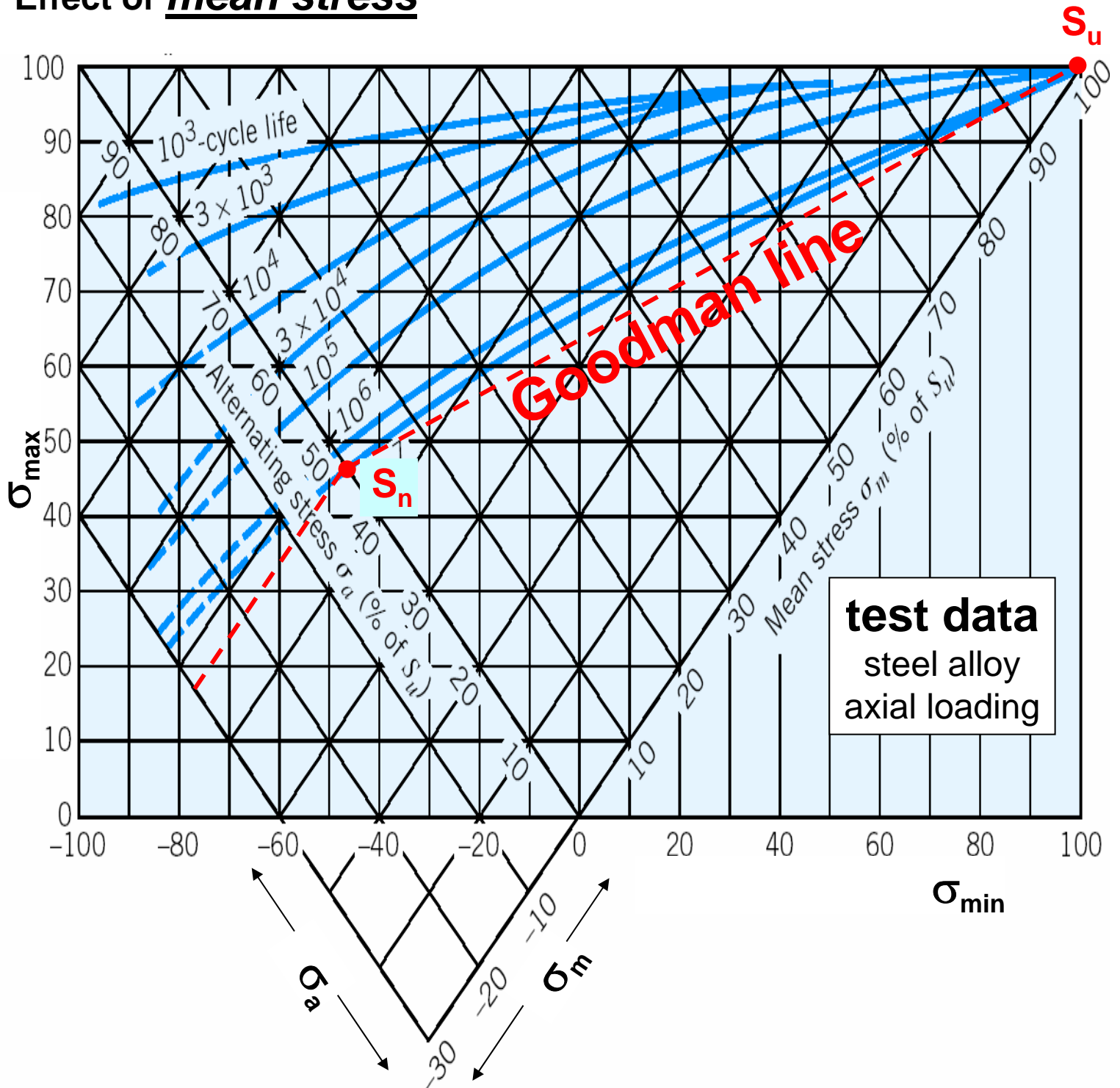
Juvinall p.312

Fig. 8.12



# Fatigue Strength

Effect of mean stress



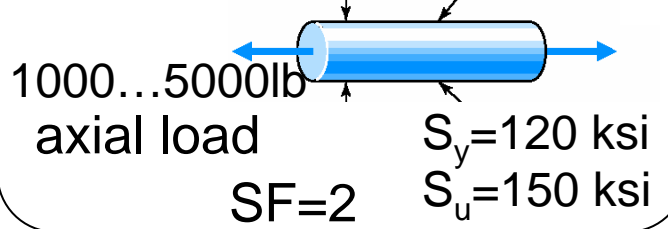
Note: Brittle materials are usually on Goodman line

# Fatigue Strength

## Effect of mean stress

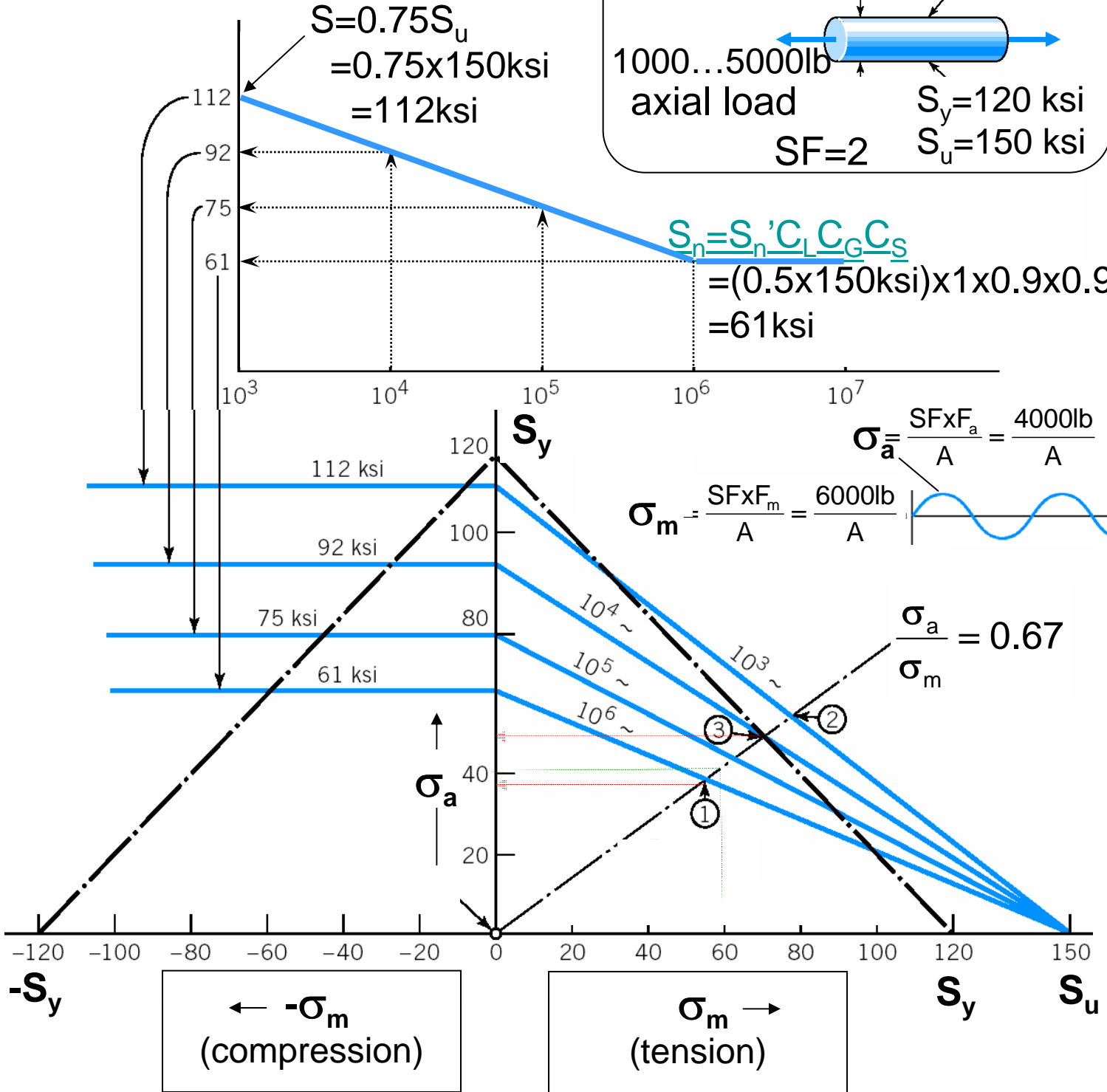
for  $N=10^6$  &  $10^3$

$d = ? < 2$  in polished



$$S = 0.75 S_u = 0.75 \times 150 \text{ ksi} = 112 \text{ ksi}$$

$$S_n = S_n' C_L C_G C_S = (0.5 \times 150 \text{ ksi}) \times 1 \times 0.9 \times 0.9 = 61 \text{ ksi}$$

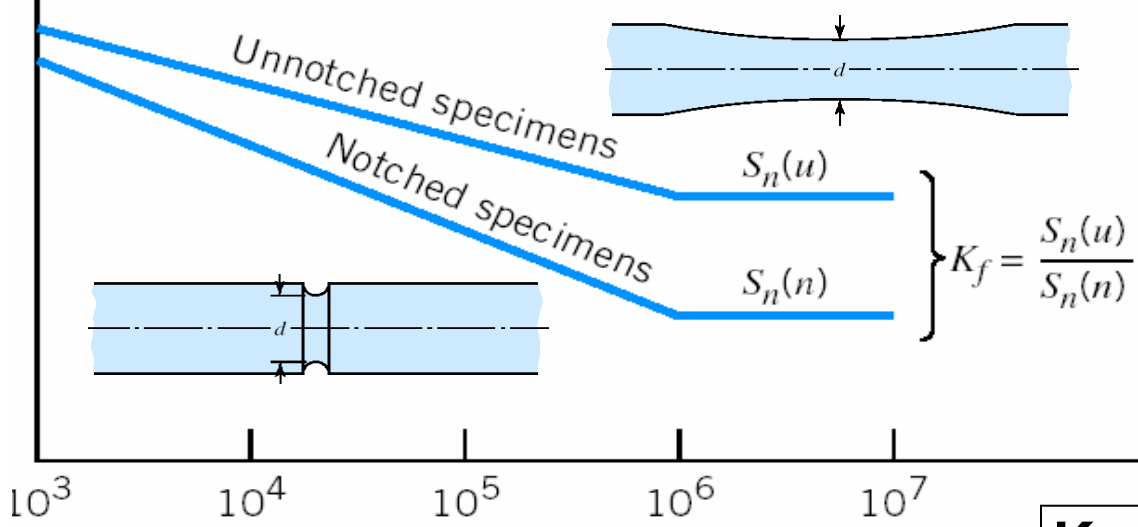


$\sigma_a(N=10^6) = 38 \text{ ksi} \rightarrow d = 0.367 \text{ in} < 3/8 \text{ in}$

$\sigma_a(S_y) = 48 \text{ ksi} \rightarrow d = 0.326 \text{ in} < 11/32 \text{ in}$

# Fatigue Strength

## Stress concentration

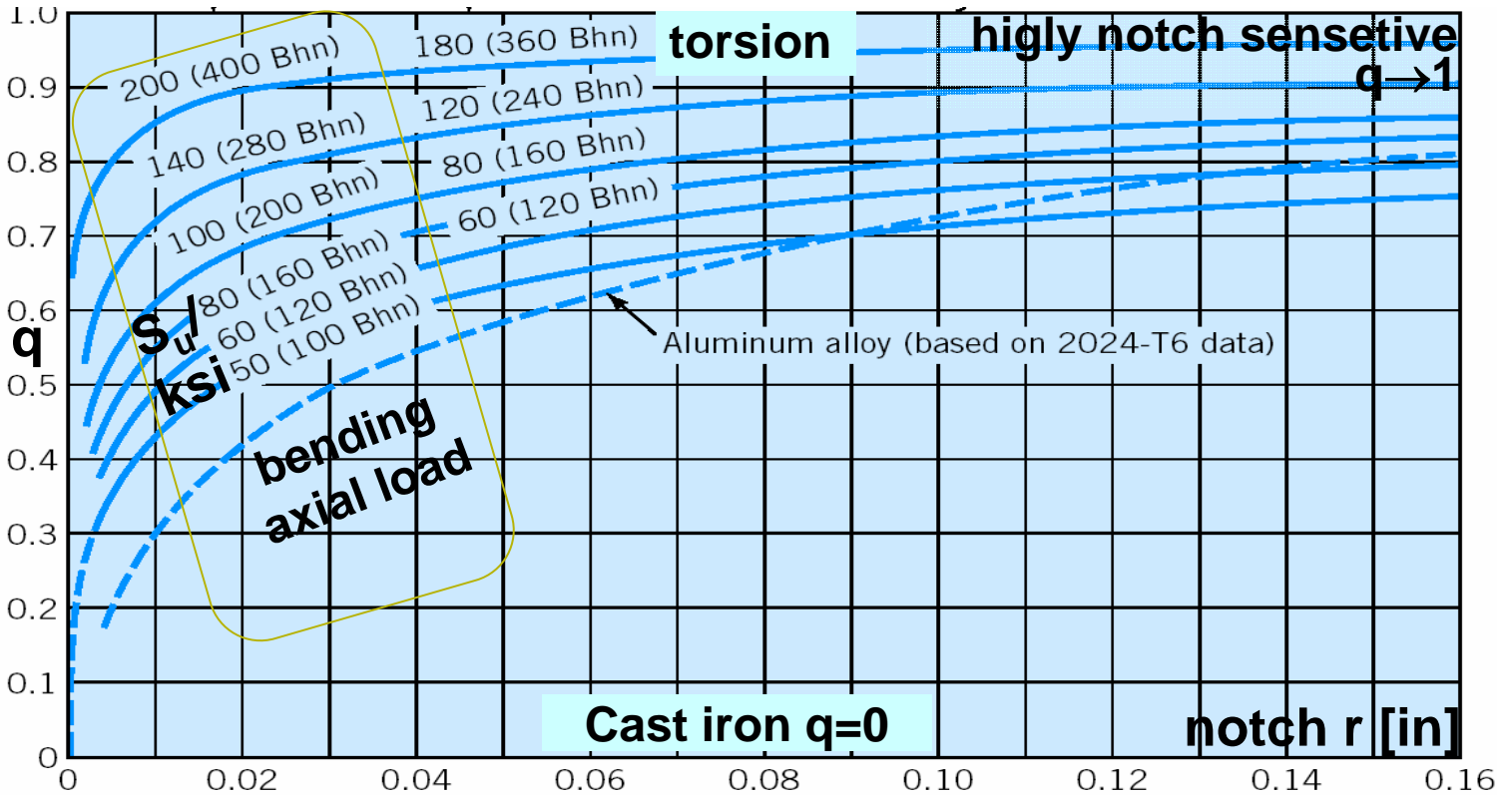


$$K_f < K_t$$

$$K_f = 1 + (K_t - 1) \cdot q$$

$q$ ...sensitivity factor

geometric or theoretic factor



Apply  $K_f$  to mean stress  $\sigma_m$  and to alternating stress  $\sigma_a$