Fatigue

Term **fatigue** introduced by Poncelet (France) 1839

**progressive fracture** is more descriptive

1. Minute crack at critical area of high local stress
   (geometric stress raiser, flaws, preexisting cracks)

2. Crack gradually enlarges
   (creating “beach marks”)

3. Final fracture
   (suddenly, when section sufficiently weakened)

Fatigue: no or only microscopic distortion

static failure: gross distortion

3. Final fracture
   (rough zone)
Fatigue

- Repeated plastic deformation
- Thousands/millions of microscopic yielding (far below conventional yield or elastic point)
- Highly localized plastic yielding (holes, sharp corners, threads, keyways, scratches, corrosion)

![Diagram showing plastic deformation and elastic behavior](Diagram.png)

- **Strengthen vulnerable location** often as effective as choosing a stronger material
- (If local yielding is sufficiently minute strain-strengthen may stop the yielding)
Standard Fatigue Strength $S_n$

Empirical data from R.R. Moore fatigue test
(Highly standardized and restricted conditions)

Rotating-beam fatigue-testing machine

Pure bending (zero traverse shear)

Test specimen

Flexible coupling

Revolution counter

various Weights

$N$ cycles of tension-to-compression-to-tension

Motor
1750 rpm

110-Volts AC
Fatigue strength, or Peak alternating stress $S$ (ksi)

- Linear coordinates (not used for obvious reason)
- Semilog coordinates
- Log-log coordinates

Ferrous materials: for life cycle $N > 10^6$ → $\sigma < S_{n'}$
Fatigue Strength

S-N curve approximation for **steel**

\[ S_n' = 0.5 \times S_u \]

(0.4 \times S_u \text{ for cast iron})

\[ S_u / \text{ksi} = 0.5 \times H_B \ldots \text{Brinell Hardness} \ (\text{also Bhn}) \]

Hence \[ S_n' / \text{ksi} = 0.25 \times H_B \] for \( H_B < 400 \)

10^3-cycle fatigue: \[ S_{1000} = 0.9 \times S_u \]
Endurance Limit

S-N curve for nonferrous metals

- No Sharply defined knee and
- No True endurance limit (Fatigue strength at $N=5 \times 10^8$ often used)

$$S_n' = 0.4 \times S_u$$ (for $S_u < 48$ ksi)
(like for cast iron)
## Endurance Limit

S-N curve approximation

<table>
<thead>
<tr>
<th>Material</th>
<th>Equation</th>
<th>@ N=10^6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>( S_n' = 0.5 \times S_u )</td>
<td></td>
</tr>
<tr>
<td>Titanium</td>
<td>( S_n' = 0.45\ldots0.6 \times S_u )</td>
<td></td>
</tr>
<tr>
<td>Cast Iron</td>
<td>( S_n' = 0.4 \times S_u )</td>
<td>@ N=10^8</td>
</tr>
<tr>
<td>Aluminum</td>
<td>( S_n' = 0.35 \times S_u )</td>
<td></td>
</tr>
<tr>
<td>Magnesium</td>
<td>( S_n' = 0.35\ldots0.5 \times S_u )</td>
<td></td>
</tr>
<tr>
<td>Nickel alloys</td>
<td>( S_n' = 0.35\ldots0.5 \times S_u )</td>
<td></td>
</tr>
<tr>
<td>Cooper alloys</td>
<td>( S_n' = 0.25\ldots0.5 \times S_u )</td>
<td></td>
</tr>
</tbody>
</table>
Endurance Limit

Rotating Bending (Moore testing)
maximum stresses on surface
weakest point → fatigue start

Reversed Bending (not rotating bending like in Moore testing)
maximum stresses only @ top and bottom
high probability not weakest point
Fatigue strength usually slightly greater
deliberately neglected → safe side

Reversed Axial Loading
maximum stresses entire cross section
no reserve!
Fatigue strength about 10% less
eccentric loads about 20…30% less
\[ C_G = 0.7…0.9 \] gradient factor

Reversed Torsional Loading
maximum stresses on surface
shear stresses → fatigue start
reversed biaxial stress
distortion energy theory → 58%
\[ C_L = 0.58 \] load factor
Endurance Limit

Note: Dotted portion is superfluous for completely reversed stresses.

- Reversed bending
- DE theory
- Reversed torsion
Fatigue Strength

Juvinall p.312
Fig. 8.12

$S_{10}^3 = 0.9S_u$
$S_{10}^3 = 0.75S_u$
$S_{10}^3 = 0.9S_{us} \approx 0.72S_u$

$S_n = S_n' = 0.5S_u$
$S_n = 0.9S_n' = 0.45S_u$
$S_n = 0.58S_n' = 0.29S_u$

Bending
Axial (no eccentricity)
Torsion

Steel
$S_{us} = 0.8S_u$

Other ductile material
$S_{us} = 0.7S_u$
**Fatigue Strength**

Influence of **Surface**

So far special “mirror polish” surface (only in laboratory!)

Minimizes
1. surface scratches (stress concentration)
2. differences of surface & interior material
3. residual stresses from finishing

**Commercial surfaces** have localized points of greater fatigue vulnerability.

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**Surface factor** $C_S$

Use only for endurance limit!

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Graph showing the influence of surface condition on fatigue strength, with hardness ($H_B$) and tensile strength ($S_{tu}$) axes.
Fatigue Strength

Influence of Size

Reversed Axial Loading

- Maximum stresses over entire cross section, no reserve!
- Fatigue strength about 10% less
- Eccentric loads about 20…30% less

\[ C_G = 0.7 \ldots 0.9 \]
\( \text{gradient factor} \)

Bending & Torsional Reversed Loading

- 0.3” test specimen
  - Equal surface stresses
  - \( (a) \ d = (0.3” \text{ or } 7.6 \text{ mm}) \)

- Large specimen
  - \( > 0.4” \ C_G = 0.9 \)
  - \( 2..4” \ C_G = 0.8 \)
  - \( 4..6” \ C_G = 0.7 \)
  - \( (b) \ d > (0.3” \text{ or } 7.6 \text{ mm}) \)

- Small specimen
  - \( < 0.3” \ C_G = 1 \)
  - \( (c) \ d < (0.3” \text{ or } 7.6 \text{ mm}) \)

Use equivalent round section!
# Fatigue Strength

## 10⁶-cycle strength (endurance limit)

\[ S_n = S_n' C_L C_G C_S \]

<table>
<thead>
<tr>
<th>( C_L ) (load factor)</th>
<th>Bending</th>
<th>Axial</th>
<th>Torsion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>0.58</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( C_G ) (gradient factor)</th>
<th>Bending</th>
<th>Axial</th>
<th>Torsion</th>
</tr>
</thead>
<tbody>
<tr>
<td>(&lt; 0.4'')</td>
<td>1</td>
<td>0.7...0.9</td>
<td>1</td>
</tr>
<tr>
<td>0.4''...2''</td>
<td>0.9</td>
<td>0.7...0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>2''...4''</td>
<td></td>
<td></td>
<td>reduce - 0.1</td>
</tr>
<tr>
<td>4''...6''</td>
<td></td>
<td></td>
<td>reduce - 0.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( C_S ) (surface factor)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>see Fig. 8.13</td>
</tr>
</tbody>
</table>

## 1000-cycle strength

<table>
<thead>
<tr>
<th>( C_L ) (load factor)</th>
<th>Bending</th>
<th>Axial</th>
<th>Torsion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.9(S_u)</td>
<td>0.75(S_u)</td>
<td>0.9(S_\text{us})</td>
</tr>
</tbody>
</table>

- steel: \( S_\text{us} = 0.8S_u \)
- other ductile metals: \( S_\text{us} = 0.7S_u \)
Example

Known: \( D = 25 \text{mm}, \ Su = 950 \text{MPa}, \ Sy = 600 \text{MPa}, \) reversed axially loaded, steel, hot-rolled surface

Find: \( S_n(2 \times 10^5 \text{ life cycles}) \)

**Endurance limit (10^6 cycle strength)**

\[
S_n = S_n' C_L C_G C_s
\]

For axial,

\[
S_n' = 0.5 S_u = \left[0.5(950) = 475\right] \text{MPa}
\]

\( C_L = 1 \)

\( C_G = 0.8 \) (between 0.7 and 0.9)

\( C_s = 0.475 \)

\[
S_n = \left[475(1)(0.8)(0.475) = 180.5\right] \text{MPa}
\]

**10^3 cycle strength**

For axial,

\[
0.75 S_u = \left[0.75(950) = 712.5\right] \text{MPa}
\]

**S-N curves**
Fatigue Strength

Effect of \textit{mean stress}

Fluctuating stress = static stress + completely reversed stress

mean + alternating

\begin{align*}
\sigma_m &= \text{mean stress; } \sigma_a = \text{alternating stress (or stress amplitude)} \\
\sigma_{\text{max}} &= \text{maximum stress; } \sigma_{\text{min}} = \text{minimum stress} \\
\sigma_m &= (\sigma_{\text{max}} + \sigma_{\text{min}})/2 \\
\sigma_a &= (\sigma_{\text{max}} - \sigma_{\text{min}})/2
\end{align*}
Fatigue Strength

Effect of *mean stress*

Static tensile stress *reduces amplitude* of reversed stress that can be superimposed

Microscopic Yielding

Macroscopic Yielding on first load application
Fatigue Strength

Effect of mean stress

Compressive mean stress does not reduce amplitude that can be superimposed

Values from S-N curve ($\sigma_m = 0$)

No macroscopic yielding

empiric concept

constant-life fatigue diagram

Juvinall p.318 Fig. 8.16
Fatigue Strength

\[ S_{10} = 0.75S_u \]
\[ S_{10} = 0.9S_{us} \approx 0.72S_u \]

- Bending
- Axial (no eccentricity)
- Torsion

\[ S_n = S_n' = 0.5S_u \]
\[ S_n = 0.9S_n' = 0.45S_u \]
\[ S_n = 0.58S_n' = 0.29S_u \]

Life, \( N \) (cycles (log))
Fatigue Strength

Effect of mean stress

Note: Brittle materials are usually on Goodman line

test data
steel alloy
axial loading
Fatigue Strength

Effect of **mean stress**

For $N=10^6 \& 10^3$

- Mean stress $d=\,? < 2\text{in}$
- Polished
- 1000...5000lb axial load
  - $S_y=120 \text{ksi}$
  - $S_u=150 \text{ksi}$

$S_n=S_n'C_L'C_G'C_S$

$(0.5 \times 150 \text{ksi}) \times 1 \times 0.9 \times 0.9 = 61 \text{ksi}$

$S=0.75S_u$

$=0.75 \times 150 \text{ksi} = 112 \text{ksi}$

$\sigma_m = \frac{SF \times F_m}{A} = \frac{4000 \text{lb}}{A}$

$\sigma_a = \frac{SF \times F_a}{A} = \frac{6000 \text{lb}}{A}$

$\sigma_a(N=10^6) = 38 \text{ksi}$ $\rightarrow d=0.367\text{in} < 3/8 \text{in}$

$\sigma_a(S_y) = 48 \text{ksi}$ $\rightarrow d=0.326\text{in} < 11/32 \text{in}$
Fatigue Strength

Stress concentration

\[ K_f = 1 + (K_t - 1) \cdot q \]

q...sensitivity factor

geometric or theoretic factor

Apply \( K_f \) to mean stress \( \sigma_m \) and to alternating stress \( \sigma_a \)