Impact

also called shock, sudden or impulsive loading

- driving a nail with a hammer, automobile collisions...

---

**To distinguish:**

Compare Time of Load Application $t$ with

Natural Period of Vibration $\tau = 2\pi \sqrt{\frac{m}{k}}$

<table>
<thead>
<tr>
<th>Static load $t &gt; 3,\tau$</th>
<th>“Gray area” $3,\tau &gt; t &gt; 1/2,\tau$</th>
<th>Impact load $t &lt; 1/2,\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrying loads</td>
<td>Design for</td>
<td>Absorbing energy</td>
</tr>
</tbody>
</table>
Impact

Material properties vary with loading speed

- Works favorably
- Promotes brittle fracture

In praxis:
- Static properties known
- Loading rate only approximated
  - x4 for automobile suspension parts
Linear & Bending Impact

Assumptions:

- **No mass** of structure
- **No deflection** within impacting mass
- **No damping**

Dynamic = static deflection curve only multiplied by impact factor

All energy goes into structure

Most severe impact

Elastic-strain energy stored in structure:

\[ W (h + \delta) = 0.5 F_e \delta \]

Impact Deflection:

\[ \delta = \delta_{st} \left( 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} \right) \]

Impact Load:

\[ F_e = W \left( 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} \right) \]

Impact Stress:

\[ \sigma_e = \sigma_{st} \left( 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} \right) \]
**Linear & Bending Impact**

Elastic-strain energy stored in structure = $0.5F_e\delta$

Work of falling weight = $W(h+\delta)$

\[ W(h+\delta) = 0.5F_e\delta \]

\[ F_e = k\delta \]

\[ k = \frac{W}{\delta_{st}} \]

\[ F_e = \frac{W\delta}{\delta_{st}} \]

\[ v^2 = 2gh \rightarrow h = \frac{v^2}{2g} \]

\[ \delta = \delta_{st} \left( 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} \right) \]

\[ F_e = W \left( 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} \right) \]

\[ \delta = \delta_{st} \left( 1 + \sqrt{1 + \frac{v^2}{g\delta_{st}}} \right) \]

\[ F_e = W \left( 1 + \sqrt{1 + \frac{v^2}{g\delta_{st}}} \right) \]

For $h >> \delta_{st}$

\[ \delta = \delta_{st} \left( \frac{2h}{\delta_{st}} \right) = \sqrt{2h\delta_{st}} \]

\[ F_e = W \sqrt{\frac{2h}{\delta_{st}}} = \sqrt{2Whk} \]

\[ \delta = \delta_{st} \left( \frac{v^2}{g\delta_{st}} \right) = \sqrt{\frac{v^2\delta_{st}}{g}} \]

\[ F_e = W \sqrt{\frac{v^2}{g\delta_{st}}} = \sqrt{\frac{v^2Wk}{g}} \]

\[ h = 0 \text{ or } v=0 \]

**Suddenly Applied Load**

**Impact Factor = 2**

(Double Safety Factor)
Linear & Bending Impact

Elastic-strain energy stored in structure
\[ = 0.5 F_e \delta \]

Work of falling weight
\[ = W(h+\delta) \]

**For** \( h >> \delta_{st} \)

Vertical movement

\[ F_e = k \delta \]
\[ k = W / \delta_{st} \]
\[ F_e = \frac{W \delta}{\delta_{st}} \]

\[ W = k \delta_{st} = g m \]

Impact kin. Energy … \( U = 0.5 m v^2 \)

Impact Factor

\[ \delta = \delta_{st} \sqrt{\frac{v^2}{g \delta_{st}}} = \sqrt{\frac{v^2 \delta_{st}}{g}} \]
\[ F_e = W \sqrt{\frac{v^2}{g \delta_{st}}} = \sqrt{\frac{v^2 W k}{g}} \]

Horizontal movement

\[ \delta = \sqrt{\frac{2U}{k}} \]
\[ F_e = \sqrt{2Uk} \]

As greater **Stiffness** and **kin. Energy**
as greater **Equivalent static force**
**Linear Impact of Straight Bar**

- Basic relationship → Gives optimistic results

- Calculates stress *lower* than actual stress because of
  - Non-uniform loading & stress distribution
  - Mass of the impacted rod

\[
F_e = \sqrt{2U_k}
\]

\[
\sigma = \frac{F_e}{A} \quad k = \frac{A E}{L}
\]

\[
\sigma = \sqrt{\frac{2UE}{AL}} = \sqrt{\frac{2UE}{V}}
\]

\[
U = \frac{\sigma^2 V}{2E}
\]

**Impact Energy Capacity**

of a straight rod
Linear Impact of Straight Bar

How compare Energy-Absorbing Capacities?

\[ U = \frac{\sigma^2 V}{2E} \]

Set \( \delta = S_y \)

\[ \begin{align*}
U_a &= \frac{S_y^2 V}{2E} \\
U_{b\text{lower}} &= \frac{S_y^2 \frac{V}{2}}{2E} = \frac{1}{2} U_a \\
U_{b\text{upper}} &= \frac{\left(\frac{S_y}{4}\right)^2 2V}{2E} = \frac{1}{8} U_a \\
U_b &= \frac{5}{8} U_a
\end{align*} \]

But \( V_b = 2\frac{1}{2} V_a \)

\[ \frac{U_a}{m_a} = \frac{8}{5} \frac{U_b}{2m_b} = 4 \frac{U_b}{m_b} \]

...mass specific energy capacity

Shows Importance of Section Uniformity
# Impact Absorption Capacity

\[ \delta = \frac{F_e L}{AE} \]

\[ U = 0.5 F_e \delta \]

\[ F_e = S_e A \]

\[ U = \frac{S_e^2 AL}{2E} = \frac{S_e^2 V}{E \cdot 2} \]

\[ \frac{U}{m} = \frac{S_e^2 V}{2EpV} \]

...mass specific energy capacity

<table>
<thead>
<tr>
<th></th>
<th>( S_e/\text{MPa} )</th>
<th>( E/\text{MPa} )</th>
<th>( S_e^2/E )</th>
<th>( \text{ratio} )</th>
<th>( \rho/(\text{kg/m}^3) )</th>
<th>( S_e^2/E\rho )</th>
<th>( \text{ratio} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>soft steel</td>
<td>207</td>
<td>207000</td>
<td>0.207</td>
<td>1</td>
<td>77</td>
<td>0.0027</td>
<td>1</td>
</tr>
<tr>
<td>hard steel</td>
<td>828</td>
<td>207000</td>
<td>3.312</td>
<td>16</td>
<td>77</td>
<td>0.0430</td>
<td>16</td>
</tr>
<tr>
<td>rubber</td>
<td>2.07</td>
<td>1.034</td>
<td>4.1440</td>
<td>20</td>
<td>9.2</td>
<td>0.4504</td>
<td>168</td>
</tr>
</tbody>
</table>

\( S_e \)…elastic limit
Bending Impact

2 x 4 white pine
\( E = 10^6 \text{ psi} \)
Mod. of rupture = 6 ksi

\( \delta_{st}(\text{beam}) = \frac{PL^3}{48EI} = \frac{100(60)^3}{48(10^6)(6.46)} = 0.070 \text{ in.} \)

\( \delta_{st}(\text{springs}) = \frac{P}{2k} = \frac{100}{2(100)} = 0.50 \text{ in.} \)

\( \delta_{st}(\text{total}) = 0.070 + 0.50 = 0.57 \text{ in.} \)

the impact factor is

\[ 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} = 1 + \sqrt{1 + \frac{24}{0.57}} = 7.6 \]

3. Hence, the total impact deflection is 0.57 \( \times \) 7.6 = 4.3 in., but the deflection of the beam itself is only 0.07 \( \times \) 7.6 = 0.53 in.

4. The extreme-fiber beam stress is estimated from \( F_e = 100 \times 7.6 = 760 \text{ lb} \):

\[ \sigma = \frac{M}{Z} = \frac{F_eL}{4Z} = \frac{760(60)}{4(3.56)} = 3200 \text{ psi} \]
Effect of Stress Raisers on Impact

\[ U_a = \frac{S^2V}{2E} \]
\[ U_b = \frac{(S/1.5)^2V}{2E} \]
\[ U_c = \frac{(S/12)^2V}{2E} \]

\[ U_b = \frac{U_a}{2.25} \]
\[ U_c = \frac{U_b}{64} = \frac{U_a}{144} \]

Ratio \[ 2.25 : 1 : 1/64 \]
\[ 225\% : 100\% : 1.56\% \]
Effect of Stress Raisers on Impact

Due to combination of Impact and Stress Raiser
Brittle fracture is promoted
with stress concentration factor almost equal $K_t$

\[ \sigma_{nom} = \frac{P}{A} = \frac{4P}{\pi d^2} \]
Effect of Stress Raisers on Impact

Reduce Stress Concentration and design for Uniform Stress Distribution

\[ \delta_a = \frac{S}{3.5} \left( \frac{600}{700} \right) = 0.245S \]

\[ \delta_b = \frac{S}{3.0} \left( \frac{600}{300} \right) = 0.667S \]

\[ V_b = \left( \frac{300}{700} \right) V_a = 0.429S \]

\[ U_a = \frac{S^2 V_a}{2E} \]

\[ U_b = \frac{\sigma_b^2 V_b}{U_a} = \frac{(0.667S)^2(0.429V)}{(0.245S)^2(V)} = 3.18 \]
Effect of Stress Raisers on Impact

"Very long" (>10d)

"Negligible"

Improved Design

Reduce Stress Concentration and design for Uniform Stress Distribution Raiser

Axial hole
Torsional Impact

<table>
<thead>
<tr>
<th>Linear</th>
<th>Torsional</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ...deflection [m]</td>
<td>θ...deflection [rad]</td>
</tr>
<tr>
<td>F_e...eq. static force [N]</td>
<td>T_e...eq. static Torque [Nm]</td>
</tr>
<tr>
<td>m...mass [kg]</td>
<td>I...moment of inertia [kgm²]</td>
</tr>
<tr>
<td>k...spring rate [N/m]</td>
<td>K...spring rate [Nm/rad]</td>
</tr>
<tr>
<td>v...velocity [m/s]</td>
<td>ω...velocity [rad/s]</td>
</tr>
<tr>
<td>U...kin. energy [Nm]</td>
<td>U...kin. energy [Nm]</td>
</tr>
</tbody>
</table>

\[
\delta = \sqrt{\frac{2U}{k}} \\
F_e = \sqrt{2Uk} \\
\sigma = \sqrt{\frac{2UE}{V}} \\
U = \frac{\sigma^2 V}{2E}
\]

\[
\theta = \sqrt{\frac{2U}{K}} \\
T_e = \sqrt{2UK} \\
\tau = 2\sqrt{\frac{UG}{V}} \\
U = \frac{\tau^2 V}{4G}
\]
Torsional Impact

\[ U = 0.5 \, l \, \omega^2 \quad \text{...kin. energy} \]

\[ l = 0.5 \, m \, r^2 \quad \text{...moment of inertia of wheel} \]

\[ m = \pi \, r^2 \, b \, \rho \quad \text{...mass of wheel} \]

\[ U = 0.25 \, \pi \, r^4 \, b \, \rho \, \omega^2 \]

\[ U = 0.25 \, \pi \, (0.060)^4 \, (0.002) \, (2000) \, (2400 \times 2\pi/60)^2 \, \text{Nm} \]

\[ U = 25.72 \, \text{Nm} \]

\[ \tau = 2 \sqrt{\frac{UG}{V}} = 2 \sqrt{\frac{(25.72)(79 \times 10^9)}{\pi(0.010)^2(0.250)}} \, \text{MPa} = 322 \, \text{MPa} \]

\[ T = \tau J/r \]

\[ \theta = \frac{TL}{JG} = \frac{\tau L}{rG} = \frac{(321.7 \times 10^6)(0.250)}{(0.010)(79 \times 10^9)} \, \text{rad} = 0.10 \, \text{rad} = 5.7^\circ \]
Exam

Monday April 18

Chapter 5: Elastic Strain
5.1-2, 5.5-7, 5.10-12
Deflection
Stability

Chapter 6: Failure Theories
6.1-2, 6.5-8, 6.10-12
Safety Factors

Chapter 7: Impact
7.1-2, 7.4

Chapter 8: Fatigue
8.1-11