Buckling does not vary linearly with load
- it occurs suddenly and is therefore dangerous

\[ P < P_{cr} = \frac{\pi^2EI}{L_e^2} \]

- \( E \) … modulus of Elasticity
- \( I \) … moment of inertia (smallest value)
- \( L_e \) … equivalent column length (length between zero bending)

Axis of least \( I \) and \( \rho \) becomes neutral bending axis

\[ L_e = L \]

Both ends hinged

\( L_e = 0.707L \)

\( L_e = 0.8L \)

\( L_e = 0.65L \)

\( L_e = 1.2L \)

\( L_e = 2L \)

\( L_e = 2.1L \)
Elastically Unstable

\[ P > P_{cr} = \frac{\pi^2 E I}{L_e^2} \]

with \( I = A \rho^2 \)

\[ S_{cr} = \frac{\pi^2 E}{(L_e/\rho)^2} \]

\( \rho \) …radius of gyration

\( L_e/\rho \) …slenderness ratio

Valid for all materials!
**J.B. Johnson Parabola (1900)**

\[ P > S_{cr} = S_y - \frac{S_y^2}{4\pi^2 E} \left( \frac{L_e}{\rho} \right)^2 \]

**Euler Buckling**

\[ S_{cr} = \frac{\pi^2 E}{\left( \frac{L_e}{\rho} \right)^2} \]

- \( \rho \) ...radius of gyration
- \( L_e/\rho \)...slenderness ratio

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![Graph](image.png)

**Euler Column Buckling**

- **Johnson Parabola**
  - \( E = 71 \text{ GPa}, \quad S_y = 496 \text{ MPa} \)
  - **Euler, \( E = 71 \text{ GPa} \)**

- **Euler, \( E = 203 \text{ GPa} \)**
  - \( S_y = 689 \text{ MPa} \)

- **Johnson, \( E = 203 \text{ GPa}, S_y = 689 \text{ MPa} \)**
  - Tangent points
Excentric Loaded Column

\[ e = \frac{L_e}{400} \text{ or } \frac{e c}{\rho^2} = 0.025 \text{ is recommended} \]
Equivalent Column Stress

\[ S_{cr} = \frac{S_y}{\alpha} \]

\( \alpha \)…stress multiplier

**J.B. Johnson Parabola**

\[ P > S_{cr} = S_y - \frac{S_y^2}{4\pi^2E} \left( \frac{L_e}{\rho} \right)^2 \]

\[ \alpha = \frac{S_y}{S_{cr}} = \frac{4\pi^2E}{4\pi^2E - S_y(L_e/\rho)^2} \]

**Euler Buckling**

\[ S_{cr} = \frac{\pi^2E}{(L_e/\rho)^2} \]

\[ \alpha = \frac{S_y}{S_{cr}} = \frac{S_y(L_e/\rho)^2}{\pi^2E} \]

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**Other Forms of Buckling**

(a) Wrinkling, or "accordion buckling" of thin-wall tube

(b) Typical local buckling of an externally pressurized thin-wall tube

(c) Wrinkling of thin, unsupported flanges of a channel section