Problem Set #5.

(5-1) Print-head mechanism.

A mechanical schematic of a print-head mechanism is shown in Figure 5-1.1, part (a). A bond graph model is shown in part (b). The effects and values to be considered are as follows:

- $m_3$, mass of the pin ... $m = 2.0$
- $k_1$, positioning spring ... $k_1 = 2.0$
- $k_2$, effects of the platen and hard stop ... see Figure 5-1.2.
- $b_5$, friction effect of pin sliding in its sleeve ... $b = 0.2$
- $F_4$, driving force, applied magnetically ... see discussion

(Assume all values are SI and in compatible units).

![Print-head mechanism schematic](a)

![Print-head mechanism bond graph](b)

Figure 5-1.1. Print-head mechanism
(a) schematic
(b) bond graph

The motion variables of interest are the position of the pin, $x$, and the velocity of the pin, $v$. Assume that $x=0$ position corresponds to an applied force of zero, and is at the center of the motion range.
The force-deflection characteristics of the platen (at x=1) and hard stop (at x=-1) are shown in Figure 5-1.2. The two effects are treated as a single spring effect, for convenience. Note the "dead-zone" around x=0 in this model. The slope is 20.0 outside the dead-zone.

![Figure 5-1.2. Characteristic of "spring 2."](image)

(a) Is the overall system model linear or non-linear? Explain.

(b) What constant force $F_4$ is required to maintain the pin at \{ $x=1$, $v=0$ \}? Call this value $F_{crit}$.

(c) Choose $F_4 = 0.50 \times F_{crit}$, constant. Analyze and predict the system response, for the case of initial conditions \{ $x=0$, $v=0$ \}. (Hint. Analyze means find eigenvalues, if you think that is an appropriate technique.) Show by a time simulation that your prediction was sound.

(d) Now choose $F_4 = 0.98 \times F_{crit}$, with initial conditions again set to \{ $x=0$, $v=0$ \}. Will your prediction from part (c) hold up? Explain and demonstrate by a time simulation.

(e) Now find $F_4$ constant such that the steady-state is \{ $x=1.5$, $v=0$ \}. Let the initial conditions be \{ $x= 1.4$, $v=0$ \}. Predict the behavior, based on analysis, or else explain why not. Support your explanation with simulation evidence.
Congratulations. Your bid to hang a sign outside the President’s house was approved. Now you have to do the job. A frame consists of two structural members that can be modeled by two linear springs. The sign acts like a single mass (point). You may work in the x-y plane due to the actual symmetry in the frame. Figure 5-2.1 shows the arrangement.

When links 1 and 2 are properly designed the sign will hang as shown. The attachments of the links are fixed at distance A apart vertically.

Figure 5-2.1. Hanging sign schematic.

Modeling assumptions.
- Assume that the sign has mass M and acts like a mass point at the connection point C.
- Assume that each link is massless and acts like a linear spring.
- Assume that the three connection points for the links permit free rotation.

Coordinates:
- x is the horizontal direction; y is the vertical direction. (0,0) is at the x-y origin.
- Denote the location of point C by \( x_C, y_C \). We wish to track the behavior of the sign by tracking this point.

The design variables available to you are the stiffnesses of the links, \( k_1 \) and \( k_2 \), and their free lengths, \( L_{f1} \) and \( L_{f2} \).

(a) Design the frame so that there is an equilibrium point (EP) at \( x_C = A \), \( y_C = 0 \). Can you achieve this using the same spring specifications for both links (much cheaper)?

(b) If the sign is unhooked where does joint C move to?

(c) Suppose \( L_{f2} \) is 10% too short compared to your result in part (a). How low does the sign hang?
(5-3) Linearization technique, example 1.

Given the equation \( \dot{x} = -x + 3u \) find \( u \) to put an equilibrium point (EP) at \( x=1 \). Then linearize the model about the EP.

(5-4) Linearization technique, example 2.

Given the equation \( \dot{x} = -x^3 + u \) find \( u \) to put an equilibrium point (EP) at \( x=1 \). Then linearize the model about the EP.

(5-5) Linearization technique, example 3.

Given the equation \( \dot{x} = -e^{-x} + 2u^2 \) find \( u \) to put an equilibrium point (EP) at \( x=1 \). Then linearize the model about the EP.

Remember your goal: **DOMINATION!**