Problem:

4.15. Consider the control system of a satellite given in Figure P4.5.
(a) Find the closed-loop transfer function.
(b) Find the closed-loop dc gain.
(c) If $K_v = 0$, find the closed-loop system gain (magnitude of the closed-loop frequency response) at resonance.
(d) Design specifications for the system are that the peak closed-loop gain cannot be greater than 1.25 and that the system time constant $\tau = 1$ s. Design the system by finding $K$ and $K_v$ such that the specifications are satisfied. Note that the damping of the system has been increased by the velocity (also called rate and derivative) feedback.
(e) Verify the results of (d) by plotting the frequency response using MATLAB.

\[\text{Figure P4.5}\]

(a) Write the system transfer function for this attitude control system.

Solution: 4.15

(a) From the diagram
\[\theta_r - \theta - \theta \frac{d}{dt} K_v = e \quad (1)\]
\[e K \frac{1}{2} = \theta \quad (2)\]
\[\theta \frac{d}{dt} = 0 \quad (3)\]
Solving (1)\(\rightarrow\) (2),
\[\frac{\theta}{\theta_r} = T(s) = \frac{K}{s^2 + KK_v + K}\]

(b) \[T(0) = \frac{K}{K} = 1\]
(c) \( T'(s) = \frac{K}{s^2 + K} \Rightarrow \zeta = 0, \ \omega_n = \sqrt{K} \)

\[ M_{pw} = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = \infty \]

(d) \( M_{pw} = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 1.25 \Rightarrow \zeta = 0.44 \)

\[ \tau = \frac{1}{\zeta\omega_n} = 1 \Rightarrow \omega_n = \frac{1}{\zeta} = 2.27 \]

For system \( T'(s) = \frac{K}{s^2 + KK_v s + K} \)

\[ 2\zeta\omega_n = KK_v \]

\[ \omega_n^2 = K \Rightarrow \zeta = \frac{KK_v}{2}, \ \omega_n = \sqrt{K} \]

Thus \( \frac{KK_v}{2} = 0.64 \) \( \Rightarrow \sqrt{K} = 2.27 \)

Solving (1) & (2)

\( K = 5.17 \quad KK_v = 0.39 \)

(e)
Problem (From Example 4.6) Consider the third-order system with the transfer function

\[ G(s) = \frac{8}{(s+2.5)(s^2+2s+4)} = \frac{1.5244}{s+2.5} + \frac{-1.524s + 0.762}{s^2+2s+4} \]

The system unit step response is plotted in Figure A, and the system frequency response is plotted in Figure B. The final value of the step response is 0.8, and the initial value of the frequency response is also 0.8. Will these two values have the same magnitude for any stable system?

Solution

The steady-state (final) value for a unit step response equals the system gain \( K \), and the initial value of the frequency response equals the transfer function at \( s = 0 \).

For a third-order system like this one, the standard form of the transfer function is

\[ G(s) = \frac{K\omega_n^3}{(s+\frac{1}{\tau})(s^2+2\xi\omega_n s+\omega_n^2)} \]

For this system, \( \tau = \frac{1}{2.5} = 0.4s \), \( \omega_n = \sqrt{4} = 2 \) rad/s, \( 2\xi\omega_n = 2 \rightarrow \xi = 0.5 \), and \( K\omega_n^3/\tau = 8 \rightarrow K = 8 \).

\[ G(s) = \frac{K\omega_n^3}{(s+\frac{1}{\tau})(s^2+2\xi\omega_n s+\omega_n^2)} = \frac{K\omega_n^3}{(s+0.4)(s^2+1s+4)} \]

Similarly, for a second-order system, \( G(s) = \frac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{K\omega_n^2}{s^2+2\omega_n^2 s+\omega_n^2} = \frac{K\omega_n^2}{\omega_n^2} = K \)

And for a first-order system, \( G(s) = \frac{K}{s+\omega_n} = K \)

From Section 4.6, third- and higher-order systems can be expressed as the sum of first- and second-order systems.

\[ \therefore \text{Yes, } G(o) = K \text{ for all systems.} \]
4.18. For the system of Figure P4.3(a), the input \( r(t) = 3 \cos 2t \) is applied at \( t = 0 \).

(a) Find the steady-state system response.
(b) Find the range of time \( t \) for which the system is in steady-state.
(c) Find the steady-state response for the input \( r(t) = 3 \cos 8t \).
(d) Why is the amplitude of the response in (a) much greater than that in (c), with the amplitudes of the input signals equal?

Solution:

\[
4.18 \quad (a) \quad G(j2) = \frac{5}{2.5 + j2} = \frac{5}{3.20 \angle 38.7^\circ} = 1.56 \angle -38.7^\circ \\
C(j2) = (3 \angle 0^\circ)(1.56 \angle -38.7^\circ) = 4.68 \angle -38.7^\circ \\
css(t) = 4.68 \cos(2t - 38.7^\circ)
\]

\[(b) \quad T = \frac{1}{2.5} = 0.4 \text{ sec} \quad \therefore \quad t > 4T = 1.6 \text{ sec}
\]

\[(c) \quad G(j8) = \frac{5}{2.5 + j8} = \frac{5}{8.38 \angle 72.6^\circ} = 0.597 \angle -72.6^\circ \\
C(j8) = (3 \angle 0^\circ)(0.597 \angle -72.6^\circ) = 1.79 \angle -72.6^\circ \\
css(t) = 1.79 \cos(8t - 72.6^\circ)
\]

(d) In frequency response, gain at 2 rad/s is larger than gain at 8 rad/s. This is a low-pass system.
Problem: For the system depicted in this figure:

A) Find the closed-loop transfer function.

B) Find the closed-loop DC gain.

C) If $K_v = 0$, find the closed-loop system gain (magnitude of the closed-loop transfer function) at resonance.

D) The design specifications require that the peak closed-loop gain can never be greater than 1.26 at any frequency. Find $K_v$ such that this requirement is met.

E) Verify the results of Part D using Matlab.

Solution:

A) $C(s) = \frac{1}{z} \hat{e}(t)$, $\dot{C}(s) = \frac{10}{s+10} \dot{e}(t)$, $\dot{e}(t) = R(t) - K_v \ddot{C}(t) - C(s)$ so $R(s) = \dot{C}(t) + K_v \dot{C}(s) + C(s)$

$$R(s) = \frac{s^2 + 10}{10} C(s) + K_v s \dot{C}(s) + C(s) = \left[ \frac{s^2 + s + 1}{10} + K_v s + 1 \right] C(s) = \left[ s^2 + (10K_v + 1)s + 10 \right] \frac{1}{10} C(s)$$

$$T(s) = \frac{C(s)}{R(s)} = \frac{10}{s^2 + (10K_v + 1)s + 10}$$

B) $T(0) = \frac{10}{10} = 1$

C) $\omega_n^2 = 10 \rightarrow \omega_n = \sqrt{10} \approx 3.16$ radians

$$2 \pi \omega_n = 10K_v + 1 \rightarrow \delta = \frac{1}{2 \sqrt{10}} \approx 0.158 = 15.8\%$$

From N-D4: Maximum magnitude of frequency response $M_{\omega} = \frac{1}{2 \sqrt{1-\delta^2}} = \frac{1}{2 \sqrt{1-0.01}} = 0.4143 M_{\omega} = 0.4143 \times 1.26 = 0.5217$

$$2 \pi \omega_n = 10K_v + 1 \rightarrow K_v = \frac{2 \pi \omega_n - 1}{10} = \frac{2 \pi \sqrt{10} - 1}{10} = 0.018 \rightarrow K_v = 0.018$$
>> num=10

num =

10

>> den=[1,10*0.180+1,10]

den =

1.0000  2.8000  10.0000

>> T=tf(num,den)

Transfer function:
    10
----------
   s^2 + 2.8s + 10

>> step(T)
>>
Step Response

~1.26