(1) Problem

A) The first-order plant modeled in Figure A has the unit step response shown in Figure B. Find the parameters of the transfer function. Only

\[ \frac{K}{\tau s + 1} \]

B) The plant is connected to the closed-loop system shown in Figure C. Sketch the unit step response of the closed-loop system.

Solution

A) \( \frac{\Delta \text{output}}{\Delta \text{input}} = 3 \) \( \rightarrow \) \( k = 3 \)

\( G(s) \approx \frac{3}{0.66s + 1} \)

B) \( G_c(s) = \frac{1.2}{0.66s + 1} \)

\( C(\tau) = 0.632 \) \( K = 0.632(3) = 1.896 \) \( \rightarrow \) \( \tau \approx 0.66 \)

\( k = 1.2, \tau = 0.2645 \)
Prob 4.2. The closed-loop first-order system of Fig P4.2 (a) has the unit step response given in Fig P4.2 (b). The steady-state value of \( c(t) \) is 0.96. Find the parameters \( K \) and \( \tau \).

**Solution:** From Fig P4.2 (a)

\[
(R-C) \frac{K}{\tau s + 1} = C
\]

Thus

\[
\frac{C}{R} = \frac{K}{\tau s + K + 1} = \frac{K}{\frac{K+1}{s+1}} = \frac{K}{\tau' s + 1}
\]

where \( K' = \frac{K}{K+1}, \quad \tau' = \frac{\tau}{K+1} \)

From P4.2 (b)

\[K' = \frac{K}{K+1} = 0.96 \quad (1)\]

Using Equation 4-9

\[K' \left(1 - e^{-\tau'/\tau'}\right) = 0.96 \left(1 - e^{-\frac{1}{K+1}}\right) = 0.6 \quad (2)\]

Solving (1) & (2)

\[K = 24 \quad \tau = 25.5 \text{ sec}\]
**Problem**

A) For the system modeled in Figure A, sketch the unit step response, giving approximate values on two axes.

\[
\frac{2}{s+2.5} \rightarrow \frac{c(s)}{r(s)}
\]
(A)

B) Run a simulation of the system and compare the response to your sketch in part A.

C) Repeat part A for the system modeled in Figure B.

\[
\frac{6}{s^2 + 8s + 8} \rightarrow \frac{c(s)}{r(s)}
\]

D) Repeat part B for the system modeled in Figure B and compare to part C.

**Solution**

A) \( G(s) = \frac{2}{s+25} = \frac{0.8}{0.4s+1} \rightarrow K=0.8, \tau=0.4 \ s \)

![Graph of c(t)](image)

B) See attached. The results are as expected.

C) \( G(s) = \frac{6}{s^2 + 8s + 8} \)

\( \omega_n = 8 \rightarrow \omega_n = 2.83 \text{ rad/s} \), \( K\omega_n^2 = 6 \rightarrow K = \frac{6}{\omega_n^2} = 0.75 \), \( 2\tau\omega_n = 1 \rightarrow \tau = \frac{1}{2\tau\omega_n} = 0.0625 = 0.25 \)

\( T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{\sqrt{8^2-0.25^2}} = 1.11 \ s \)

\( T = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{8}} \approx 2.22 \ s \)

![Graph of c(t)](image)

D) See attached. The results are as expected.
Part B

```matlab
num = [2]

num =

2

>> den=[1,2.5]

den =

1.0000  2.5000

>> G=tf(num,den)

Transfer function:

2

---

s + 2.5
```

Part D

```matlab
>> num=[6]

num =

6

>> den=[1,1,8]

den =

1   1   8

>> G=tf(num,den)

Transfer function:

6

---------

s^2 + s + 8
```

```matlab
>> step(G)
```

```matlab
>>
```
Part B
Part D
Prob 4.4. For the system shown in Fig. P4.4, sketch the unit step response of the system without mathematically solving for the time response \( c(t) \). Indicate approximate numerical values on both the amplitude axis and time axis.

\[
\begin{align*}
R(s) + & \quad \frac{10}{s^2 + 65 + 26} \quad \rightarrow \quad C(s) \\
\text{Fig P4.4}
\end{align*}
\]

**Solution:** From Fig P4.4,

\[
(R - C) \frac{10}{s^2 + 65 + 26} = C
\]

thus

\[
\frac{C}{R} = \frac{10}{s^2 + 65 + 36} = \frac{K w_n^2}{s^2 + 2\zeta w_n s + w_n^2}
\]

\[
\therefore \quad w_n^2 = 36 \quad (1) \quad 2\zeta w_n = 6 \quad (2)
\]

From (1) & (2)

\[
\begin{align*}
\omega_n &= 6 \text{ rad/s} \\
\zeta &= 0.5
\end{align*}
\]

\[
\therefore \quad \tau = \frac{1}{\zeta \omega_n} = 0.333 \text{ sec}
\]

From Equation 4-32

\[
\text{percent overshoot} = e^{-5\pi\sqrt{1-\zeta^2}} \times 100 = 16.3
\]

From Equation 4-33

\[
T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{3.14}{6.11 - 0.5^2} = 0.6 \text{ sec}
\]

\[
T_s = 4\tau = 1.3 \text{ sec} \quad \text{DC gain} = \frac{10}{36} = 0.28
\]
Problem
Consider the satellite model developed in Example 2.13 (Sec. 2.6):

\[ \tau(t) = J\dot{\theta} \quad \text{and} \quad G(s) = \frac{\theta(s)}{\tau(s)} = \frac{1}{js^2} ; \quad J=1 \]

Shown below is a model of the satellite attitude control system:

A) Write the transfer function for this control system.
B) The system is commanded to assume an attitude of 10° (θ_r(t) = 10°u(t)). When the system reaches steady-state, what will be the attitude of the satellite (θ_s, t)?
C) The closed-loop system is to respond to a step input in minimum time with no overshoot, which requires that \( s = i \). Find \( K_v \) as a function of \( K \) such that this requirement is satisfied.
D) The system in Part C must reach steady-state in approximately 65. Find \( K \) that satisfies this requirement.
E) Verify the results of Parts B-D using a Matlab simulation.
F) The rate signal is measured using a rate gyro. If this gyro fails (in which case \( K_v = 0 \)), what will be the nature of the system?
G) Verify the result of Part F using a Matlab simulation.

Solution

A) \( T(s) = \frac{K}{1 + K \frac{1}{2} \kappa_v + K \frac{1}{2}} \) → \( T(s) = \frac{K}{s^2 + KK_v s + K} \)
B) \[ G(s) = T(s) = \frac{K}{K} = 1 \quad \text{→} \quad \theta_{ss} = 10° \]
C) Since the standard form of 2nd-order transfer functions is \( G(s) = \frac{K\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} \), \( 2\omega_n = KK_v \) and \( \omega_n = K \)
So \( 2\sqrt{K} = K \cdot K_v \) → \( K_v = \frac{2}{\sqrt{K}} \)
D) Settling time is proportional to the time constant: \( T_s = K\gamma \), where \( K \) depends on how close to steady-state is considered "settled." (The book uses \( K = 1 \)) → \( \gamma = \frac{T_s}{K} = \frac{6.5}{4} = 1.5s \)
By definition, \( \gamma = \frac{1}{\sqrt{K}} \). \( \sqrt{K} = 1.5 \) → \( K = \left( \frac{2}{1.5} \right)^2 = 0.444 \) → \( K_v = \frac{2}{\sqrt{K}} = \left( \frac{2}{1.5} \right) \)
E) See attached.
F) If \( K_v = 0 \), then \( T(s) = \frac{K}{s^2 + K} \), which means \( s = 0 \)
G) See attached.
\[ E \]

```matlab
>> K = 1 / 1.5^2

K =

0.4444

>> Kv = 3

Kv =

3

>> G = tf([K], [1, K*Kv, K])

Transfer function:

0.4444
------------------
\( s^2 + 1.333 \ s + 0.4444 \)

>> step(G)

>> G = tf([K], [1, 0, K])

Transfer function:

0.4444
--------
\( s^2 + 0.4444 \)

>> step(G)
```

```matlab
>>
```
Step Response

Time (sec)

Amplitude

Part E
Part G
Consider the system with transfer function \( \frac{C(s)}{R(s)} = \frac{K_i}{s+\alpha} \).

A) Find the region of allowable s-plane pole locations such that the system settling time is less than 10 s.

B) Solve for the allowable ranges of \( K_i \) and \( \alpha \) for Part A.

**Solution**

A) The pole locations are where the denominator of the transfer function is zero. In this case, \( s = -\alpha \).

In standard form, the transfer function is \( \frac{C(s)}{R(s)} = G(s) = \frac{K_i}{s+\alpha} \) so \( K = \frac{K_i}{\alpha} \) and \( \tau = \frac{1}{\alpha} \).

The settling time is directly proportional to the time constant: \( T_s = k\tau \), where \( k \) depends on how close to steady-state is considered “settled.” (The book uses \( k = 4 \)).

And so \( 10s < 4\tau \) where \( \tau = \frac{1}{\alpha} \rightarrow \alpha < \frac{4}{10} = 0.4 \) so \( 0.4 \leq \alpha \).

B) From the above analysis, \( \alpha < 0.4 \).

\( K = \frac{K_i}{\alpha} \rightarrow \frac{K_i}{0.4} \) Gain does not affect settling time, so \( K_i \) is unbounded.
Prob 4.14. Consider the system with the transfer function

\[
\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\]

(a) Find the region of allowable s-plane pole locations such that the system settling time is less than 2s and an overshoot for a step response is less than 10 percent.

(b) Solve for the allowable ranges of \( \zeta \) and \( \omega_n \) for (a).

Solution: From Equation 4.32

\[
\tan \alpha = \frac{\pi}{\ln \text{percent overshoot}} \leq \frac{\pi}{\ln 10\%} = 1.36
\]

\[
\alpha \leq 53.7^\circ
\]

\[
\tau = \frac{T_s}{k} \leq \frac{2}{4} = 0.5 \text{ sec}
\]

\[
\therefore \text{Re}(s) \leq -\frac{1}{\tau} = -2
\]

(b) \( \zeta = \cos \alpha \geq \cos 53.7^\circ = 0.59 \)

For \( 0.59 \leq \zeta \leq 1 \)

\[
\zeta \omega_n = \frac{1}{\tau} \geq 2
\]

\[
\therefore \omega_n \geq \frac{2}{\zeta}
\]

For \( 1 < \zeta \)

\[
\text{poles} = \omega_n \left[ -\zeta \pm \sqrt{\zeta^2 - 1} \right] \leq -2
\]

\[
\therefore \omega_n \left[ \zeta - \sqrt{\zeta^2 - 1} \right] \geq 2
\]

\[
\therefore \omega_n \geq \frac{2}{\zeta - \sqrt{\zeta^2 - 1}}
\]