ME 451 : Control Systems Laboratory

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ME451 Laboratory
Experiment #6

Air Temperature Control
1. Objective

The ability to accurately control a process is vital to numerous design efforts. For example, the automatic pilot in an airplane would not prove very useful, and potentially quite dangerous, if large deviations from the desired path could not be avoided. The Continuous Process Control laboratory will provide "hands on" experience with proportional control techniques. A performance evaluation of these methods of control will be conducted.

The process to be controlled in this laboratory session is the temperature of a flowing fluid (air). The PT 326 Process Trainer will be employed in this investigation. The PT 326 is built by Feedback Instruments Ltd. and is discussed in section 2 of this document.

2. Definitions

Figure 1 represents the basic components of a closed-loop process control system.

The following list provides the definition of terms and information adapted from the PT 326 Process Trainer Manual:

**Process**
The term process is used to describe a physical or chemical change or the conversion of energy. The temperature of air flowing in a tube is the process this laboratory is concerned with.
Detecting Element
The detecting element is a bead thermistor connected to one leg of a bridge. The resistance of the thermistor changes as the temperature of its surroundings change.

Measuring Element
This change in resistance (due to change in temperature) causes the bridge output voltage to change. Thus, the bridge output voltage may be used to measure temperature.

Measured value, \( q_0 \)
This is the voltage signal from the measuring element, which corresponds, to the value of the controlled condition.

Set Value, \( q_i \)
This is the desired value of the controlled condition. The set value may be adjusted using a turn pot on the front panel or externally by providing a voltage between 0 and -10 volts to socket D. A decrease in voltage at socket D will cause a rise in process temperature.

Deviation, \( D \)
The deviation, \( D \), is the difference of the measured value and the desired value.
\[ D = q_0 - q_i \]

Set value Disturbance
A step change in the desired value may be introduced when the INTERNAL SET VALUE DISTURBANCE is applied.

Comparing Element
The measured value from the bridge and the set value are compared with a summing amplifier. The internal signals of this equipment have been arranged to be of opposite sign. Therefore, the output from the summing amplifier represents a deviation. Socket B on the front panel may be use to monitor this deviation.

Controlling Element
A signal proportional to deviation is applied to the controlling element. A correcting signal is generated and sent to the correcting element. The PT326 is capable of continuous or two step control.

Continuous Control Modes
1. **Internal** This provides proportional control only. Proportional control may be varied on the PT326 from 5 to 200 percent. Where the percent proportional control reflects the gain applied to the deviation signal.

   \[ \text{Gain } K_p = \frac{100}{\%\text{Proportional}} \]

   Therefore, a 100% proportional setting provides a gain of one to the deviation signal.
2. **External** The internal proportional band adjustment can be switched off to allow external control.

**Motor Element**
The motor element produces an output that is adjusted in response to the signal from the controlling element. In the PT326, the motor element supplies power (between 15 and 80 watts) to the correcting element.

**Correcting Element**
The correcting element directly affects the controlled condition. The correcting element in the PT326 is a wire grid, which heats the flowing air.

3. **Equipment**

The **PT 326** is a self-contained process control trainer. It incorporates a plant and control equipment in a single unit. In this equipment, a centrifugal blower draws air from the atmosphere and forces it through a heater grid. The air temperature is detected downstream of the grid by a bead thermistor before being returned to the atmosphere. The detecting (bead thermistor) and correcting (heater) elements have been placed sufficiently far apart to facilitate the investigation of “lag” time. The air stream velocity may be adjusted by means of an inlet throttle attached to the blower. The desired temperature may be set in a range from 30ºC to 60ºC. A toggle switch provides an internal step increase to the desired temperature signal. The PT 326 may be configured to run with either open- or closed-loop control. The process trainer also allows the connection of an external controller (the PID150Y).

![Figure 2- PT 326](image)

Air Temperature Control
4. Theory

Figure 3 Model of the heater-flow system.

A mathematical model of the temperature response for the system must be developed. A diagram of the system to be modeled is presented in Figure 3. An energy balance yields

\[ \text{Heat stored} = \text{Heat in} - \text{Heat out} \] (1)

Replacing the terms of equation (1) by their thermodynamic equivalents yields

\[
\frac{d}{dt}\left[ \rho V c_p T \right] = \rho V c_p \frac{dT}{dt} = P - \rho c_p Q T
\] (2)

where \( c_p = \) specific heat of air [energy/(mass×temperature)] (Joule per Kilogram Kelvin)

\( Q = \) flow rate [volume/time] (cubic meter/sec,..)

\( \rho = \) density [mass/volume] (kilograms per cubic meter, gms/cm\(^3\)...)

\( P = \) power [energy/time] (Joules per second, Watts,..)

\( V = \) volume from heater to thermistor [volume] (cubic meter, litres,..)

\( T = \) temperature above ambient [temperature] (Kelvin, Centigrade, Fahrenheit...)

Combining terms yields the first-order differential equation for the exhaust temperature:

\[
\frac{dT}{dt} = \dot{T} = -\frac{Q}{V} T + \left[ \frac{1}{\rho V c_p} \right] P
\] (3)

Solving (3) yields:

\[
T(t) = (T_0 - T_f) e^{-\frac{t}{\tau}} + T_f
\] (4)

Taking the Laplace of equation (3) gives:

\[
sT(s) = \left( -\frac{Q}{V} \right) T(s) + \left[ \frac{1}{\rho V c_p} \right] P
\]
Factoring then gives:

\[
\left( s + \frac{Q}{V} \right) T(s) = \begin{bmatrix} 1 \\ \frac{1}{\rho V c_p} \end{bmatrix} P = \frac{1}{\frac{\rho V c_p}{s + \frac{Q}{V}}}
\]

Which in standard form yields:

\[
\frac{T}{P} = \frac{1}{\rho Q c_p} \left( \frac{1}{s + \frac{Q}{V}} \right)
\]

(5)

\[
\tau = \text{the time constant of the plant}
\]

\[
C(s) = \text{Laplace transform of the response temperature}
\]

\[
E(s) = \text{the deviation of the output temperature from the desired temperature}
\]

\[
G_p = \text{the steady-state gain of the heater system}
\]

\[
K_p = \text{proportional gain}
\]

\[
R(s) = \text{Laplace transform of the desired output temperature}
\]

One case of closed-loop control will be investigated: proportional control (P). For the analysis of the steady-state accuracy of the control, an equation to describe the system steady-state error, \( E(s) \), is first developed. The following set of equations may be written from the block diagram of Figure 4.

\[
E(s) = R(s) - C(s)
\]

(6)

\[
C(s) = E(s)G(s)
\]

(7)

Equations (5) and (6) are combined to yield the error, \( E(s) \).
The final value theorem is now used to determine the steady-state accuracy.

\[ e_{s.s.} = \lim_{s \to 0} [sE(s)] \]  

(9)

The subscript s.s. has been used to denote a steady-state condition. If the input is taken as a unit step function, \( R(s) = 1/s \), Equation (8) may be written

\[ e_{s.s.} = \frac{1}{1 + \lim_{s \to 0} G(s)} \]  

(10)

where,

\[ G(s) = \left[ K_p \right] \left[ \frac{G_p}{s \tau + 1} \right] \]  

(11)

With proportional only control, \( K_i \) and \( K_d \) are zero and Equation (10) becomes

\[ e_{s.s.} = \frac{1}{1 + K_p G_p} \]  

(12)

For the proportional control only condition, the closed-loop transfer function, \( T(s) \), may be written

\[ T(s) = \left[ \frac{K_p G_p}{1 + K_p G_p} \right] \left[ \frac{\tau}{s + 1} \right] \]  

(13)

The denominator of Equation (12) is first-order. Hence, oscillations are not anticipated for proportional control.
Pre-Lab Sample Questions

Use the block diagram below to answer the following questions:

1) Using the block diagram, find the closed-loop DC gain of the above system.
   \[ \text{Answer: DC Gain} = 0.6 \]

2) Using the block diagram, find the closed-loop time constant of the above system.
   \[ \text{Answer: } \tau = 0.2 \text{ s} \]

3) Using the block diagram, determine the steady-state error of the above system.
   \[ \text{Answer: } e_{ss} = 0.4 \]
4. Experimental Procedures

Note: $K_p = 100 / \% Proportional$

Therefore, a 100% proportional setting provides a gain of one to the deviation signal.

Part A: Determination of the gain and time constant of the open-loop system without Proportional Controller

Procedure:

a) Make the following connections on the PT326:
   - Turn Proportional control off
   - Turn Continuous control on
   - No jumper between X and Y (open-loop)
   - Put a jumper between socket A and B (To complete the circuit)

b) Set the blower intake to 40°.

c) Set Temperature value to 30° Celsius by adjusting the set value knob.

d) Use a coaxial cable, banana plug, and pigtailed to connect channel A of the Oscilloscope to the Output socket ‘Y’ (voltage reading of thermistor) and to the ground of the PT326. (Note: the pigtail has the ground plug listed on it) Also connect the socket “trigger CRO” and ground to channel B on the oscilloscope. This will measure the magnitude of the step input.

d) Turn the PT326 on.

e) Give step input using the ‘internal’ switch and observe the response on the oscilloscope.

f) From the oscilloscope, measure the steady-state time to therefore obtain the time constant, and also measure the steady-state gain (output/input).

g) A second method of determining the steady-state gain is the ratio of the change in “Measured Temp” to the change in “Set Temp” once the step input is applied. These values can be read directly from the PT 326. Calculate the gain this way as a check to your measurement found in part f.

Questions to answer in the short form:

A.1. What is the DC gain ($G_p$) using both methods mentioned in the procedure and time constant ($\tau$) for this first-order system that you found in Part A?
Part B: Open-loop response with Proportional Controller

Procedure:
   a) Make the following connections on the PT326:
      • Turn Proportional control on.
      • Keep Continuous control on.
      • No jumper between X and Y (open-loop).
      • Remove the jumper that was put between A and B.

   b) Repeat (b) through (f) as in (A) for Proportional Controller value of 200%, 150%, 100%, 50% and 30% and record your data. (be conscious of what values of gain \( K_p \) these percentages represent). At 30% proportional control the gain is rather high, and may be high enough to saturate the heater. Look at the “Measured Value” temperature on the PT 326 when set at 30% to see if it is too high to rely on the measurement.

Questions:
B.1. Fill out the table given in the short form.
B.2. What did you predict to happen to the gain and time constant of the open-loop system response as you increased the proportional gain \( K_p \)? Does the data match your predictions?

Part C: Closed-loop feedback response with Proportional Controller

Procedure:
   a) Make the following connections on the PT326:
      • Turn Proportional control on.
      • Turn Continuous control on.
      • Jumper between X and Y (closed-loop).

   b) Repeat (b) through (f) as in (A) for Proportional Controller value of 200%, 150%, 100%, 50% and 30% and record your data. For each also measure the steady-state error in temperature. This is read directly from the difference in the values of “Set Value” and “Measured Value” with the internal switch ON.

Observe how Percent Overshoot varies with Proportional control.

Questions:
C.1. Fill out the table given in the short form.
C.2. What is the effect of increasing the proportional gain on the closed-loop system response (system gain and time constant) and on the steady-state error?
C.3. Assuming no oscillations and proportional control only (closed-loop), what must the gain \( K_p \) be in order to reduce the steady-state error to zero?
Part D: Matlab simulation

Proportional Control

The Matlab simulation will consist of examining the effect of proportional only control on our closed-loop system.

The Matlab script "HEATER_CONTROL", will be used for this simulation. Download and run the script. You will be prompted to enter the system DC gain \( G_p \) and time constant \( \tau \) found from part A, as well as the \( K_p \) gains corresponding to proportional bands of 200\%, 150\%, 100\%, 50\%, and 30\%.

Each group member should turn in a plot of your time response with your short form report. Note how the measured (simulated) response compares to the desired response (a unit-step input).

After running the script and obtaining the plot, conclude how the steady-state error depends on the value of \( K_p \) (Proportional Gain).

Questions:

Answer questions D.1. to D.5. in the short form.
A.1. What is the DC gain $G_p$, (using both methods mentioned in the procedure) and time constant ($\tau$) for this first-order system?

B.1. Open-loop response with Proportional Controller Part B. Fill out Table

<table>
<thead>
<tr>
<th>proportional band</th>
<th>200%</th>
<th>150%</th>
<th>100%</th>
<th>50%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>time constant</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>system gain</td>
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</tr>
</tbody>
</table>

Can you consider your measurements taken at 30% proportional control accurate?

B.2. What did you predict to happen to the gain and time constant of the open-loop system response as you increased the proportional gain $K_p$? Does the data match your predictions?
C.1. Closed-loop response with Proportional Controller Part C. Fill out Table

<table>
<thead>
<tr>
<th>Proportional band</th>
<th>200%</th>
<th>150%</th>
<th>100%</th>
<th>50%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
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<tr>
<td>Time constant</td>
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<tr>
<td>System gain</td>
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<tr>
<td>Steady-state error (temp)</td>
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</tbody>
</table>

C.2. What is the effect of increasing the proportional gain on the closed-loop system response (system gain and time constant) and on the steady-state error?

C.3. Assuming no oscillations and proportional control only (closed-loop), what must the gain $K_p$ be in order to reduce the steady-state error to zero?
D.1. Compute the closed-loop transfer function for proportional control ($K_p$) only. Substitute your calculated value of the time constant ($\tau$) and the DC gain ($G_p$) from question A.1. into the transfer function, leaving $K_p$ as a variable.

D.2. Substitute in each value of $K_p$ in the table below into the closed-loop transfer function found in question D.1. above. Also, compare your transfer functions to the ones generated in the Matlab script “HeaterControl.m.” Note, you will need to convert the Matlab transfer functions into standard form. Comment on any differences.

<table>
<thead>
<tr>
<th>proportional band</th>
<th>200%</th>
<th>150%</th>
<th>100%</th>
<th>50%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>transfer function</td>
<td></td>
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<td></td>
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<tr>
<td>transfer function from Matlab script</td>
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<tr>
<td>standard form of Matlab transfer function</td>
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</table>
D.3. From the Matlab plots obtained make an analysis on how steady-state error depends on the value of $K_p$ (proportional gain)?

D.4. Do you recall seeing any percent overshoot on the oscilloscope? How did the percent overshoot vary with proportional control?

D.5. Do your observations of this percent overshoot match with the theory of first-order systems (look at Matlab simulation)? Speculate on why you think there may be some percent overshoot in the actual system.