Name: ________________________________

Lecture Section (Radcliffe or Ermer): ________________

ME 451 - Spring 1994

Midterm Examination #1

2/16/94

Score:

1) _______________
2) _______________
3) _______________
4) _______________

Total: __________
1. a. (15 pts) Derive the ordinary differential equation model of the following mechanical system (assume \(x_1\) and \(x_2\) are measured from the equilibrium position, the bar at \(x_1\) is massless, and there is viscous friction between \(m_2\) and the ground).

\[ \begin{align*}
\text{Diagram:} & \quad x_1, x_2, m_2, b_1, k_1, b_2, k_2, F(t) \\
\end{align*} \]

b. (10 pts) Find the transfer function \(G(s) = \frac{X_1(s)}{F(s)} = \)
2. The following differential equation describes the behavior of an electrical circuit:

\[
\dddot{q} + 2\dot{q} + 4q = 3e_0(t)
\]

a. (15 pts) Draw a simulation diagram for this equation.

b. (10 pts) Derive a set of state variable equations for the model. Express the final results in matrix form.

3. Given the following block diagram:
a. (10 pts) Find the transfer function $G_p(s) = \frac{C(s)}{X(s)} = \boxed{}$

b. (15 pts) Find the transfer function $T(s) = \frac{C(s)}{R(s)} = \boxed{}$

4. Given the following non-linear differential equation:
\( f(x,u) = \ddot{x} = -3\sin x + u(t) \)

a. (15 pts) Linearize the equation about the operating point \( x = \pi/4 \) (Hint: use the new variables \( x' = x - \pi/4 \) and \( u' = u - u(\pi/4) \)).

b. (10 pts) Solve the linearized equations for \( x(t) \) when \( u'(t) \) is a unit impulse function.