1. Given the non-linear differential equation for cutter displacement

\[ \dot{x} = 1.5u^2 - 3.5 - 3x - x^2 \]

where \( x = x(t) \) is the cutter displacement in meters and \( u = u(t) \) is the drive voltage

(a) Find an appropriate operating point value \( u(t) = u_o \) to linearize the above differential equation for the cutter at the operating point, \( x(t) = x_o = 2 \) meters.

\[ u_o = \]

(b) Linearize the non-linear differential equation at the \( x(t) = x_o = 2 \) meter operating point.

(Enter result in the box below)

Linearized Equation
(10 pts) 2. Give the MatLab statement(s) required to define the vector, \( \mathbf{x} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \) and the matrix \( \mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix} \) and then compute the product \( \mathbf{y} = \mathbf{Ax} \)

3. An electrical system has the differential equation and initial condition model below,
\[
2\ddot{e} + 5\dot{e} + 20e = 3i(t)
\]
\( e(0) = 2, \dot{e}(0) = 1 \)

(10 pts) a) Give a set of 1st order state equations \( \dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{bu} \) in matrix form which are equivalent to the above 2nd order differential equation. Define the relationship between your states \( \mathbf{x} \) and the electrical voltage \( e(t) \)

(10 pts) b) Give the transfer function, \( \frac{E(s)}{I(s)} \) that relates the electrical voltage to the applied current.

\[
\frac{E(s)}{I(s)} = 
\]
4. Given the following block diagram:

\[
\begin{pmatrix}
\begin{array}{c}
R(s) \\
K \\
\end{array}
\end{pmatrix} + \begin{pmatrix}
\begin{array}{c}
2 \\
\frac{2}{s + 1}
\end{array}
\end{pmatrix} \rightarrow \begin{pmatrix}
\begin{array}{c}
C(s)
\end{array}
\end{pmatrix}
\]

(15 pts) Find the transfer function \( T(s) = \frac{C(s)}{R(s)} \).

\[
\frac{C(s)}{R(s)} = \]

\[
\frac{2}{s + 1}
\]
5. Given the following transfer function:

\[ \frac{Y(s)}{U(s)} = \frac{2s + 4}{s^2 + s + 3} \]

(15 pts) Draw a simulation diagram below
(15 pts) 6. a) Find the transfer function, $E_2/E_1$, for the circuit shown below and enter it in the box.

\[ E_2/E_1 = \]

(5 pts) b) What is the (minimum) number of states required to model the above circuit?.

Enter a number in the box

\[ \quad \]