1. For the vibratory system shown,

(12 pts) a) Derive the differential equations needed to model the mechanical system response to force, \( f(t) \). Assume \( x_1 \) and \( x_2 \) are measured from the equilibrium position, and the point at \( x_1 \) is massless.

(13 pts) b) Give the transfer function, \( \frac{X_2(s)}{F(s)} \) that relates the displacement of the weight, \( X_2(s) = \mathcal{L} \{ y(t) \} \), to the applied force, \( F(s) = \mathcal{L} \{ f(t) \} \).

\[
\frac{X_2(s)}{F(s)} =
\]
2. The following transfer function describes the behavior of an electrical circuit:

\[ \frac{Y(s)}{U(s)} = \frac{3s + 2}{s^2 + 6s + 3} \]

(15 pts) a) Draw a simulation diagram for this equation.

(10 pts) b) Derive a set of state equations for the model. Express the final results in matrix form.
3. Given the following block diagram:

![Block Diagram](image)

(10 pts) a) Find the transfer function \( \frac{E_2(s)}{X(s)} = \)

\[
\frac{E_2(s)}{X(s)} =
\]

(15 pts) b) Find the transfer function \( T(s) = \frac{C(s)}{R(s)} \).

\[
\frac{C(s)}{R(s)} =
\]
4. Given the following non-linear differential equation:

\[ f(x,u) = \ddot{x} = -4x^2 + u(t) \]

(5 pts) a) Find the equilibrium value of \( x(t) = x_o \) at the operating point input, \( u(t) = u_o = 1 \).

\[ x_o = \]

(20 pts) b) Linearize the equation about the equilibrium operating point for \( u(t) = u_o = 1 \).

(Enter result in the box below)