1. Given the non-linear differential equation for automobile velocity

\[ 1000 \ddot{v} = 100 \theta + v \dot{\theta} - 0.1v^2 \]

where \( v = v(t) \) is automobile velocity (m/sec) and \( \theta = \theta(t) \) is the throttle angle (rad)

(5 pts) a) Find an appropriate operating point value \( \theta(t) = \theta_o \) to linearize the above differential equation for automobile velocity at the operating point speed, \( v(t) = v_o = 25 \) m/sec.

\[ \theta_o = \]

(15 pts) b) Linearize the non-linear differential equation at the operating point.

(Enter result in the box below)

\[ \text{Linearized Equation} \]
(5 pts) 2. a) Give the MatLab statement required to define the vector, \( a = [0.0 \ 0.5 \ 1.0 \ldots \ 100] \)

(5 pts) b) Give the MatLab statement required to solve the equation
\[
\begin{bmatrix}
1 & 0 \\
2 & 4
\end{bmatrix}
x = \begin{bmatrix}
1 \\
2
\end{bmatrix}
\]

3. An mechanical rotation system, its differential equation and initial condition model are,

\[
\ddot{\theta} + 2\ddot{\theta} + 90\theta = \tau(t)
\]
\[
\theta(0) = 2, \ \dot{\theta}(0) = 1
\]

(10 pts) a) Give a set of 1st order state equations in the form \( \dot{X} = AX + Bu \) which are equivalent to the above 2\textsuperscript{nd} order differential equation. State how your states are related to the displacement \( \theta(t) \)

(10 pts) b) Give the transfer function, \( \Theta(s)/\tau(s) \) that relates the displacement of the weight, \( \Theta(s) = \mathcal{L}[\theta(t)] \), to the applied torque, \( \tau(s) = \mathcal{L}[\tau(t)] \).

\[
\Theta(s)/\tau(s) =
\]
(30 pts) 4. Given the following block diagram:

\[
\begin{align*}
R(s) & \rightarrow + \rightarrow E_1(s) \rightarrow 3 \rightarrow + \rightarrow X(s) \rightarrow + \rightarrow E_2(s) \rightarrow 1/0.5s+1 \rightarrow C(s) \\
\frac{4}{s+4} & \rightarrow - \rightarrow E_1(s) \rightarrow - \rightarrow X(s) \rightarrow - \rightarrow E_2(s) \rightarrow - \rightarrow C(s)
\end{align*}
\]

Find the transfer function \( T(s) = \frac{C(s)}{R(s)} \).

\[
\frac{C(s)}{R(s)} =
\]
(15 pts) 5. a) Find the transfer function, \( V_2(s) / V_1(s) \), for the circuit shown below and enter it in the box provided.

\[
V_2(s) / V_1(s) =
\]

(5 pts) b) What is the (minimum) number of states required to model the above circuit?

Enter a number in the box