1. a) For each system’s differential equation model given below, write the system’s transfer function in the box provided.
   
   i) \[ 5 \frac{dy}{dt} + 12y = 2u \]

   ii) \[ \frac{dy^3}{dt^3} + 5 \frac{dy^2}{dt^2} + 40 \frac{dy}{dt} + y = \frac{du^2}{dt^2} + 2 \frac{du}{dt} + 3u \]

   b) For each transfer function given below, write the corresponding differential equation in the box provided.
   
   i) \[ \frac{Y(s)}{U(s)} = \frac{1}{s + 2} \]

   ii) \[ \frac{Y(s)}{U(s)} = \frac{3s + 1}{s^2 + 9} \]
(25 pts) 2. For the circuit shown below, find the transfer function $V_2(s)/V_1(s)$ in standard form as the ratio of two polynomials. Enter the transfer function $V_2(s)/V_1(s)$ in the box

$$V_2(s)/V_1(s) =$$
(30 pts) 3. For the mechanical rotation system shown, a) Write the equations of motion. Express them in matrix form in the Laplace domain and enter your result in the box provided.

Laplace transform of the equations of motion in matrix form (Not the transfer function yet

\[
\begin{bmatrix}
\theta_1 & \theta_2
\end{bmatrix}
\begin{bmatrix}
K_2
\end{bmatrix}
\begin{bmatrix}
J_1 & J_2
\end{bmatrix}
\begin{bmatrix}
f_v
\end{bmatrix}
\]
(15 pts) b) Find the Transfer Function, $\theta_2(s)/T(s)$. Enter your result in the box provided.

<table>
<thead>
<tr>
<th>Transfer Function, $\theta_2(s)/T(s)$</th>
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4. Given the non-linear differential equation for production line speed

\[ \dot{v} + 10v u + 10v^2 = 200u \]

where \( v = v(t) \) is production line speed in meters/minute and \( u = u(t) \) is the input drive in Amps

(10 pts)  
(a) Find an appropriate operating point value \( u(t) = u_o \) to linearize the above differential equation for production line speed at the operating point speed, \( v(t) = v_o = 10 \) m/min.

\[ u_o = \]

(15 pts)  
(b) Linearize the non-linear differential equation at the 10 m/min operating point.  
(Enter result in the box below)

Linearized Equation