IMPORTANT NOTE...

These collected problems ARE NOT the only type of problems that could appear. Missing are problems in this document that we have covered include (at least)

1) Drawing block diagrams (Chapter 1)
2) Mechanical Rotation system models
3) DC motor system models
and others

You should review your homework and the posted solutions. Any problem in your homework is a possible midterm problem.

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Fall 1992

1) Find the differential equation that relates the displacement $x(t)$ to the force $f(t)$ for the mechanical system shown below is:

![Mechanical System Diagram](image)

b) The transfer function associated with the differential equation

$$\ddot{x}(t) + 3\dot{x}(t) + 2x(t) = 4f(t)$$

with input $f(t)$ and output $x(t)$ is:
5) You are asked to study the dynamics of a system governed by the ordinary differential equation

\[ \ddot{y}(t) + 0.1\dot{y}(t) + \sin[y(t)] = u(t) \]

which may also be written in the form

\[ f(u, y, \dot{y}, \ddot{y}) = \ddot{y}(t) + 0.1\dot{y}(t) + \sin[y(t)] - u(t) = 0 \]

Find the following

a) \( \frac{\partial f}{\partial u} = \)

b) \( \frac{\partial f}{\partial y} = \)

c) \( \frac{\partial f}{\partial \dot{y}} = \)

d) \( \frac{\partial f}{\partial \ddot{y}} = \)

e) Let \( v(t) = u(t) - u_o \), \( x(t) = y(t) - y_o \) and \( \dot{x}(t) = \dot{y}(t) - \dot{y}_o \) and write down the linearized differential equation relating \( v(t) \) to \( x(t) \) for the equilibrium given by \( u_o = 1/\sqrt{2} \), \( y_o = \pi/4 \), \( \dot{y}_o = 0 \).
Fall 1992:
1. The vibratory system shown below,

\[ W = 9.8 \text{ N} \]
\[ c = 98 \text{ N-sec/m} \]
\[ k = 98 \text{ N/cm} \]

has the second order differential equation of motion,

\[ \ddot{y} + 10\dot{y} + 10000y = F(t) \quad y(0) = 2, \quad \dot{y}(0) = 1 \]

(15 pts) Give the transfer function, \( Y(s)/F(s) \) that relates the displacement of the weight, \( Y(s) = \mathcal{L}[y(t)] \), to the applied force, \( F(s) = \mathcal{L}[f(t)] \).

\[ Y(s)/F(s) = \]

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Fall 1992
(10 pts) 2. A control system has Laplace domain transfer function,

\[ G(s) = \frac{Y(s)}{F(s)} = \frac{3s + 1}{(s + 1)(s + 2)} \]

What is the system’s impulse response, ie. \( y(t) \) for \( f(t) = \) unit impulse?

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Fall 1995
(15 pts) 5. a) Find the transfer function, \( V_2(s)/V_1(s) \), for the circuit shown below and enter it in the box provided.

\[ V_2(s)/V_1(s) = \]
Fall 1995

1. Given the non-linear differential equation for cutter displacement

$$\dot{x} = 1.5u^2 - 3.5 - 3x - x^2$$

where $x = x(t)$ is the cutter displacement in meters and $u = u(t)$ is the drive voltage

(5 pts) a) Find an appropriate operating point value $u(t) = u_o$ to linearize the above differential equation for the cutter at the operating point, $x(t) = x_o = 2$ meters.

$$u_o =$$

(15 pts) b) Linearize the non-linear differential equation at the $x(t) = x_o = 2$ meter operating point. (Enter result in the box below)

Linearized Equation

(10 pts) 2. Give the MatLab statement(s) required to define the vector, $x = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and the matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ and then compute the product $y = Ax$

3. An electrical system has the differential equation and initial condition model below,

$$2\ddot{e} + 5\dot{e} + 20e = 3i(t)$$

$e(0) = 2$, $\dot{e}(0) = 1$

(10 pts) b) Give the transfer function, $E(s)/I(s)$ that relates the electrical voltage to the applied current.

$$E(s)/I(s) =$$
3. A hydraulic system has the differential equation and initial condition model below, 
\[2 \ddot{p} + 5 \dot{p} + 20p = 3v(t)\]
\[p(0) = 2, \ \dot{p}(0) = 1\]
(10 pts) b) Give the transfer function, \(P(s)/V(s)\) that relates the pressure \(p\) to the input \(v\).