Frequency Response Example:
For the Open-Loop Transfer Function $KGH = K \left[ \frac{12}{(s+1)(s+2)(s+3)} \right]$
At zero frequency, this system has a DC (steady-state) gain of
$$GH(0) = \left( \frac{12}{1 * 2 * 3} \right) = 2$$
In a decibel (dB) scale, this gain is expressed.
$$GH(0) = 20 \log_{10} (2) = 20(0.3010) = 6.02 \text{ dB}$$
The two are equivalent but one is on a linear scale and the other on a logarithmic scale

Note: For the record, “deci-Bel” dB is the $10*\log( \text{ power ratio})$

$$dB = 10\log_{10} \left( \frac{P}{P_{ref}} \right) = 10\log_{10} \left( \frac{ky^2}{ky_{ref}^2} \right) = 10\log_{10} \left( \frac{y^2}{y_{ref}^2} \right) = 10\log_{10} \left[ \left( \frac{y}{y_{ref}} \right)^2 \right]$$

$$= 20\log_{10} \left( \frac{y}{y_{ref}} \right) = 20\log_{10} (y) \text{ for } y_{ref} = 1 \text{ because } \log_{10}(1) = 0$$

The Root Locus showing the closed-loop pole locations is

The Open-Loop Frequency Response (Bode Diagram) is plotted with the command

EDU» num=[12]; den=conv([1 1], conv([1 2], [1 3])); G=tf(num,den)
EDU» bode(G)
And the Gain and Phase margins are computed with the command

EDU» margin(G)

Yielding the plot...

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**Bode Diagram**

Gm=13.979 dB (at 3.3166 rad/sec), Pm=75.636 deg. (at 1.2232 rad/sec)

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This plot show a Gain Margin (GM) = 13.979 dB and a Phase Margin of 75.636 degrees.

The Gain Margin indicates that the control gain can be increased by 13.979 dB ($10^{(13.979/20)} = 5$) from $K = 1 = 0\text{dB}$ before the system will go unstable.
The Phase Margin indicates that at the current gain \((K = 1)\), the system can absorb an addition phase lag of 75.636 before it will go unstable. 

For the system 
\[
KGH = K \left[ \frac{12}{s + 1} \left( \frac{1}{s + 2} \right) \left( s + 3 \right) \right]
\]
the GM=14. dB and Phase Margin = 76 degrees with \(K=1\) indicate a very stable system.

The step response for the Closed-Loop system 
\[
T = \frac{12}{s^3 + 6s^2 + 11s + 18}
\]
confirms this prediction.

Although the step response is stable, the steady state error is 33%. Could we reduce it by increasing the gain \(K\) from \(K = 1\) (0 dB) to \(K = 2\) (6.02 dB)? Yes, just add 6.02 dB to the frequency response shift it up 6.02 dB at all frequencies. Let’s plot it with the Matlab commands

\[
\text{EDU} \rightarrow \text{num}=[24]; \text{den}=[1 \ 6 \ 11 \ 6]; \text{G}=\text{tf}(	ext{num}, \text{den}); \text{margin}(\text{G})
\]

The new open-loop transfer function 
\[
G_2(s) = \frac{24}{s^3 + 6s^2 + 11s + 6}
\]
has a DC gain of \(G_2(s) = 4\) (= 12 dB) reducing the error to \(e(\infty) = 1/(1 + 4) = 0.2 = 20\%\) but sacrifices stability with the new GM=8 dB and PM = 35 degrees.

The system’s closed-loop transfer function
has the predicted, less stable response with better steady-state error.

\[ T_2 = \frac{24}{s^3 + 6s^2 + 11s + 30} \]

Bode Diagram

\( Gm=7.9588 \, \text{dB} \, \text{(at 3.3166 rad/sec)}, \, Pm=35.425 \, \text{deg.} \, \text{(at 2.0639 rad/sec)} \)