Prerequisites: Complex Numbers

- **Ordered pair** of two real numbers
  \[ s = x + jy \quad j = \sqrt{-1} \]
- **Conjugate**
  \[ \bar{s} = s^* = x - jy \]
- **Addition**
  \[ s_1 + s_2 = (x_1 + x_2) + j(y_1 + y_2) \]
- **Multiplication**
  \[ s_1 s_2 = (x_1 + jy_1) (x_2 + jy_2) = (x_1 x_2 - y_1 y_2) + j(y_1 x_2 + x_1 y_2) \]

Complex Numbers: Polar vs. Rectangular Form

- **Euler’s identity**
  \[ e^{j\theta} := \cos \theta + j \sin \theta \]
- **Polar form**
  \[ s := x + jy = r e^{j\theta} \]
- **Magnitude**
  \[ r = \sqrt{x^2 + y^2} \]
- **Phase**
  \[ \theta = \tan^{-1}(y/x) \]

Complex Numbers: Polar Arithmetic

- For two polar complex numbers...
  \[ s_1 = r_1 e^{j\theta_1}, \quad s_2 = r_2 e^{j\theta_2} \]
- **Multiplication is easy**
  \[ s_1 s_2 = r_1 r_2 e^{j(\theta_1 + \theta_2)} \]
- **Division is also**
  \[ \frac{s_1}{s_2} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)} \]
- Addition and subtraction are a mess...
  - Convert to rectangular, add then convert back

Matrices

- An nxm array of numbers is a matrix
  - nx1 arrays are vectors (a column)
    \[ A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \]
  - 1xn arrays are transposed vectors

Matrix Multiply

- Multiply rows by columns
  \[ A \mathbf{x} = [a_{11} a_{12} \cdots a_{1n}] [x_1 x_2 \cdots x_n]^T = a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n \]
- There is no matrix divide, only inverses
Matrix Determinant

• Pick any row or column...
  - The determinant is the sum of the “signed row elements” times the determinants of there “minors”
  - The $i,j$ minor is the matrix remaining after the $i^{th}$ row and $j^{th}$ column are deleted from the matrix

\[
\begin{vmatrix}
1 & 2 & 0 \\
3 & 4 & 0 \\
0 & 3 & 1 \\
\end{vmatrix} = [4(0) - 3(0)] - 2(0) - 0(0) = 0 \\

= 1 - 6 + 0 = -5
\]

- There is NO OTHER WAY!!!! (No Short cuts!!)

Kramer’s Rule

• For the $n \times n$ linear equation $Ay = f$

The solution is

\[
y_j = \frac{A^*}{|A|}
\]

- Where $A*$ is the matrix $A$ with $f$ substituted for the $j^{th}$ column

\[
\begin{vmatrix}
1 & 3 & 2 \\
0 & 2 & 3 \\
\end{vmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
\end{bmatrix} =
\begin{bmatrix}
1 \\
2 \\
\end{bmatrix}
\]

\[
y_1 = \frac{3}{2} [1] - \frac{3}{2} [2] = (2 - 6)/2 = -2
\]

A Little Matlab

Comment

>> Define the matrix $A$ ($A$ is echoed)

$A =
\begin{bmatrix}
1 & 3 \\
0 & 2 \\
\end{bmatrix}$

>> Define the vector $f$ ($f$ is echoed)

$f =
\begin{bmatrix}
1 \\
2 \\
\end{bmatrix}$

>> compute $y_j = (A^*/|A|) f$ ($y_j$ is echoed)

with $y_1 = -2$

and $y_2 = 1$

Matlab

$\begin{bmatrix}
1 & 3 \\
0 & 2 \\
\end{bmatrix}$

$\begin{bmatrix}
1 \\
2 \\
\end{bmatrix}$

$\begin{bmatrix}
2 \\
-2 \\
\end{bmatrix}$

Logarithm

• Definition

\[ a = b^{\log_b(a)} \]

\[ a = 10^{\log_{10}(a)} \]

\[ a = e^{\ln(a)} = e^{\log_e(a)} \]

• Use

\[ \log(ab) = \log(a) + \log(b) \]

Next Time …

• **Laplace Transforms**
  - ODE’s become Linear Algebraic Equations

• Read the Appendices

• Take a look at the Old Math Quizzes

• Work on “2004B” Math Quiz homework