That is satisfied at the operating point. Find an equilibrium where all input and output signals are zero.

Identify the system model.

**Six Steps to Linearization**

1. Identify the system model’s inputs and outputs
2. Express the model in the form $f(r, \dot{r}, \ddot{r}, \ldots, \dot{c}, \ddot{c}, \ldots) = 0$
3. Find an equilibrium where all input and output derivatives are zero and the original model is satisfied at the operating point $(r_0, c_0)$
   
   $f(r, \dot{r}, \ddot{r}, \ldots) = f(r_0, 0, 0, \ldots) = 0$
   
   That is … Solve $f(r_0, c_0) = 0$

Typically you are given the output $c$ and must find the input $r$.

4. Perform a Taylor Series expansion about the operating point retaining only 1st derivative terms.
   
   
   
   $f(r, \dot{r}, \ddot{r}, \ldots) = f(r_0, 0, 0, \ldots)$

   
   $\frac{\partial f}{\partial r} (r - r_0) + \frac{\partial f}{\partial \dot{r}} (\dot{r} - r_0) + \frac{\partial f}{\partial \ddot{r}} (\ddot{r} - r_0) + \ldots$

5. Change variables
   
   $r = (r - r_0)$, $\dot{r} = (\dot{r} - \dot{r}_0)$, $\ddot{r} = (\ddot{r} - \ddot{r}_0)$, etc.
   
   $\dot{c} = (\dot{c} - \dot{c}_0)$, $\ddot{c} = (\ddot{c} - \ddot{c}_0)$, etc.

6. Rewrite the Taylor Expansion in the new variables

   $f(r, \dot{r}, \ddot{r}, \ldots) = f(r_0, 0, 0, \ldots)$

   $\frac{\partial f}{\partial r} (r_0 - r) + \frac{\partial f}{\partial \dot{r}} (\dot{r}_0 - \dot{r}) + \frac{\partial f}{\partial \ddot{r}} (\ddot{r}_0 - \ddot{r}) + \ldots$

   
   $\frac{\partial f}{\partial c} (c_0 - c) + \frac{\partial f}{\partial \dot{c}} (\dot{c}_0 - \dot{c}) + \frac{\partial f}{\partial \ddot{c}} (\ddot{c}_0 - \ddot{c}) + \ldots$

**Linearization of a Magnetic Bearing model**

**The magnetic bearing**

- The magnetic bearing model

- The magnetic bearing's magnetic field relationship in the new variables

**What is a linear system?**

- A system having **Principle of Superposition**

  $u_1(t) \to y_1(t)$
  $u_2(t) \to y_2(t)$

  $\Rightarrow \alpha_1 u_1(t) + \alpha_2 u_2(t) \to \alpha_1 y_1(t) + \alpha_2 y_2(t)$

  $\forall \alpha_1, \alpha_2 \in \mathbb{R}$

A nonlinear system does not satisfy the principle of superposition.
The magnetic bearing model

1) Identify model input(s) and output(s)

2) Express the model in the $f(u_0) = 0$ form

3) Find an equilibrium for $x_0 = 2$ mm

4) Taylor Expansion about $(i_x, x_0) = (1A, 2 \text{ mm})$

5) Change Variables

**System Responses**

Chapter 4

- Responses of Linear Time-invariant Systems
  - Important input functions
    - Step Response:
      - Constant input, "easy to make", clear transient
    - Ramp Response:
      - Constant change in input, process control
    - Sinusoidal Response:
      - Constant Frequency, for vibration
      - Enough frequencies -- complete system description
  - All have observed properties that engineers use

**0th Order systems**

- Not in the book, but ...
- Much of Engineering design
  - Done with "instantaneous" algebraic models
  - The model is a simple "Gain"
- Output is always the input times a constant
  - The model is easy, so is the response computation

**1st Order Time Response**

- 1st order systems have a single derivative of the output variable
- The TF has a first order polynomial in the denominator

- For Engineering Conversation...
  - the second is more common - has physical meaning
1st Order Time Response
(Model has two parameters)

- For Engineering Conversation…
  - the second is more common - has physical meaning
  \[ G(s) = \frac{C(s)}{R(s)} = \frac{b_0}{s + a_0} \quad \text{OR} \quad G(s) = \frac{C(s)}{R(s)} = \frac{K}{\tau s + 1} \]
- Time Constant, \( \tau \) indicates “speed of response”
- (Steady-State) Gain \( K \) indicates “strength of response”
  \[ \tau = \frac{1}{a_0} \quad \text{and} \quad K = b_0/a_0 \]

1st Order Step Response

- For the 1st order system with TF
  \[ \frac{C(s)}{R(s)} = \frac{K}{\tau s + 1} \]
- And step unit input
  \[ R(s) = \frac{1}{s} \]
- The output is...
  \[ C(s) = \left( \frac{K}{\tau s + 1} \right) \frac{1}{s} = \frac{K}{s(\tau s + 1)} = \frac{K/\tau}{s(1/\tau)} = \frac{K}{s} + \frac{-K}{s(1/\tau)} \]

Summary and Exercises

- Modeling of
  - Nonlinear systems with linearization
- Next
  - 2nd Order System Time Response