"Objectives" of Lecture on DSP

• Introduction to laboratory #1
• The need for DSP
• Resolution, Aliasing & Windowing
• Introduction to FFT.
Why **Digital** Signal Processing?

- Allows fast & complex calculations, usually without loss of information.

**E.G.**

- MRIs
- Communication (TV, CD, MP3, etc.)
- Signal diagnostics (machine health, “looking” for hidden details)
A/D converters  
(Analogue/Digital)

This lecture, 3 concepts & 1 process.

- A/D resolution
- Aliasing (sampling rate)
- Windowing (leakage)

Time domain

FFT

Frequency domain
Getting it into the computer!

(A/D)
Getting it into the computer!
(A/D)

We now have a sequence of numbers, i.e., a vector.
Some definitions

- Sampling rate
  \[ f_s = \frac{1}{\Delta t} \]
- Window length or Time Record length, T.R.
Mention of one other issue: 

Resolution

For a specific voltage range there will only be a certain number of "divisions" available (see later).
Mention of one other issue: Resolution

For a specific voltage range there will only be a certain number of "divisions" available (see next slide).
Example of a A/D spec.

“12bit A/D converter with a sampling rate of 15kHz over a range of -/+ 5 volts”

Voltage increments:
\[ \Delta \text{voltage} = \frac{10}{2^{12}} = \frac{10}{4096} = 2.441 \text{mV} \]

Time increment:
\[ \Delta t = \frac{1}{15,000} \text{ sec.} \]
Aliasing

• How fast do we have to sample?
• What happens if we don't sample fast enough?
• What can we do about it?
Fast enough?

1Hz analogue signal

10Hz sampling frequency
Or is this fast enough?

1Hz analogue signal

5 Hz sampling frequency
Aliased signal

1Hz analogue signal or a 4Hz analogue signal
5 Hz sampling frequency
“Onset” of aliasing

We must sample AT LEAST twice as fast as the highest frequency that is present.
e.g. here, 4Hz signal therefore sample > 8Hz.
SUMMARY

• To avoid aliasing, the analogue signal is first FILTERED (anti-aliasing filters) to ensure that all frequencies higher than 1/2 the sampling frequency have been removed.

• The maximum detectable frequency is sometimes called the Nyquist frequency.
Windowing

There must be a finite record length.
Windowing

Finite record length, hence “stop” and “start”.

T.R.
Windowing

The Fourier series algorithm believes this finite record, of length T.R., to be the fundamental part of a periodic function. It attempts to recreate a function as sketched below. The sudden “stop” and “start”, shown in red, causes the creation of additional components in the frequency domain. These additional frequencies are known as LEAKAGE. This can be minimized by tapering the beginning and ending of each T.R. to zero.
Windowing function
Quick introduction to Fast Fourier Transform (FFT)
Recap Fourier series.

Periodic signal of period $\tau$, \[ x(t) = x(t + \tau) \]

\[
x(t) = \frac{1}{T} \left\{ a_0 + \sum_{n=1}^{\infty} a_n \cos(\omega_n t) + b_n \sin(\omega_n t) \right\} \quad \text{where} \quad \omega_n = \frac{2n\pi}{\tau} \quad n = 1, 2, 3, \ldots
\]

and where \[
a_n = \frac{2}{\tau} \int_{-\tau/2}^{\tau/2} x(t) \cos(\omega_n t) dt
\]

\[
b_n = \frac{2}{\tau} \int_{-\tau/2}^{\tau/2} x(t) \sin(\omega_n t) dt
\]

See eq(1.2.1-3)
Fourier series and FFT

Recall that for the following square wave:

\[ x(t) = \frac{4}{\pi} \sin(\omega_1 t) + \frac{4}{3\pi} \sin(3\omega_1 t) + \frac{4}{5\pi} \sin(5\omega_1 t) + \ldots \]
Another way of viewing this:

Magnitude of Sine component

Time domain

Frequency domain

FFT
Lab. Example
FFT summary

N data points

N/2 Mag. data points
N/2 Phase. data points