ME451: Control Systems

Lecture 8
Modeling of DC motors

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Course roadmap

Modeling
- ✓ Laplace transform
- ✓ Transfer function
- ✓ Models for systems
  - ✓ electrical
  - ✓ mechanical
  - • electromechanical
- ✓ Block diagrams
- ✓ Linearization

Analysis

Time response
- Transient
- Steady state

Frequency response
- Bode plot

Stability
- Routh-Hurwitz
- Nyquist

Design
- Design specs
- Root locus
- Frequency domain
- PID & Lead-lag
- Design examples

(Matlab simulations &) laboratories

What is DC motor?

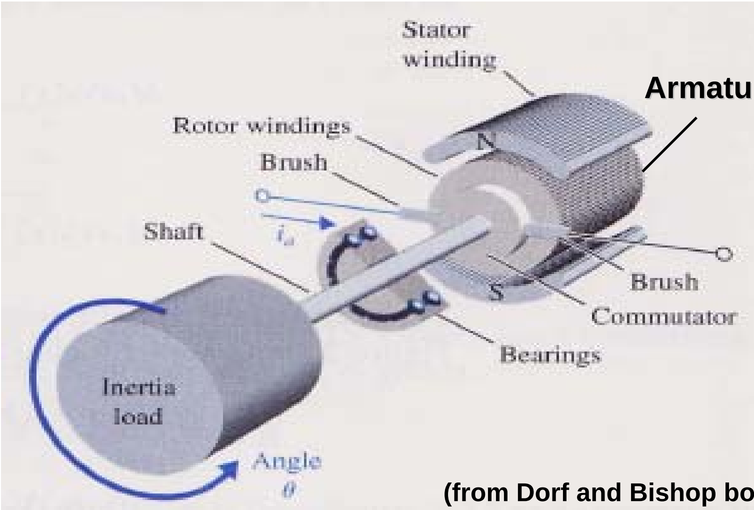
An actuator, converting electrical energy into rotational mechanical energy

(You will see DC motor during Lab 1 and 4.)

Why DC motor?

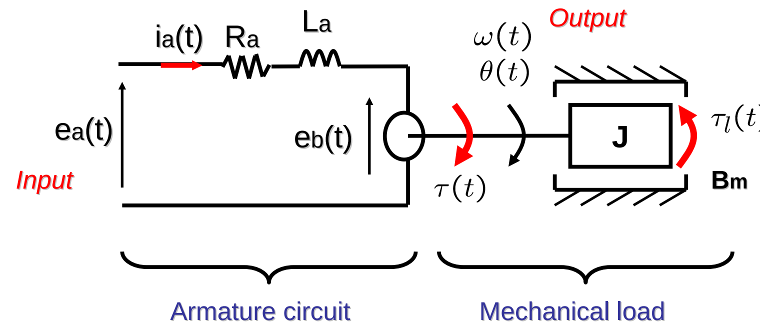
- Advantages:
  - high torque
  - speed controllability
  - portability, etc.
- Widely used in control applications: robot, tape drives, printers, machine tool industries, radar tracking system, etc.
- Used for moving loads when
  - Rapid (microseconds) response is not required
  - Relatively low power is required

How does DC motor work?



(from Dorf and Bishop book)

Model of DC motor



"a" :armature

e_a :applied voltage

i_a :armature current

"b" :back EMF

mechanical

θ :angular position

ω :angular velocity

J : rotor inertia

B : viscous friction

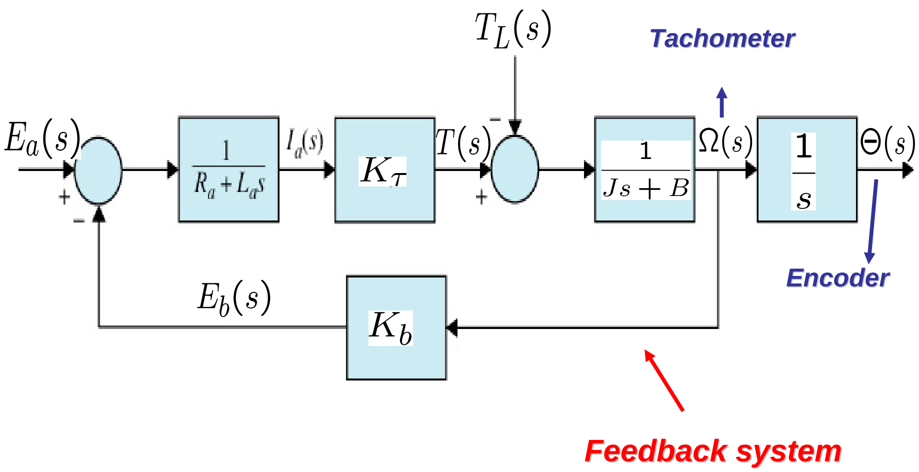
Modeling of DC motor: time domain

- Armature circuit $e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_b(t)$
- Connection between mechanical/electrical parts
  - Motor torque $\tau(t) = K_\tau i_a(t)$
  - Back EMF $e_b(t) = K_b \omega(t)$
- Mechanical load $J\ddot{\theta}(t) = \tau(t) - B\dot{\theta}(t) - \tau_l(t)$ (Load torque: $\tau_l(t)$)
- Angular position $\omega(t) = \dot{\theta}(t)$

Modeling of DC motor: s-domain

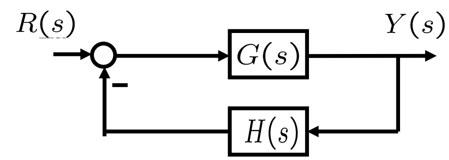
- Armature circuit $I_a(s) = \frac{1}{R_a + L_a s}(E_a(s) - E_b(s))$
- Connection between mechanical/electrical parts
  - Motor torque $T(s) = K_\tau I_a(s)$
  - Back EMF $E_b(s) = K_b \Omega(s)$
- Mechanical load $\Omega(s) = \frac{1}{Js + B}(T(s) - T_L(s))$
- Angular position $\Theta(s) = \frac{1}{s}\Omega(s)$

DC motor: Block diagram



Useful formula for feedback

Negative feedback system



$$Y(s) = G(s)(R(s) - H(s)Y(s)) \Rightarrow (1 + G(s)H(s))Y(s) = G(s)R(s)$$

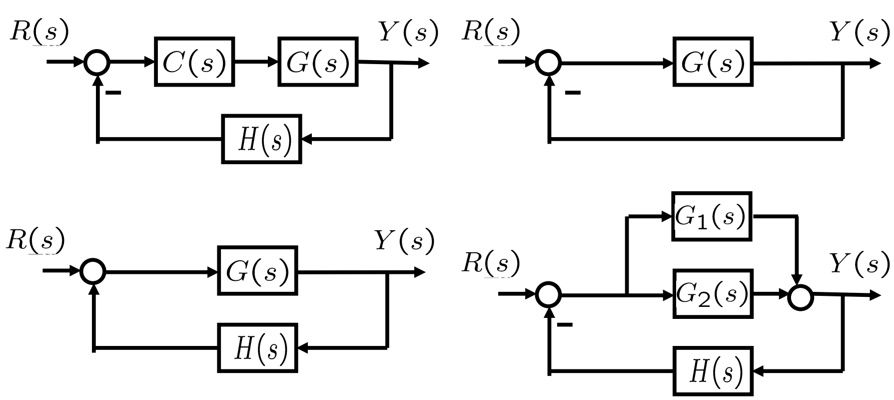
$$\Rightarrow \frac{Y(s)}{R(s)} = \frac{F_g}{1 - L_g} = \frac{G(s)}{1 + G(s)H(s)}$$

Memorize this!

$\left(\begin{array}{ll} G(s) & \text{: forward gain} \\ G(s)H(s)(-1) & \text{: loop gain} \end{array} \right)$

Ex: Derivation of transfer functions

Compute transfer functions from R(s) to Y(s).



DC motor: Transfer functions (TF)

$$\frac{\Omega(s)}{E_a(s)} = \frac{\frac{K_\tau}{(L_a s+R_a)(Js+B)}}{1+\frac{K_b K_\tau}{(L_a s+R_a)(Js+B)}} = \frac{K_\tau}{(L_a s + R_a)(Js+B) + K_b K_\tau} =: G_1(s)$$

$$\frac{\Omega(s)}{T_L(s)} = \frac{-\frac{1}{Js+B}}{1+\frac{K_b K_\tau}{(L_a s+R_a)(Js+B)}} = -\frac{L_a s + R_a}{(L_a s + R_a)(Js+B) + K_b K_\tau} =: G_2(s)$$

2nd order system

$$\Rightarrow \Omega(s) = G_1(s)E_a(s) + G_2(s)T_L(s)$$

$$\Rightarrow \Theta(s) = \frac{1}{s}\Omega(s) = \frac{1}{s}(G_1(s)E_a(s) + G_2(s)T_L(s))$$

DC motor: Transfer functions (cont'd)

Note: In many cases La<<Ra. Then, an approximated TF is obtained by setting La=0.

$$\frac{\Omega(s)}{E_a(s)} = \frac{K_\tau}{(L_a s + R_a)(Js+B) + K_b K_\tau} \approx \frac{K_\tau}{R_a(Js+B) + K_b K_\tau}$$

$$=: \frac{K}{Ts+1} \quad \left(K := \frac{K_\tau}{R_a B + K_b K_\tau},\ T = \frac{R_a J}{R_a B + K_b K_\tau} \right)$$

2nd order system ⟹ *1st order system*

$$\frac{\Theta(s)}{E_a(s)} = \frac{K}{s(Ts+1)}$$

Summary and Exercises

- Modeling of DC motor
  - What is DC motor and how does it work?
  - Derivation of a transfer function
  - Block diagram with feedback
- Next
  - ***Stability*** of linear control systems, one of the most important topics in feedback control
- Exercises
  - Go over the derivation for DC motor transfer functions by yourself. Obtain T(s)/E_a(s).

Main message until this point

Many systems
can be represented
as transfer functions!

Using the transfer functions, ….
(to be continued)