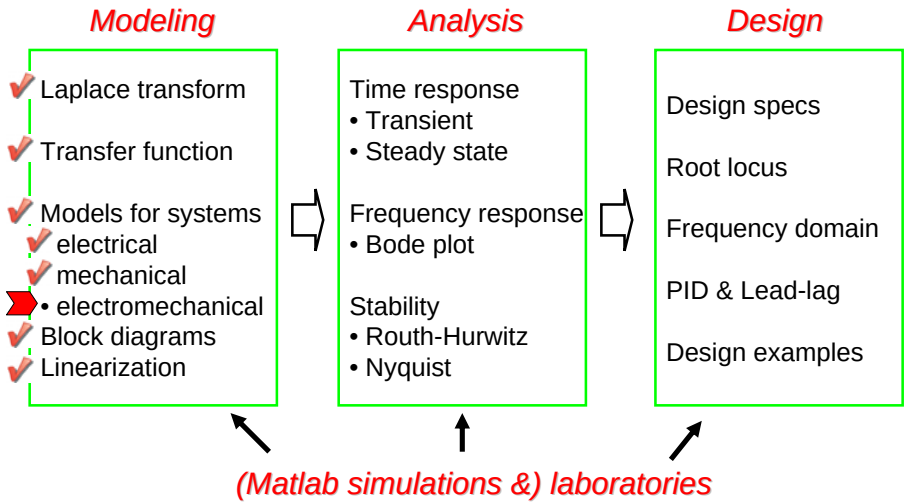
ME451: Control Systems

Lecture 8
Modeling of DC motors

Dr. Jongeun Choi
Department of Mechanical Engineering
Michigan State University

Course roadmap



What is DC motor?

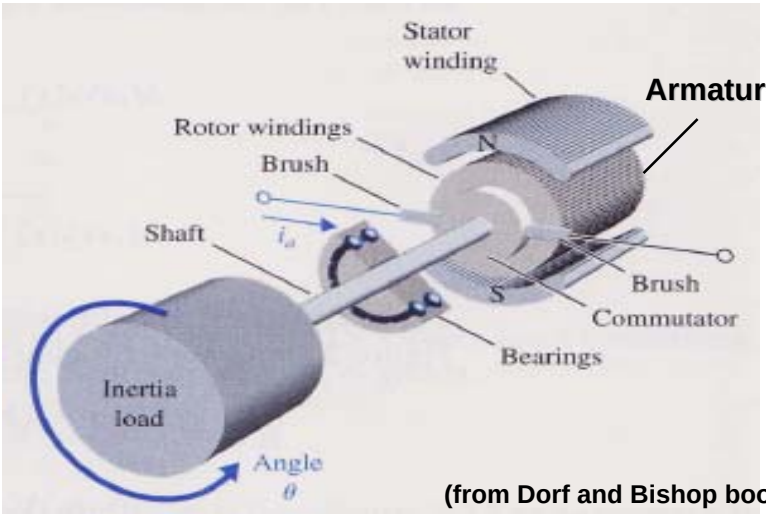
An actuator, converting electrical energy into rotational mechanical energy

(You will see DC motor during Lab 1 and 4.)

Why DC motor?

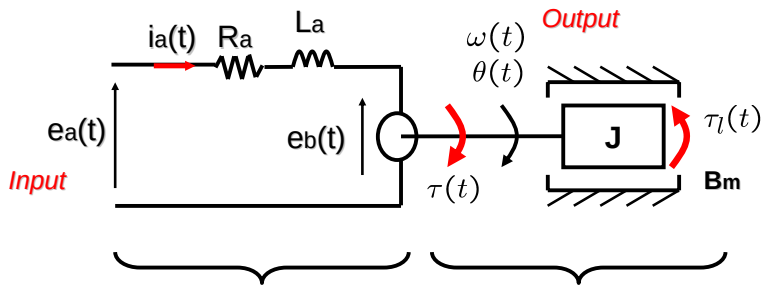
- Advantages:
  - high torque
  - speed controllability
  - portability, etc.
- Widely used in control applications: robot, tape drives, printers, machine tool industries, radar tracking system, etc.
- Used for moving loads when
  - Rapid (microseconds) response is not required
  - Relatively low power is required

How does DC motor work?



(from Dorf and Bishop book)

Model of DC motor



"a" :armature
- e_a :applied voltage
- i_a :armature current

"b" :back EMF

mechanical
- θ :angular position
- ω :angular velocity
- J : rotor inertia
- B : viscous friction

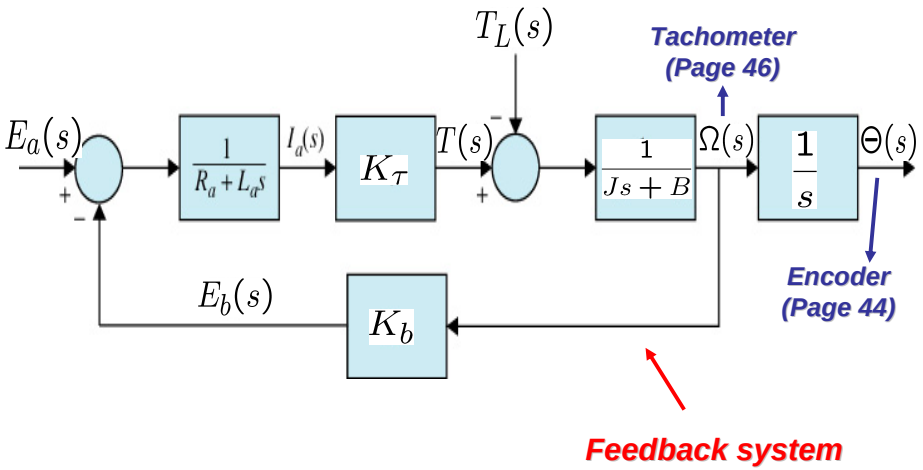
Modeling of DC motor: time domain

- Armature circuit $e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_b(t)$
- Connection between mechanical/electrical parts
  - Motor torque $\tau(t) = K_\tau i_a(t)$
  - Back EMF $e_b(t) = K_b \omega(t)$
- Mechanical load $J\ddot{\theta}(t) = \tau(t) - B\dot{\theta}(t) - \tau_l(t)$ (Load torque)
- Angular position $\omega(t) = \dot{\theta}(t)$

Modeling of DC motor: s-domain

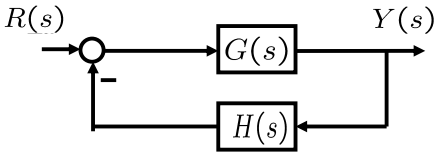
- Armature circuit $I_a(s) = \frac{1}{R_a + L_a s}(E_a(s) - E_b(s))$
- Connection between mechanical/electrical parts
  - Motor torque $T(s) = K_\tau I_a(s)$
  - Back EMF $E_b(s) = K_b \Omega(s)$
- Mechanical load $\Omega(s) = \frac{1}{Js + B}(T(s) - T_L(s))$
- Angular position $\Theta(s) = \frac{1}{s}\Omega(s)$

DC motor: Block diagram



Useful formula for feedback

Negative feedback system



$$Y(s)=G(s)(R(s)-H(s)Y(s)) \Rightarrow (1+G(s)H(s))Y(s)=G(s)R(s)$$

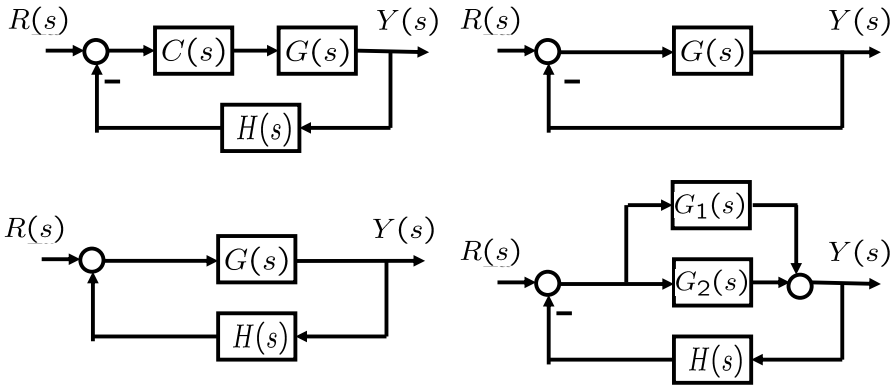
$$\Rightarrow \frac{Y(s)}{R(s)}=\frac{F_g}{1-L_g}=\frac{G(s)}{1+G(s)H(s)}$$

Memorize this!

$$\begin{pmatrix} G(s) & \text{: forward gain} \\ G(s)H(s)(-1) & \text{: loop gain} \end{pmatrix}$$

Ex: Derivation of transfer functions

Compute transfer functions from R(s) to Y(s).



DC motor: Transfer functions (TF)

$$\frac{\Omega(s)}{E_a(s)} =$$

$$\frac{\Omega(s)}{T_L(s)} =$$

2nd order system

$$\Rightarrow \Omega(s)=G_1(s)E_a(s)+G_2(s)T_L(s)$$

$$\Rightarrow \Theta(s)=\frac{1}{s}\Omega(s)=\frac{1}{s}(G_1(s)E_a(s)+G_2(s)T_L(s))$$

DC motor: Transfer functions (cont'd)

Note: In many cases La<<Ra. Then, an approximated TF is obtained by setting La=0.

$$\frac{\Omega(s)}{E_a(s)} = \frac{K_\tau}{(L_a s + R_a)(Js + B) + K_b K_\tau} \approx \frac{K_\tau}{R_a(Js + B) + K_b K_\tau}$$

$$=: \frac{K}{Ts + 1} \quad \left(K := \frac{K_\tau}{R_a B + K_b K_\tau},\ T = \frac{R_a J}{R_a B + K_b K_\tau} \right)$$

2nd order system ⟹ *1st order system*

$$\frac{\Theta(s)}{E_a(s)} = \frac{K}{s(Ts + 1)}$$

Summary and Exercises

- Modeling of DC motor
  - What is DC motor and how does it work?
  - Derivation of a transfer function
  - Block diagram with feedback
- Next
  - ***Stability*** of linear control systems, one of the most important topics in feedback control
- Exercises

  Go over the derivation for DC motor transfer functions by yourself. Obtain $T(s)/E_a(s)$.

Main message until this point

Many systems can be represented as transfer functions!

Using the transfer functions, ….
(to be continued)