ME451: Control Systems

Lecture 5
Modeling of mechanical systems

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Course roadmap

Modeling
- ✓ Laplace transform
- ✓ Transfer function
- ✓ Models for systems
  - ✓ electrical
  - • mechanical
  - • electromechanical
- Block diagrams
- Linearization

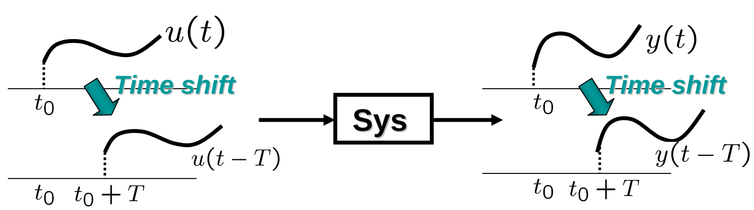
Analysis
- Time response
  - • Transient
  - • Steady state
- Frequency response
  - • Bode plot
- Stability
  - • Routh-Hurwitz
  - • Nyquist

Design
- Design specs
- Root locus
- Frequency domain
- PID & Lead-lag
- Design examples

(Matlab simulations &) laboratories

Time-invariant & time-varying

- A system is called ***time-invariant (time-varying)*** if system parameters do not (do) change in time.
- Example: Mx''(t)=f(t) & M(t)x''(t)=f(t)
- For time-invariant systems:

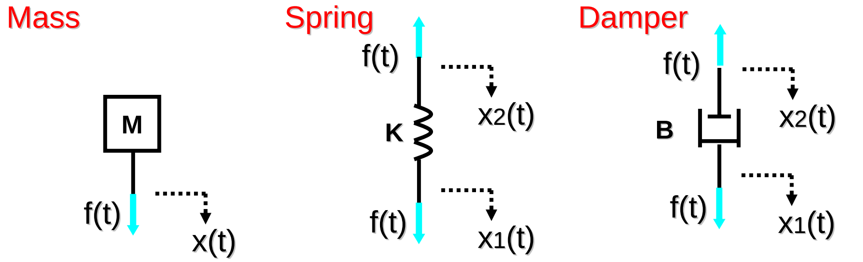


- This course deals with time-invariant systems.

Newton's laws of motion

- 1st law:
  - A particle remains at rest or continues to move in a straight line with a constant velocity if there is no unbalancing force acting on it.
- 2nd law:
  - $\sum F_i(t) = m\frac{d^2x}{dt^2}$: translational
  - $\sum \tau_i(t) = I\frac{d^2\theta}{dt^2}$: rotational
- 3rd law:
  - For every action has an equal and opposite reaction

Translational mechanical elements: (constitutive equations)

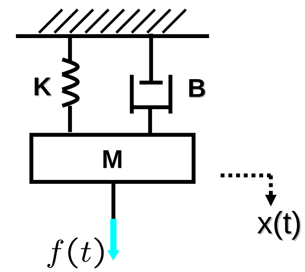


Mass: $f(t) = Mx''(t)$ $\;\Downarrow\;$ $\begin{pmatrix} x(0)=0 \\ \dot{x}(0)=0 \end{pmatrix}$ $\;\Rightarrow\;$ $F(s) = Ms^2X(s)$

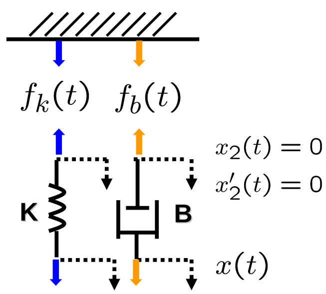
Spring: $f(t) = K(x_1(t) - x_2(t))$ $\;\Downarrow\;$ $F(s) = K(X_1(s) - X_2(s))$

Damper: $f(t) = B(x_1'(t) - x_2'(t))$ $\;\Downarrow\;$ $\begin{pmatrix} x_1(0)=0 \\ x_2(0)=0 \end{pmatrix}$ $\;\Rightarrow\;$ $F(s) = Bs(X_1(s) - X_2(s))$

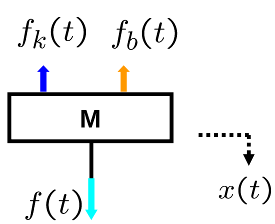
Mass-spring-damper system



Free body diagram

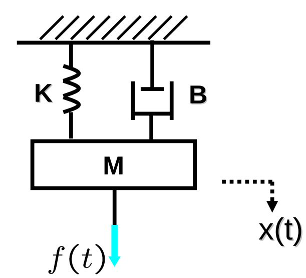


Direction of actual force will be automatically determined by the relative values!



- Newton's law: F=ma

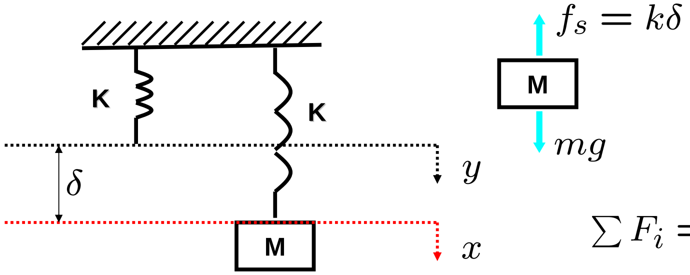
Mass-spring-damper system



- Equation of motion
- By Laplace transform (with zero initial conditions),

(2nd order system)

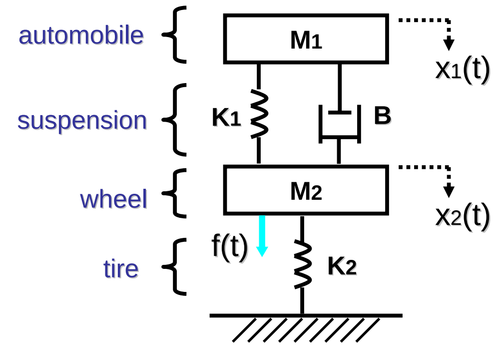
Gravity?



$$\sum F_i = ma$$

- At rest,
- y coordinate:
- x coordinate:

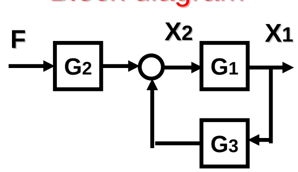
Automobile suspension system



Automobile suspension system

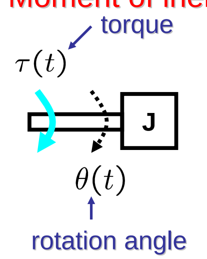
Laplace transform with zero ICs

Block diagram



Rotational mechanical elements (constitutive equations)

Moment of inertia

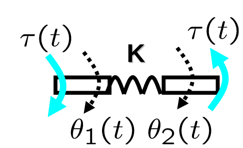


$$\tau(t) = J\theta''(t)$$

$$\begin{pmatrix} \theta(0) = 0 \\ \dot{\theta}(0) = 0 \end{pmatrix}$$

$$T(s) = Js^2\Theta(s)$$

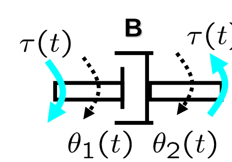
Rotational spring



$$\tau(t) = K(\theta_1(t) - \theta_2(t))$$

$$T(s) = K(\Theta_1(s) - \Theta_2(s))$$

Friction

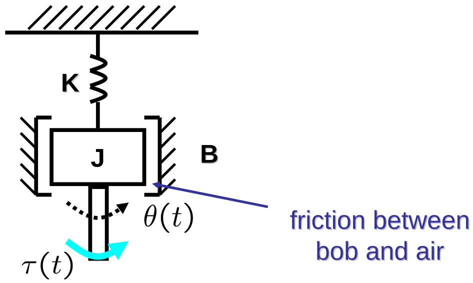


$$\tau(t) = B(\theta_1'(t) - \theta_2'(t))$$

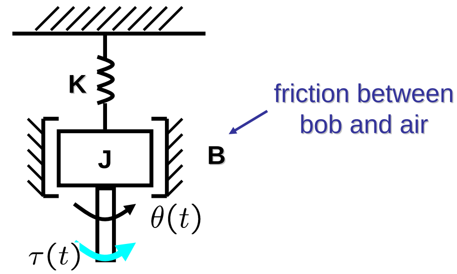
$$\begin{pmatrix} \Theta_1(0) = 0 \\ \Theta_2(0) = 0 \end{pmatrix}$$

$$T(s) = Bs(\Theta_1(s) - \Theta_2(s))$$

Torsional pendulum system Ex.2.12



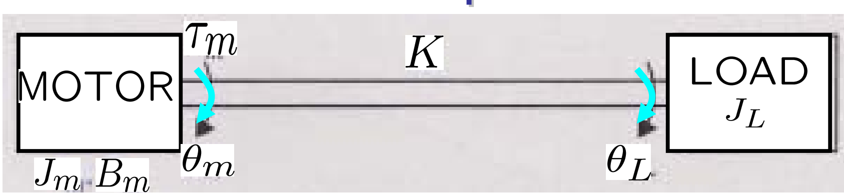
Torsional pendulum system



- Equation of Motion
- By Laplace transform (with zero ICs),

(2nd order system)

Example



- By Newton's law
- By Laplace transform (with zero ICs),

Example (cont'd)

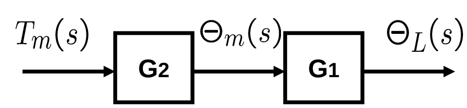
- From second equation:

(2nd order system)

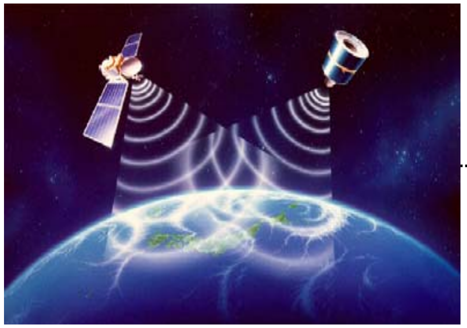
- From first equation:

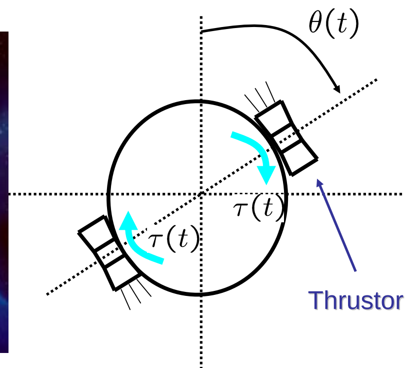
(4th order system)

Block diagram



Rigid satellite Ex. 2.13





- Broadcasting
- Weather forecast
- Communication
- GPS, etc.

$$\tau(t) = J\ddot{\theta}(t)$$

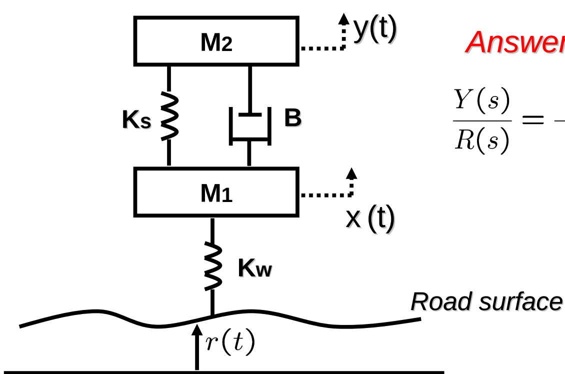
$$\Rightarrow G(s) = \frac{\Theta(s)}{T(s)} = \frac{1}{Js^2}$$ *Double integrator*

Summary & Exercises

- Modeling of mechanical systems
  - Translational
  - Rotational
- Next, block diagrams.
- Exercises
  - Derive equations for the automobile suspension problem.

Exercises (Franklin et al.)

- *Quarter car model*: Obtain a transfer function from R(s) to Y(s).



Answer

$$\frac{Y(s)}{R(s)} = \frac{\frac{k_w b}{m_1 m_2}\left(s + \frac{k_s}{b}\right)}{s^4 + \left(\frac{b}{m_1} + \frac{b}{m_2}\right)s^3 + \left(\frac{k_s}{m_1} + \frac{k_s}{m_2} + \frac{k_w}{m_1}\right)s^2 + \left(\frac{k_w b}{m_1 m_2}\right)s + \frac{k_w k_s}{m_1 m_2}}$$