ME451: Control Systems

Lecture 14
Time response of 1st-order systems

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Course roadmap

Modeling

- ✓ Laplace transform
- ✓ Transfer function
- ✓ Models for systems
  - ✓ electrical
  - ✓ mechanical
  - ✓ electromechanical
- ✓ Block diagrams
- ✓ Linearization

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Analysis

- ✓ Time response
  - ➤ • Transient
  - ✓ Steady state
- Frequency response
  - • Bode plot
- ✓ Stability
  - ✓ • Routh-Hurwitz
  - • Nyquist

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Design

- Design specs
- Root locus
- Frequency domain
- PID & Lead-lag
- Design examples

(Matlab simulations &) laboratories

Performance measures (review)

- Transient response ← **(Today's lecture)**
  - Peak value
  - Peak time
  - Percent overshoot
  - Delay time
  - Rise time
  - Settling time
- Steady state response
  - Steady state error ← **(Done)**

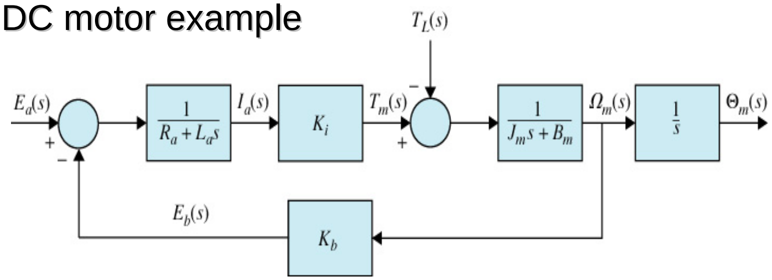
Next, we will connect these measures with s-domain.

First-order system

- A standard form of the first-order system:

$$G(s) = \frac{K}{Ts+1}$$

- DC motor example



DC motor example (cont'd)

- If La<<Ra, we can obtain a 1st-order system

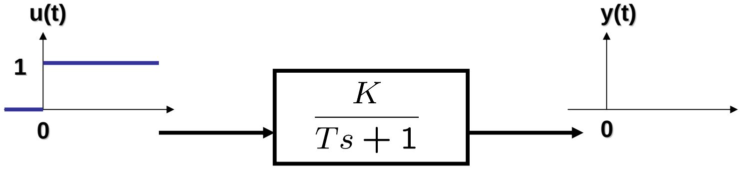
$$\frac{\Omega_m(s)}{E_a(s)} = \frac{K_i}{(L_a s + R_a)(J_m s + B_m) + K_b K_i} \approx \frac{K_i}{R_a(J_m s + B_m) + K_b K_i}$$

$$=: \frac{K}{Ts+1} \quad \left(K := \frac{K_i}{R_a B_m + K_b K_i},\ T := \frac{R_a J_m}{R_a B_m + K_b K_i} \right)$$

- TF from motor input voltage to
  - motor speed is 1st-oder
  - motor position is 2nd-order

Step response for 1st-order system

- Input a unit step function to a first-order system. Then, what is the output?



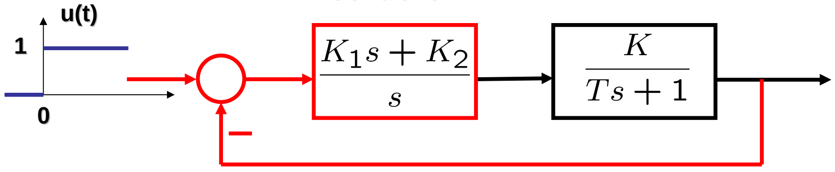
$$\begin{aligned} Y(s) &= G(s)U(s) \\ &= \frac{K/T}{s+1/T} \cdot \frac{1}{s} \\ &= \frac{K}{s} + \frac{-K}{s+1/T} \end{aligned} \quad \xrightarrow{\mathcal{L}^{-1}} \quad \begin{aligned} y(t) &= \mathcal{L}^{-1}\{Y(s)\} \\ &= \underline{K(1-e^{-t/T})} \\ &\quad (t>0) \end{aligned}$$

(Partial fraction expansion)

How to eliminate steady-state error

- Make a feedback system with a controller having an integrator (*copy of Laplace transform of a unit step function*):

Controller



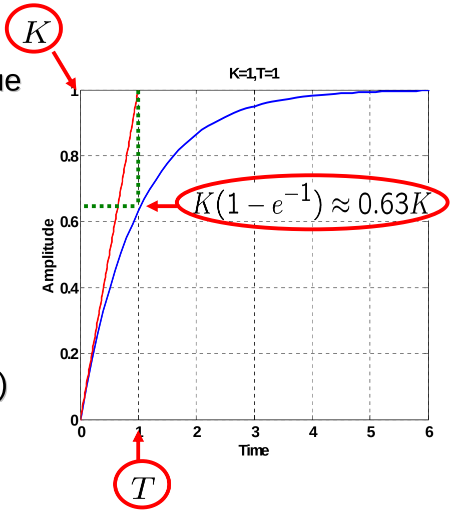
One has to select controller parameters to stabilize the feedback system. Suppose K=T=1, and obtain such parameters!

Meaning of K and T

- K : Gain
  - Final (steady-state) value

$$\lim_{t\to\infty} y(t) = K$$

- T : Time constant
  - Time when response rises 63% of final value
  - Indication of speed of response (convergence)
  - Response is faster as T becomes smaller.



DC gain for a general system

- DC gain : Final value of a unit step response
  - For first-order systems, DC gain is K.
  - For a general stable system G, DC gain is G(0).

$$\lim_{t\to\infty} y(t) \underset{\uparrow}{=} \lim_{s\to 0} sG(s)\frac{1}{s} = G(0)$$

Final value theorem

- Examples

$$G(s) = \frac{3}{2s+5} \qquad G(0) = \frac{3}{5}$$

$$G(s) = \frac{7}{s^2+2s+3} \qquad G(0) = \frac{7}{3}$$

Settling time of 1st-order systems

$$y(t) = K(1 - e^{-t/T})$$

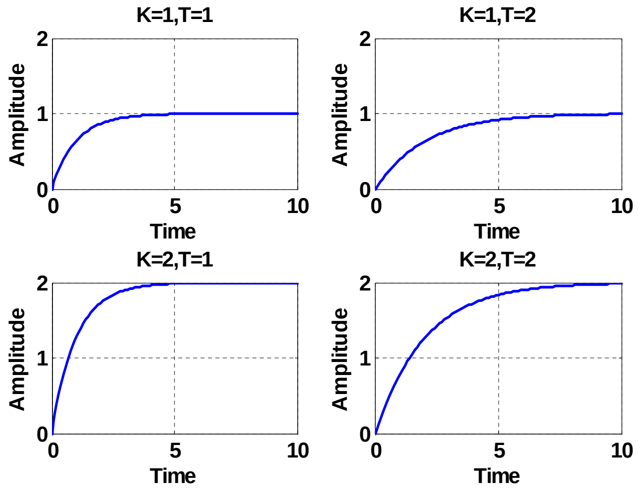
- Relation between time and exponential decay

t	$e^{-t/T}$
0	1
T	0.3679
$2T$	0.1353
$3T$	0.0498
$4T$	0.0183
$5T$	0.0067

5% settling time is about 3T! (← $3T$)

2% settling time is about 4T! (← $4T$)

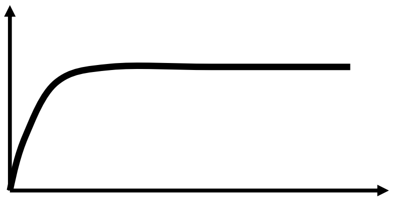
Step response for some K & T



System identification

- Suppose that we have a “black-box” system

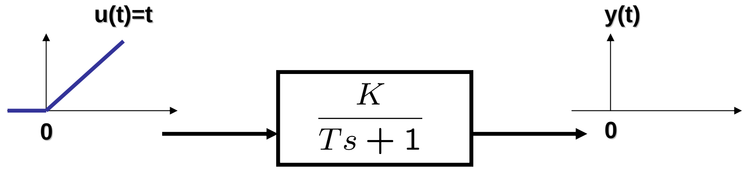
- Obtain step response



- Can you obtain a transfer function? How?

Ramp response for 1st-order system

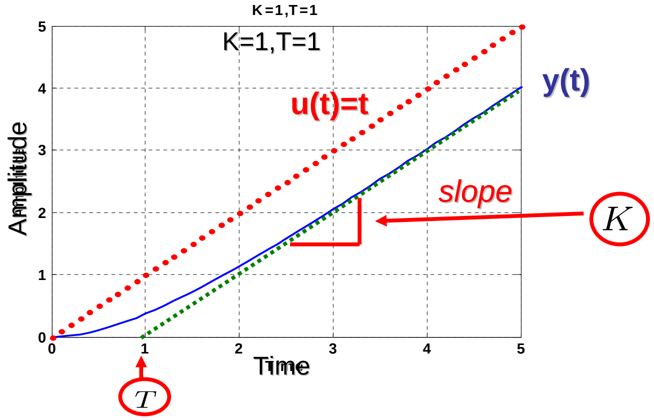
- Input a unit ramp function to a 1st-order system. Then, what is the output?



$$\begin{aligned} Y(s) &= G(s)U(s) \\ &= \frac{K/T}{s+1/T}\cdot\frac{1}{s^2} \\ &= \frac{K}{s^2} + \frac{-KT}{s} + \frac{KT}{s+1/T} \end{aligned} \quad \xrightarrow{\mathcal{L}^{-1}} \quad \begin{aligned} y(t) &= \mathcal{L}^{-1}\{Y(s)\} \\ &= K(t-T+KTe^{-t/T}) \\ & (t>0) \end{aligned}$$

(Partial fraction expansion)

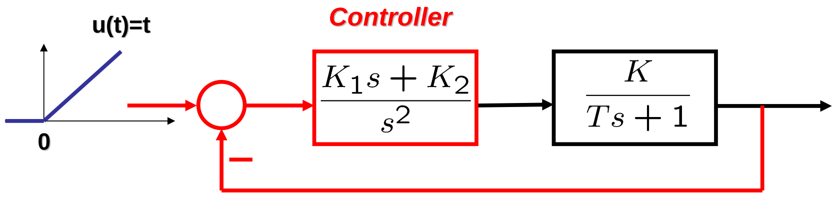
Ramp response for 1st-order system



- Steady state response $y_{ss}(t) = K(t-T)$
- We may want to modify the system s.t. $y_{ss}(t) = u(t)$

How to eliminate steady-state error

- Make a feedback system with a controller having a double integrator (*copy of Laplace transform of ramp function*):



One has to select controller parameters to stabilize the feedback system.
Suppose K=T=1, and obtain such parameters!

Summary and exercises

- Time response for 1st-order systems
  - Step and ramp responses
  - Time constant and DC gain
  - System identification
- Next, time response for 2nd-order systems
- Exercises
  - Review examples in this lecture.