**Steady-state error: unity feedback**

Suppose that we want output $y(t)$ to track $r(t)$.

- **Error** $e(t) = r(t) - y(t)$
- **Steady-state error**
  \[
  e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s - \frac{1}{1 + G(s)}R(s)
  \]

We assume that the CL system is stable!

Unity feedback!

Next, we will connect these measures with s-domain.

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**Performance measures (review)**

- **Transient response**
  - Peak value
  - Peak time
  - Percent overshoot
  - Delay time
  - Rise time
  - Settling time
- **Steady state response**
  - Steady state error

(From next lecture)

(Today’s lecture)
Error constants

- Step-error (position-error) constant
  \[ K_p := \lim_{s \to 0} G(s) \]

- Ramp-error (velocity-error) constant
  \[ K_v := \lim_{s \to 0} sG(s) \]

- Parabolic-error (acceleration-error) constant
  \[ K_a := \lim_{s \to 0} s^2 G(s) \]

- \( K_p, K_v, K_a \): ability to reduce steady-state error

Steady-state error for step \( r(t) \)

\[ r(t) = R u_s(t) \Rightarrow e_{ss} = \frac{R}{1 + K_p} \]

Steady-state error for ramp \( r(t) \)

\[ r(t) = R t u_s(t) \Rightarrow e_{ss} = \frac{R}{K_v} \]

Steady-state error for parabolic \( r(t) \)

\[ r(t) = \frac{R t^2}{2} u_s(t) \Rightarrow e_{ss} = \frac{R}{K_a} \]
**System type**

- **System type of** $G$ **is defined as the order** (number) **of poles of** $G(s)$ **at** $s=0$.
- **Examples**

  \[
  G(s) = \frac{K(1 + 0.5s)}{s(1 + s)(1 + 2s)(1 + s + s^2)} \quad \Rightarrow \quad \text{type 1}
  \]

  \[
  G(s) = \frac{K(1 + s)}{s^2}e^{-Ts} \quad \Rightarrow \quad \text{type 2}
  \]

  \[
  G(s) = \frac{K(1 + 2s)}{s^3} \quad \Rightarrow \quad \text{type 3}
  \]

**Zero steady-state error**

- If error constant is infinite, we can achieve zero steady-state error. (Accurate tracking)
  - For step $r(t)$
    \[
    K_p = \lim_{s \to 0} G(s) = \infty \Leftrightarrow G(s) \text{ is of at least type 1}
    \]
  - For ramp $r(t)$
    \[
    K_v = \lim_{s \to 0} sG(s) = \infty \Leftrightarrow G(s) \text{ is of at least type 2}
    \]
  - For parabolic $r(t)$
    \[
    K_a = \lim_{s \to 0} s^2G(s) = \infty \Leftrightarrow G(s) \text{ is of at least type 3}
    \]

**Example 1**

- $G(s)$ of type 2
  \[
  G(s) = \frac{K}{s^2(s + 12)}
  \]
  - Characteristic equation
    \[
    1 + G(s) = 0 \Leftrightarrow s^2(s + 12) + K = 0 \Leftrightarrow s^3 + 12s^2 + K = 0
    \]
  - CL system is NOT stable for any $K$.
  - $e(t)$ goes to infinity. (Don’t use today’s results if CL system is not stable!!!)

**Example 2**

- $G(s)$ of type 1
  \[
  G(s) = \frac{K(s + 3.15)}{s(s + 1.5)(s + 0.5)}
  \]
  - By Routh-Hurwitz criterion, CL is stable iff
    \[
    0 < K < 1.304
    \]
  - Step $r(t)$
    \[
    e_{ss} = \frac{R}{1 + K_p} = 0
    \]
  - Ramp $r(t)$
    \[
    e_{ss} = \frac{R}{K_v} \quad K_v := \lim_{s \to 0} sG(s) = \frac{3.15K}{0.75} = 4.2K
    \]
  - Parabolic $r(t)$
    \[
    e_{ss} = \frac{R}{K_a} = \infty \quad K_a := \lim_{s \to 0} s^2G(s) = 0
    \]
Example 3

- G(s) of type 2
  \[ G(s) = \frac{5(s + 1)}{s^2(s + 12)(s + 5)} \]
- By Routh-Hurwitz criterion, we can show that CL system is stable.
- Step \( r(t) \)
  \[ e_{ss} = \frac{R}{1 + K_p} = 0 \]
- Ramp \( r(t) \)
  \[ e_{ss} = \frac{R}{K_v} = 0 \]
- Parabolic \( r(t) \)
  \[ e_{ss} = \frac{R}{K_a} = 12R \quad K_a := \lim_{s \to 0} s^2 G(s) = \frac{1}{12} \]

A control example

- Closed-loop stable?
- Compute error constants
  \[ K_p = \quad K_v = \quad K_a = \]
- Compute steady state errors
  \[ e_{ss} = \quad e_{ss} = \quad e_{ss} = \]

Summary and Exercises

- Steady-state error
  - For unity feedback (STABLE!) systems, the system type of the forward-path system determines if the steady-state error is zero.
  - The key tool is the final value theorem!
- Next, time response of 1st-order systems
- Exercises
  - Go over the examples in this lecture.