ME451: Control Systems

Lecture 11
Routh-Hurwitz criterion: Control examples

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Course roadmap

Modeling

- ✔ Laplace transform
- ✔ Transfer function
- ✔ Models for systems
  - ✔ electrical
  - ✔ mechanical
  - ✔ electromechanical
- ✔ Linearization

Analysis

Time response
- Transient
- Steady state

Frequency response
- Bode plot

✔ Stability
- Routh-Hurwitz
- Nyquist

Design

Design specs

Root locus

Frequency domain

PID & Lead-lag

Design examples

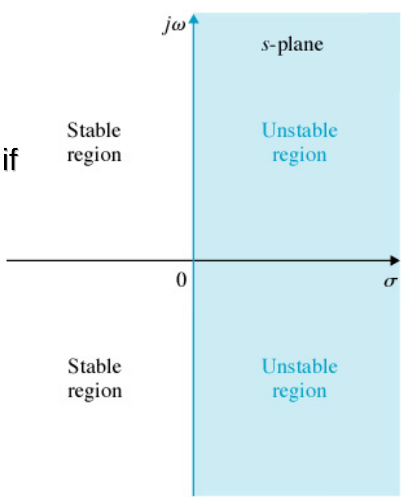
(Matlab simulations &) laboratories

Stability summary (review)

Let s_i be poles of rational G. Then, G is …

- (BIBO, asymptotically) stable if $Re(s_i)<0$ for all i.
- marginally stable if
  - $Re(s_i)<=0$ for all i, and
  - simple root for $Re(s_i)=0$
- unstable if

  it is neither stable nor marginally stable.



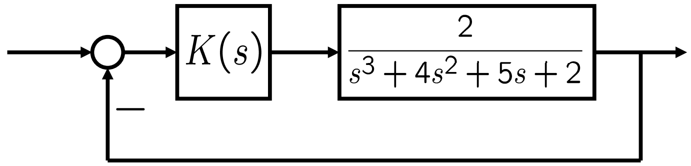
Routh-Hurwitz criterion (review)

$$Q(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0$$

s^n	a_n	a_{n-2}	a_{n-4}	a_{n-6}	$\cdots$
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	a_{n-7}	$\cdots$
s^{n-2}	b_1	b_2	b_3	b_4	$\cdots$
s^{n-3}	c_1	c_2	c_3	c_4	$\cdots$
$\vdots$	$\vdots$	$\vdots$			
s^2	k_1	k_2			
s^1	l_1				
s^0	m_1				

The number of roots in the right half-plane is equal to the number of sign changes in the first column of Routh array.

Example 1



- Design K(s) that stabilizes the closed-loop system for the following cases.
  - K(s) = K (constant)
  - K(s) = K_P+K_I/s (PI (Proportional-Integral) controller)

Example 1: K(s)=K

- Characteristic equation

$$1 + K\frac{2}{s^3+4s^2+5s+2} = 0$$

$$\Rightarrow \quad s^3+4s^2+5s+2+2K = 0$$

- Routh array

s^3
s^2
s^1
s^0

Example 1: K(s)=K_P+K_I/s

- Characteristic equation

$$1 + \left(K_P + \frac{K_I}{s}\right)\frac{2}{s^3+4s^2+5s+2} = 0$$

$$\Rightarrow \quad s^4+4s^3+5s^2+(2+2K_P)s+2K_I = 0$$

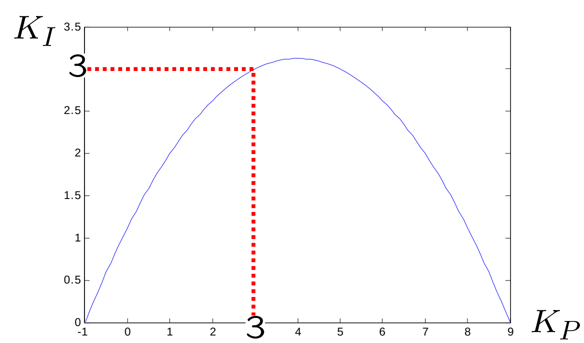
- Routh array

s^4
s^3
s^2
s^1
s^0

Example 1: Range of (K_P,K_I)

- From Routh array,

$$K_P < 9$$
$$K_I > 0$$
$$(1+K_P)(9-K_P) - 8K_I > 0$$



Example 1: K(s)=KP+KI/s (cont'd)

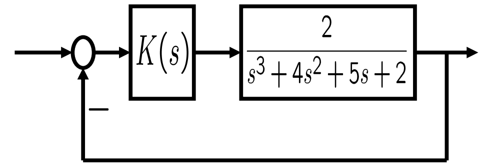
- Select KP=3 (<9)
- Routh array (cont'd)

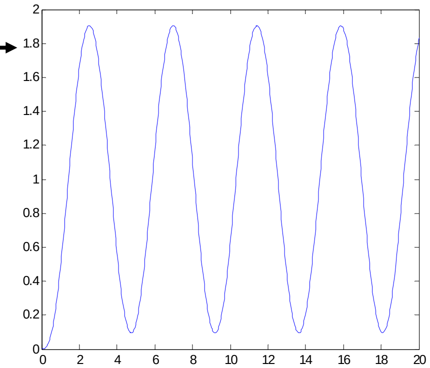
s^4
s^3
s^2
s^1
s^0

- If we select different KP, the range of KI changes.

Example 1: What happens if KP=KI=3

- Auxiliary equation $3s^2 + 6 = 0 \Leftrightarrow s = \pm\sqrt{2}j$





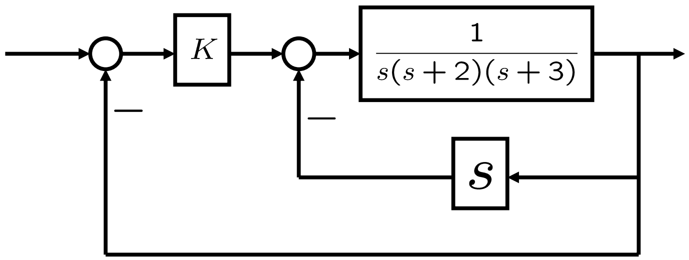
Unit step response

- Oscillation frequency

  $\sqrt{2}$(rad/sec)

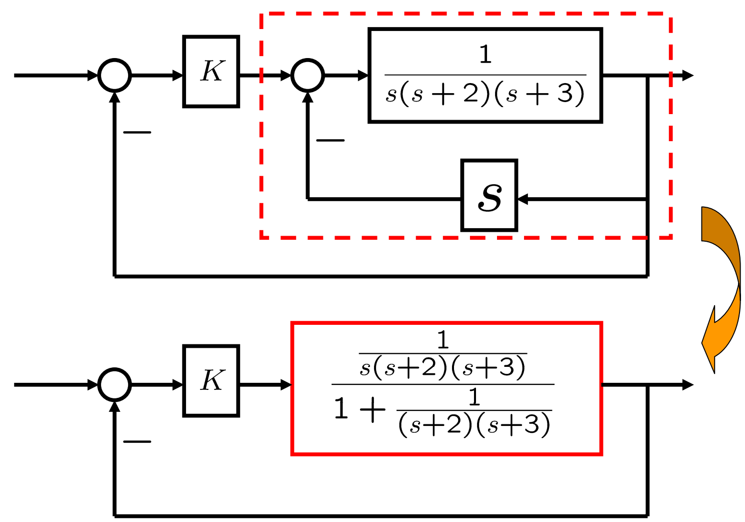
- Period $\frac{2\pi}{\sqrt{2}} \approx 4.4$(sec)

Example 2



- Determine the range of K and a that stabilize the closed-loop system.

Example 2 (cont'd)



Example 2 (cont'd)

- Characteristic equation

$$1+K\frac{\frac{1}{s(s+2)(s+3)}}{1+\frac{1}{(s+2)(s+3)}}=0$$

$$\Rightarrow \quad 1+\frac{K}{s}\cdot\frac{1}{(s+2)(s+3)+1}=0$$

$$\Rightarrow \quad s(s+2)(s+3)+s+K=0$$

$$\Rightarrow \quad s^3+5s^2+7s+K=0$$

Example 2 (cont'd)

- Routh array $\quad s^3+5s^2+7s+K=0$

s^3	
s^2	
s^1	
s^0	

- If K=35, oscillation frequency is obtained by the auxiliary equation

$$5s^2+35=0 \Leftrightarrow s=\pm\sqrt{7}j$$

Summary and Exercises

- Control examples for Routh-Hurwitz criterion
  - P controller gain range for stability
  - PI controller gain range for stability
  - Oscillation frequency
  - Characteristic equation
- Next
  - Time domain specifications
- Exercises
  - Read Chapter 6 again.
  - Redo Examples 1 and 2
  - Do Problem 6.6-(a) and 6.7-(b)-Find the range of K for which the system is stable.

More example 1

$$Q(s)=s^3+s^2+s+1 \ \ (=(s+1)(s^2+1))$$

Routh array

s^3	
s^2	
s^1	
s^0	

No sign changes in the first column $\Rightarrow$ No root in OPEN(!) RHP

More example 2

$$Q(s) = s^5 + s^4 + 2s^3 + 2s^2 + s + 1 \ (= (s+1)(s^2+1)^2)$$

Routh array

s^5	
s^4	
s^3	
s^2	
s^1	
s^0	

More example 3

$$Q(s) = s^4 - 1 \ (= (s+1)(s-1)(s^2+1))$$

Routh array

s^4	
s^3	
s^2	
s^1	
s^0	

One sign changes in the first column → One root in OPEN(!) RHP