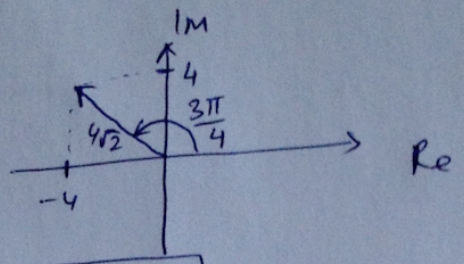
ME-451
Math Quiz Solutions

1. (i) $(1-j)-(5-5j)$

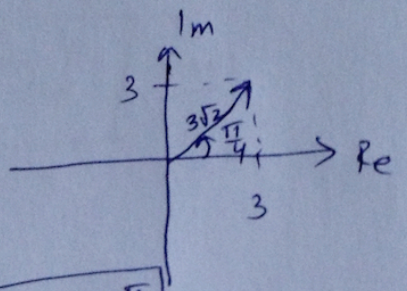
$= \boxed{-4+4j}$ $=$



$= \boxed{4\sqrt{2}\, e^{i\frac{3\pi}{4}}}$

2. (ii) $(1+2j)+(2+1j)$

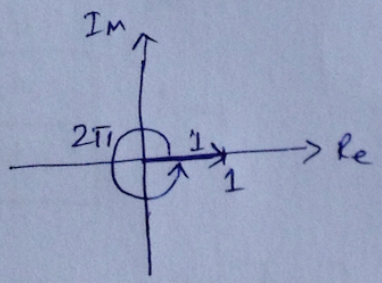
$= \boxed{3+3j}$ $=$



$= \boxed{3\sqrt{2}\, e^{i\frac{\pi}{4}}}$

3. (iii) $e^{\pi j}\, e^{\pi j}$

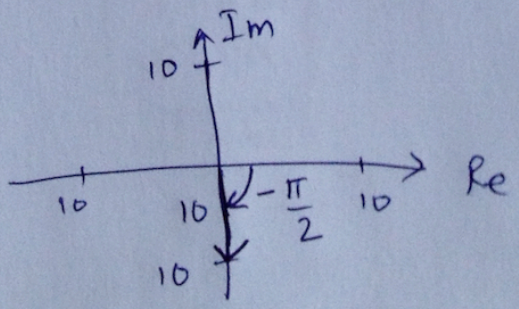
$= \boxed{1e^{2\pi j}}$ $\longrightarrow$



$= \boxed{1+0j}$

4. (iv) $\dfrac{10\, e^{\frac{\pi}{2}j}}{e^{\pi j}}$

$= \boxed{10\, e^{-\frac{\pi}{2}j}}$ $\longrightarrow$



$= \boxed{0-1j}$

2. (i) $\log_{10} X^{15} Y^{-6}$

$$= \boxed{15 \log_{10} X - 6 \log_{10} Y} \quad \text{Ans}$$

(ii) $\log_{10} \frac{Y X^{25}}{Y^{50}}$

$$= \log_{10} Y^{-49} X^{25}$$

$$= \boxed{25 \log_{10} X - 49 \log_{10} Y} \quad \text{Ans}$$

3. $\frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$$\frac{1}{x} = \frac{a+b}{ab}$$

Or $\boxed{x = \frac{ab}{a+b}}$ Ans

4. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{\text{adj}[\ ]}{\text{Det}[\ ]}$

$$= \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{(ad - bc)}$$

$$= \boxed{\left(\frac{1}{ad-bc}\right) \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}} \quad \text{Ans}$$

5. $\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = 5u_5(t)$, $y(0) = 1$, $\dot{y}(0) = 2$.

we know,

$$\mathcal{L}(\ddot{y}(t)) = s^2 y(s) - sy(0) - \dot{y}(0)$$

$$\mathcal{L}(\dot{y}(t)) = sy(s) - y(0).$$

$\therefore$ Taking laplace transform of equation we get,

$$s^2 y(s) - sy(0) - \dot{y}(0) + 3[sy(s) - y(0)] + 2y(s) = \frac{5}{s}$$

or, $(s^2 + 3s + 2)y(s) - s - 5 = \frac{5}{s}$

or, $(s^2 + 3s + 2)y(s) = \frac{5}{s} + s + 5$

or, $(s^2 + 3s + 2)y(s) = \frac{5 + s^2 + 5s}{s}$

or, $y(s) = \frac{5 + s^2 + 5s}{s(s^2 + 3s + 2)}$

$$= \frac{5(s+1) + s^2}{s(s+2)(s+1)}$$

$$= \frac{5}{s(s+2)} + \frac{s}{(s+2)(s+1)}$$

$$= \left[\frac{A}{s+2} + \frac{B}{s}\right] + \left[\frac{C}{s+2} + \frac{D}{s+1}\right]$$

By partial fraction.

$$= \frac{-2.5}{(s+2)} + \frac{2.5}{s} + \frac{2}{s+2} - \frac{1}{s+1}$$

$$= \frac{2.5}{s} - \frac{0.5}{s+2} - \frac{1}{s+1}$$

$\therefore$ $y(t) = \mathcal{L}^{-1}\left[\frac{2.5}{s} - \frac{0.5}{s+2} - \frac{1}{s+1}\right]$

$$= \boxed{2.5u_5(t) - 0.5e^{-2t} - e^{-t}}$$ Ans